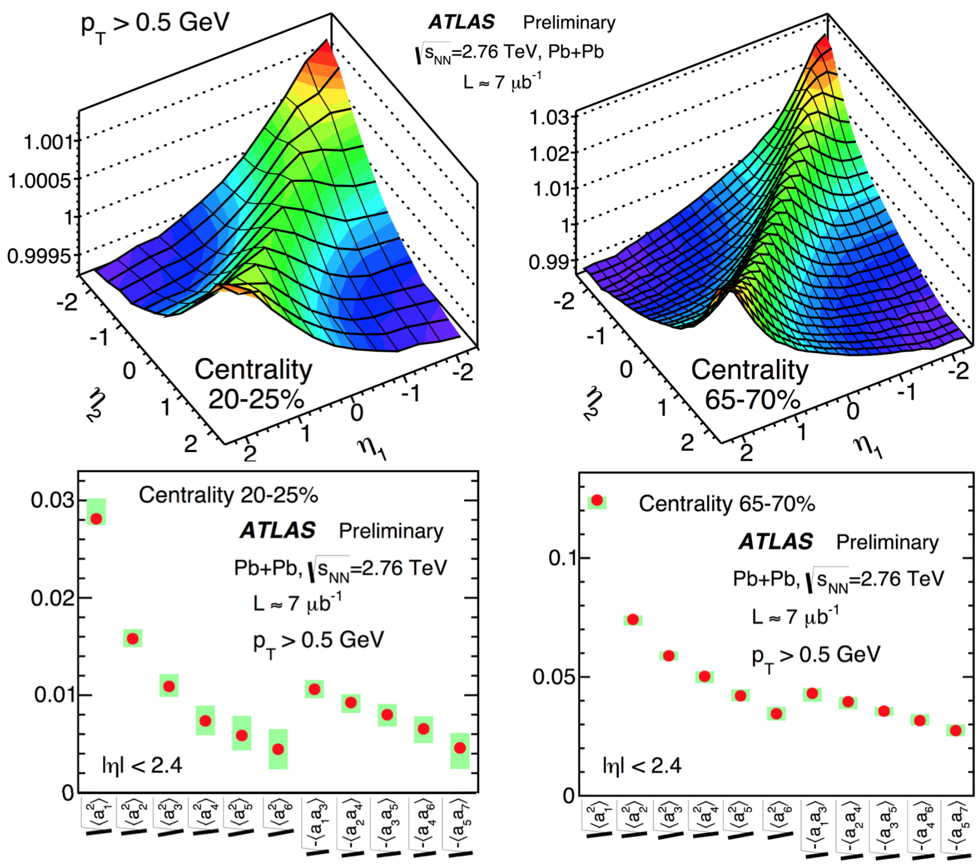
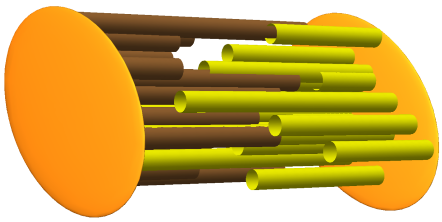
Longitudinal correlations in ultra-relativistic nuclear collisions

Research Project Objectives (scientific problem aimed to be solved by the proposed project, project's research hypotheses)

Very recently, new exciting data for the *longitudinal* (i.e., in the pseudorapidity variable) two-body correlations were released by the ATLAS Collaboration [1, 2] at the Large Hadron Collider (see Fig. 1 for an example). Although the field of this type of correlations has a long history, the groundbreaking experimental studies of the Pb-Pb, p-Pb, and p-p reactions at the highest-available LHC energies, together with new proposed methodologies in data analysis, open completely new opportunities to verify relevant physical hypothesis. It is well known that the long-range rapidity correlations carry information on the earliest stages of the collision, thus provide us with access to the QCD mechanisms of the early entropy (or energy) production. Whereas in elementary collisions commonly accepted models of particle production exist, mainly based on the string-breaking process [3–5], for the A-A or p-A collisions, where initial densities are very high, the situation is open. Moreover, new experimental phenomena, such as, e.g., the event-plane angle decorrelation [6] or the reaction-independent scaling behavior reported in Ref. [2], pose real theoretical challenges, requiring deeper understanding, and, perhaps, even a revision of the presently assumed paradigms used in model building in the early phase.



