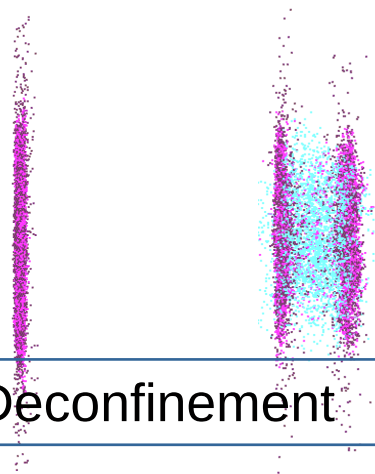
QCD $SU_c(3)\times SU_f(3)\ : L(q_f^c, \bar{q}_f^c, A_\mu^c, m_f)$

Global symmetries

$U_B^1 \times U_S^1 \times U_Q^1$

$m \to \infty$ $\qquad$ $m \to 0$

$Z(3)$ symmetry $\qquad$ Chiral $SU_L(2)\times SU_R(2)$ symmetry



QGP $\qquad$ HG

Quark Potential

Deconfinement

Debye screened $\qquad$ confined