Rysunek 1. Sample preliminary correlation data from the ATLAS Collaboration [1]. The top panels show the correlation function, and the bottom panels give the coefficients of its decomposition in the two-dimensional basis of the Legendre polynomials.



Rysunek 2. Cartoon of the fluctuating source model. The strings (tubes) correspond to the longitudinally-extended sources attached to nucleus A or B (indicated with ovals) and having the other end-point randomly distributed over the available rapidity range.

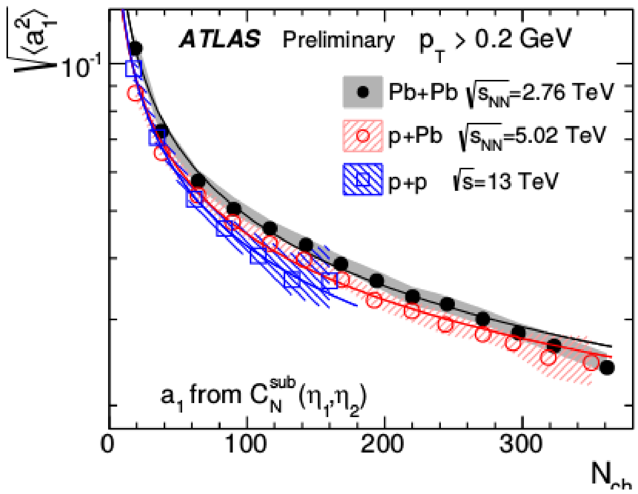
In Ref. [7, 8] Bożek and the PI of the present proposal have come up with a very simple schematic model of the early entropy production, which is motivated by the string-breaking mechanism. An important element of this model is the assumption that the end-points of strings take a random location from string to string (see Fig. 2 for the basic concept), thus contributing to fluctuations. The model has yielded promising results, with fair qualitative agreement with the new experimental results, which proves feasibility of the approach. In the current proposal we plan to pursue this successful line of research by carrying out novel theoretical analyses of the initial-state correlations for hitherto unexplored cases. We also plan to apply more elaborate methods which will allow for better analysis of the physical mechanisms, not sensitive to spurious effects not related to the initial-state dynamics.

In general, correlations are seen in event-by-event fluctuations of observable quantities, thus the question of the nature of correlations is related to the question about fluctuations: *In relativistic heavy-ion collisions, where do fluctuations come from?* It can be shown, that in the independent-source model explored here, there are two main components: the event-by-event fluctuations of the number of sources in the two colliding nuclei, and the genuine dynamical correlations in the particle production mechanism. Here emerges the first major objective of the project, namely, the separation of the dynamical correlations from the trivial fluctuations in the number of sources, and their realistic modeling in state-of-the-art approaches.

The second main objective is very basic, namely, we wish to *determine the relevant degrees of freedom* in the early-state dynamics. There are indications (already put forward when interpreting the RHIC data [9, 10]) that the wounded quark model [11, 12] works better than the popular wounded nucleon model [13, 14], in particular in describing the multiplicity distributions. Essentially, the agreement with the data may be achieved without the introduction of the binary-collision component [15, 16], which introduces nonlinearity between the number of participating nucleons and the multiplicity of the produced hadrons. Basically, one assumes here that the number of the produced particles N is proportional to the number of the wounded quarks N_q,

$$N \sim N_q. \qquad (1)$$

We plan to explore this basic question of the nature of relevant degrees of freedom in the early phase with the help of the recently released LHC data [1, 2], applying a novel analysis based on the fluctuating-source model. A particular attention will be brought to the intriguing question of scaling [2] of the correlation measures on the inverse number of measured charged hadrons (Fig. 3).



Rysunek 3. Approximate universal scaling of the a_{11} coefficient with the inverse number of detected charged particles N_{ch} [2].

In fact, we will argue that this scaling raises serious questions about the degrees of freedom in the initial state and suggests the relevance of subnucleonic constituents in the early dynamics.

Finally, the third major question concerns the modeling of the *production mechanism from sources*. Our hope is that a joint consideration of various correlation observables, with the help of novel methods, will allow to put relevant constraints on the elementary (i.e., coming from a single source) production of particles in the early phase. The new data will certainly pose challenges to theoretical approaches, including the independent-source model.

The significance of the project, as well as our detailed tasks and methods are described in the sections below.

Significance of the project (state of the art, justification for tackling specific scientific problems by the proposed project, pioneering nature of the project, the impact of the project results on the development of the research field and scientific discipline, economic and societal impact)

The understanding of the dynamics of the early phase has long been though as one of the most important problems of the field of ultra-relativistic heavy-ion collisions. The reason, apart for the basic dynamical aspects discussed in the preceding section, is that the effect of this stage of the reaction can be thought of as the "preparation" of the initial state for the subsequent collective evolution (hydrodynamics, transport). Therefore, it directly affects such fundamental questions as the extraction of the transport coefficients (e.g., viscosities, conductivities, etc.) of the quark-gluon plasma, and concerns the issues related to the spatial anisotropy of the system, also in the longitudinal direction. More generally, it is also important for the still not fully understood isotropization and the early-thermalization puzzles.

There are competing theoretical approaches on the market of modeling the initial state, with calculations based on relativistic hydrodynamics or transport on the one side, and Color Glass Condensate (CGC) [17–19] on the other side. The initial conditions for hydrodynamics typically come from the Glauber framework [15, 20, 21] or the Kharzeev-Levin-Nardi (KLN) model [22], motivated with the

QCD saturation. However, the modeling of the initial condition for the one-body observables (multiplicities, the rapidity- and transverse-momentum spectra) were sensitive to event-averaged initial conditions only. Important exception (but this, in essence, is a correlation observable), was the harmonic flow, in particular, the famous triangular flow. One should also mention here the transverse-momentum fluctuations, studied previously at various energies, but these studies were typically constrained to a single pseudorapidity bin.

A key feature behind our project is that the imprints of the initial state can be clearly seen in fluctuations of the final hadron distributions, and this fact is in the core of our methodology. Moreover, our project will provide the initial conditions not only in three dimensions, but also fluctuating from collision to collision in such a way that various correlation data are (hopefully) jointly reproduced. This will have a large practical impact on the field, as all phenomenological simulations use models of the initial state. We are planning (see the tasks listed in the section below) to analyze a vast number of different correlation observables, and if our scheme works, it will also be helpful for the experimentalists in providing the model expectations for the upcoming results (we plan to make predictions for hitherto not studied quantities).

Some of our results will have analytic or semi-analytic character, which is always very useful in theoretical models, as physical features may be assessed exactly. Elaborations will be carried out with state-of-the art numerical simulations of the dynamics of the early phase and the hadronization process. This will allow for estimates of unwanted effects, such as various short-distance correlations which hide the relevant dynamics of the early phase.

While carrying out the project, we will be responsive to upcoming results, which undoubtedly will become available in the near future.

In general, through realization of the main goal listed in section *Research Project Objectives*, our investigations will contribute to better understanding of fundamental questions concerning the dynamics of hot and dense strongly interacting matter.

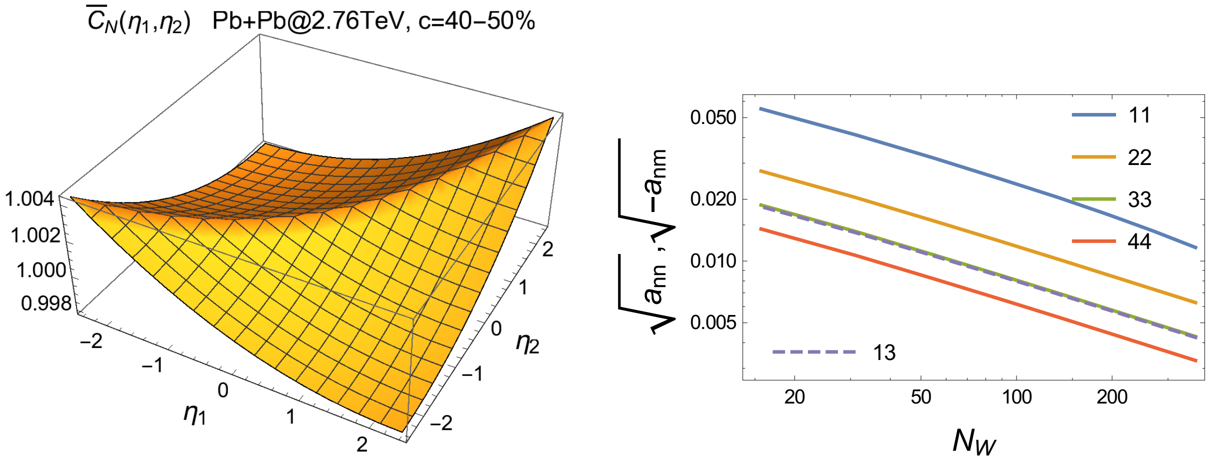
From the social point of view, the project would have a positive impact on the energetic activity in the field of high-energy and nuclear physics in the Institute of Physics of the Jan Kochanowski University in Kielce. The employment of a post-doc and a fellowship for a PhD student would importantly contribute to the development of the local academic community, where the physics or ultra-relativistic heavy ion collisions has long been one of its specialties.

Work plan (outline of the work plan, critical paths, state of preliminary and initial research indicating feasibility of research objectives)

Some preliminary results, proving the feasibility of the proposed project, have been very recently published in Refs. [8], with the PI of the present proposal as a co-author. In particular, we were able to derive simple analytic expressions for the basic correlation function

$$C(\eta_1, \eta_2) = \langle \rho_2(\eta_1, \eta_2) \rangle / [\langle \rho_1(\eta_1) \rangle \langle \rho_1(\eta_2) \rangle], \tag{2}$$

where $\rho_{1,2}$ indicate the one- and two-particle distributions and $\langle . \rangle$ denotes event-wise averaging. Related quantities are the coefficients of the expansion of the correlation function (2) in a basis of



Rysunek 4. Preliminary predictions of the schematic model of Ref. [8] in the wounded-nucleon model. Left: two-particle correlation function for the wounded nucleon model with fluctuating longitudinally-extended sources. Right: dependence of the coefficients of the expansion of the correlation function in Legendre polynomials on the number of wounded nucleons.

orthonormal polynomials on the experimentally covered range of pseudorapidities, $[-Y, Y]$, defined as [23]

$$a_{nm} = \int_{-Y}^{Y} \frac{d\eta_1}{Y} \int_{-Y}^{Y} \frac{d\eta_2}{Y} C(\eta_1, \eta_2) T_n\left(\frac{\eta_1}{Y}\right) T_m\left(\frac{\eta_1}{Y}\right), \tag{3}$$

where T_n are proportional to the Legendre polynomials (for details of the definitions, see Ref. [8]). The a_{nm} coefficients are more convenient in quantitative comparisons and play the role analogous to the v_n coefficients in studies of the harmonic flow.

Our starting point is a schematic model which is well motivated for the sufficiently narrow pseudorapidity coverage, as in the LHC experiments. Figure 4 shows examples of typical calculations for the correlation function and for its decomposition coefficients in the orthonormal basis. The comparison of the results of the schematic model with the experiment [1, 2] is very encouraging. This opens a possibility of further wide-ranging correlation analyses in the proposed model and its extensions, which, as mentioned in the previous section, will allow for deeper verification of the model assumptions, estimates of associated effects, as well as may serve as baseline for future experimental analyses.

The work plan of the project consists of the following tasks:

1. **Analysis of multiplicity correlations with higher-order cumulants**

   The analysis of the early-stage correlations based on the two-particle functions of Ref. [1, 2] is sensitive to various effects which obscure the picture of the initial state, such as charge conservation or resonance decays after hadronization, which generate a sharp peak in the relative pseudorapidity of the two particles. Such effects may be eliminated by looking at cumulants of n-particle correlations [24]. That way, we can focus on the long-range pseudorapidity components. It should be possible to derive analytic expressions for the cumulants in the schematic model of Ref. [8], which will be interesting, as will shed light in the dependence of these quantities on the fluctuation of sources and on genuine correlations in the entropy production in the initial state. The cumulant measures will undoubtedly be used in future experimental data analysis, so our results will acquire direct practical significance.

We will also carry out the Principal Component Analysis (PCA) [25] of the problem, which in the present case amounts to using the eigenfunctions of the correlation matrix rather than the Legendre polynomials in the expansion of the correlation matrix. The details of this technique are specified in the next section. Importantly, PCA allows for identification of the eigenmodes of collective fluctuations when their eigenvalues are large well separated from remaining modes. For that reason this analysis is important for the physical interpretation of the results.

A very intriguing issue, observed experimentally in the a_{11} coefficient measured by the ATLAS Collaboration [2], concerns the scaling with the inverse number of the observed charged particles, $\sim 1/N_{\rm ch}$, with the proportionality coefficient independent of the reaction, i.e., approximately the same for Pb-Pb, p-Pb, and p-p reactions (cf. Fig. 3). We suspect that this behavior may be related to the fact that $N_{\rm ch}$ is proportional to the number of sources. Such an explanation would also require a larger number of sources in the proton, which opts for the constituent degrees of freedom rather than "larger" wounded nucleons. We plan to carefully investigate this hypothesis, based on the correlation data.

2. **Analysis of the forward-backward transverse-momentum correlations**

In Ref. [26] it was noticed that the size fluctuations of the initial fireball lead to event-by-event transverse momentum fluctuations in the measured hadron distributions. The mechanism follows from the fact that more compressed matter expands faster and provides stronger boost to the created hadrons, and vice versa. The analysis of Ref. [26] was carried out for averages in the central rapidity bin, as done in the measurements at RHIC. In this proposal we plan to make predictions, based on the mentioned "compression-boost" mechanism, for pseudorapidity correlations of the (averaged over hadrons in a given pseudorapidity bin) transverse momentum. Indeed, if the physical picture is as depicted in Fig. 2, then the system should be event-by-event smaller or larger in the transverse plane along the whole pseudorapidity range. This would lead to a positive correlation in the transverse (radial) flow velocity, and, in consequence, to positive and strong transverse momentum correlations. Because of the end-point fluctuations, decorrelation will occur, and the amount of this decorrelation will serve as a probe of the model. We will provide semi-analytic and numerical estimates of the effect, which is accessible to experimental analyses. It will allow for further verification of the length-fluctuation model of the generation of correlations in the initial state. With the help of `GLISSANDO` [27, 28] we will carry out detailed simulations in the wounded-quark and wounded-nucleon models. This will allow us for a comparison of the two approaches, helping to resolve the issue of the relevant degrees of freedom in the early phase of the fireball formation. Our predictions will be made for the systems studied and the LHC, i.e., Pb-Pb, p-Pb, and p-p. The PCA method will also be used within this task.

Our results in this task will be complementary to the analysis of the decorrelation of harmonic flow discussed below.

3. **Analysis of forward-backward correlations of event-plane angles and eccentricity magnitudes in the initial state**

The effect of the event-plane angle decorrelation in distant pseudorapidity bins, known as the *torque effect*, was predicted several years ago [29]. Only recently the phenomenon was confirmed by the CMS Collaboration [6]. In the proposed project we plan to extend the analysis of the event-plane angle decorrelation reported in Refs. [30, 31] to the case of the wounded-quark model. A novel analysis will be made for the correlation of the magnitude of the eccentricities at various pseudorapidities, which manifests itself experimentally as correlation of the harmonic flow coefficients at various pseudorapidities. We expect here a similar effect of decorrelation as for the event-plane angles, due to the fluctuations in the longitudinally-extended source model. As in other tasks, we will explore the wounded quark model, as well as carry out the PCA and higher-cumulant analyses, important to eliminate undesired non-flow effects. As a related issue, we also plan to explore cross-correlations between the eccentricities, size, and multiplicity. Such detailed information will help to verify the specific model assumptions. As in the case of other observables, the calculations will serve as benchmarks for future experimental studies.

4. **Formulation of the fluctuating source model in the fragmentation region**

   The simple preliminary model proposed in Ref. [7, 8] uses assumptions which are good over relatively small span of the pseudorapidity around the central region. Specifically, the random distribution of the end-point locations was set to be uniform. In more accurate studies this need not be the case; in particular, significant departures are expected in the fragmentation region. In this part of the project we are planning to use the quark parton distribution functions (PDFs) to appropriately fluctuate the end-point of the strings and repeat the topics of the tasks specified above. The relevant effects sensitive to the quark distribution in rapidity will occur in the fragmentation region. It will be interesting to explore various correlations between the left- and right-going fragmentation regions, where according to our model they should be small and of specific behavior on the number of sources. These correlation effects may stem only from very long strings, extending through the whole range of rapidity.

   Our theoretical predictions will be possible to be tested in experiments where forward and backward detectors are present (such as forward and backward TPCs). A specific feature of the fragmentation regions is sizable net baryon density. We will also look at correlations of this quantity, proposing appropriate correlation measures and exploring them in our model.

5. **Analysis of longitudinal correlations in asymmetric nuclear collisions**

   In the final task, we will apply the methods and objectives of the four previously specified tasks of the project to collisions of asymmetric systems, such as d-Au, ^{3}He-Au, or ^{12}C-Au. Such systems are interesting, as (similarly to the p-A case to be explored within the previous tasks) they probe the limits of collectivity in ultra-relativistic nuclear collisions. We have experience in studying such systems concerning the flow observables [32, 33]. The analysis of correlation observables will provide further predictions of our approach to the initial-state dynamics. We recall that to explain the torque effect in p-A collisions [7] it was crucial to incorporate the length fluctuations of sources. A systematic study of this effect, and other correlations, for light-heavy ultra-relativistic nuclear collisions will test this hypothesis further and provide predictions for possible experimental analyses.

All tasks of the proposal are sharply focused on a compact, well defined, timely, and very important aspect of ultra-relativistic nuclear collisions. Predictions will be new and relevant for the on-going analysis of the LHC data. Our analyses will cover a very rich spectrum of cases and apply several state-of-the art methods developed specifically for the correlation studies, hence the time span of three years for a three-person team (a post-doc for two years and a PhD student for 30 month) seems appropriate. The PI of the project has a long experience in the field, documented in numerous publications.

Research Methodology (underlying scientific methodology, data reduction and treatment schemes, type and degree of access to the equipment to be used in the proposed research)

Our methodology will be based on both analytic calculations in sufficiently simple models, as well as on numerical simulations to be carried out in more involved variants of the models. At any case, numerical simulations are necessary to obtain the fluctuation measures for the number of sources in Glauber-related approaches. The specific methods to be used in the project can be grouped as follows:

- **Analytic models**

  We plan to further develop our analytic model and methods. In particular, the derivation of the multi-particle cumulants should be straightforward to accomplish in the simple schematic model of Ref. [8]. The formulas will contain higher-cumulant coefficients for the fluctuating numbers of sources in the colliding nuclei A and B, as well as moments of the overlaid distribution of strength of sources, included in the original formulation [8].

  Another analytic extension concerns the distribution of the end-points. In the original formulation of the model [8] it was assumed to be uniform. Departures from uniformity may be incorporated in an analytic fashion, since the distribution function is proportional to the derivative of the average density of sources belonging to a given nucleus with respect to rapidity. As long as we know this average density (e.g., from fits to data on asymmetric collisions, as made in Ref, [34]), we can phenomenologically obtain the distribution of the end-point locations. It will be instructive to trace how such modifications affect the a_{nm} expansion coefficients.

  More involved analytic calculations will be carried out with the help of Mathematica.

- **Numerical simulations**

  Even for the analytic calculations mentioned above, we need to run numerical simulations to obtain the necessary statistical moments for the fluctuations of the numbers of sources. As we are planning to apply the Glauber approach, we will use `GLISSANDO` [27, 28], extended to the case of the wounded quark model. Further ramifications which may be useful in the project is the incorporation of diquarks and gluons as active degrees of freedom. Realistic parton distribution functions can be used to determine the distribution of the end-point of the strings in rapidity for collisions of various parton pairs. This feature will also be built into `GLISSANDO`.

  Software development, code testing and pilot simulations will be carried out on PC's, including the laptop bought as equipment for this project. Full scale numerical simulations, where very high statistics will be necessary, will be performed on the computer cluster of the Institute of Physics of Jan Kochanowski University. It comprises of 12 multicore Intel XEON processors, 94GB RAM, and 30TB RAID disk matrices, which is sufficient for the numerical tasks of the project. The license for Mathematica 10 (or higher) will be provided by Jan Kochanowski University.

- **Higher cumulants, multi-bin measures**

  The observables to be used throughout the project go beyond the standard measures of multiplicity correlations in pseudorapidity, and involve higher-particle correlations as quantified with cumulants. They will be generalized appropriately for the cases involving transverse-momentum correlations (task 2) and flow correlations (task 3). With a special choice of the orthonormal functions (step functions selecting specific bins), the cumulants become equivalent to multi-bin correlation measures, used, e.g., in Refs. [35, 36]. We will address this methodologically interesting issue in the project.

- **Principal Component Analysis**

  In PCA, one finds eigenmodes of the multi-bin correlation matrix, where in our case the bins are labeled by the position in pseudorapidity. Two issue are important here. First, trivial factors of correlations (such as the overall multiplicity fluctuations due to finite width of the centrality selection) are separated, and we are left with "physical" correlations. Second, if some modes have large eigenvalues well separated from other eigenvalues, these modes are dominant (principal components) in the correlations. In fact, if we wish to determine *collectivity*, we should search for such well separated principal components.

  When the eigenvectors are used, the correlation matrix is by definition diagonalized and we have a very concise representation. It is is better than the projection on (arbitrary) orthonormal polynomials, as in that case we have non-diagonal structures. This is particularly important for asymmetric collisions. We will apply PCA in tasks 1, 2, 3, and 5.

- **Elimination of short-distance components**

  Short-distance components in the correlation function have different physical origin than the long-range correlations from in the earliest stages of the collision. Such "unwanted" effects come from the local charge conservation, jets, or resonance decays, which generate a peak in the relative rapidity difference with the width of the order of one unit.

  Although the experimental procedure used in Ref. [2] is rather involved and seems a bit ad hoc, we will repeat it one-to-one in our model for the sake of better comparison with experiment, A better way of removing the unwanted short-distance components, not related to the fireball shape fluctuations, is via the higher-cumulant technique [24], which will be widely used in the tasks of the project.

  A related issue, improving the model, is to include realistic string fragmentation function for massive partons. Originally, massless partons were used, where string-breaking probability was uniform and uncorrelated. Parton mass will effectively reduce the short-distance correlation in the emission process.

- **Symmetry constraints**

  The role of symmetry constraints and their implementation will also be carefully studied. Energy conservation in the string fragmentation mechanism leads to reduction of the short-distance component, as particles with similar rapidities have lower emission probability. This effect is lower in a large collectively expanding fireball, but is essential in elementary processes. Transverse-momentum conservation is very important for the directed flow, thus we need to incorporate it when studying correlations of this observable.

- **Role of hydrodynamics**

  Our final methodological remark concerns the absence of hydrodynamic simulations in our study. The general question here is how one can infer information about final hadron distributions from the configuration of the initial state. The point is that hydrodynamics, or transport evolution, are to a good approximation *linear* in the perturbation of the initial conditions. For that reason, for example, we have (approximate) proportionality of harmonic flow coefficients of rank n to the corresponding eccentricities, $v_n = \kappa_n \epsilon_n$, which holds event-by-event. The coefficient κ_n depends on n and conditions (energy, centrality), but when we take ratios of statistical moments, this dependence cancels out from considerations and useful information may be extracted in a "hydro-independent" way.

  A similar consideration applies to the initial distributions of entropy in space-time rapidity and final distributions of hadrons in pseudorapidity. The role of the intermediate expansion phase (hydrodynamics, transport) is, essentially, to provide an event-by-event mapping between the initial distribution of entropy in spatial rapidity, and the final distribution of hadrons in pseudorapidity. The initial conditions of the fluid are subject to the longitudinal Bjorken scaling flow [37]. Subsequent longitudinal and transverse expansion influences strongly the transverse momentum spectra, but the longitudinal push is mild. Moreover, the longitudinal pressure is reduced when non-equilibrium corrections are taken into account [38, 39].

  As a result of the longitudinal expansion of the fireball, a rescaling of rapidity between the initial state and the fluid at freeze-out is introduced. This leads to moderate change in the correlation function and reduction of the coefficients a_{nm} in the final state as compared to the initial state. The effect may be estimated by examining the longitudinal dynamics in the intermediate stage, but no time consuming event-by-event studies with hydrodynamics are necessary for that purpose.

  Hydrodynamic evolution generates extra fluctuations [40] whose origin essentially follows from the fluctuation-dissipation theorem. However, as shown in recent Ref. [41] on the example of the Gubser flow, these fluctuations are expected to be negligible at sufficiently high multiplicities, at least for the eccentricity fluctuations.

Conclusion

The proposed project addresses an important topic, which has been recently actively pursued in ultra-relativistic heavy-ion analysis at the LHC, namely *longitudinal particle correlations*. We plan to carry out a detailed and well specified analysis of various correlation observables in several systems, which will be useful for the future experimental analysis. On the fundamental side, carrying out the tasks the project will contribute to our knowledge and understanding of the early dynamics of the collision.

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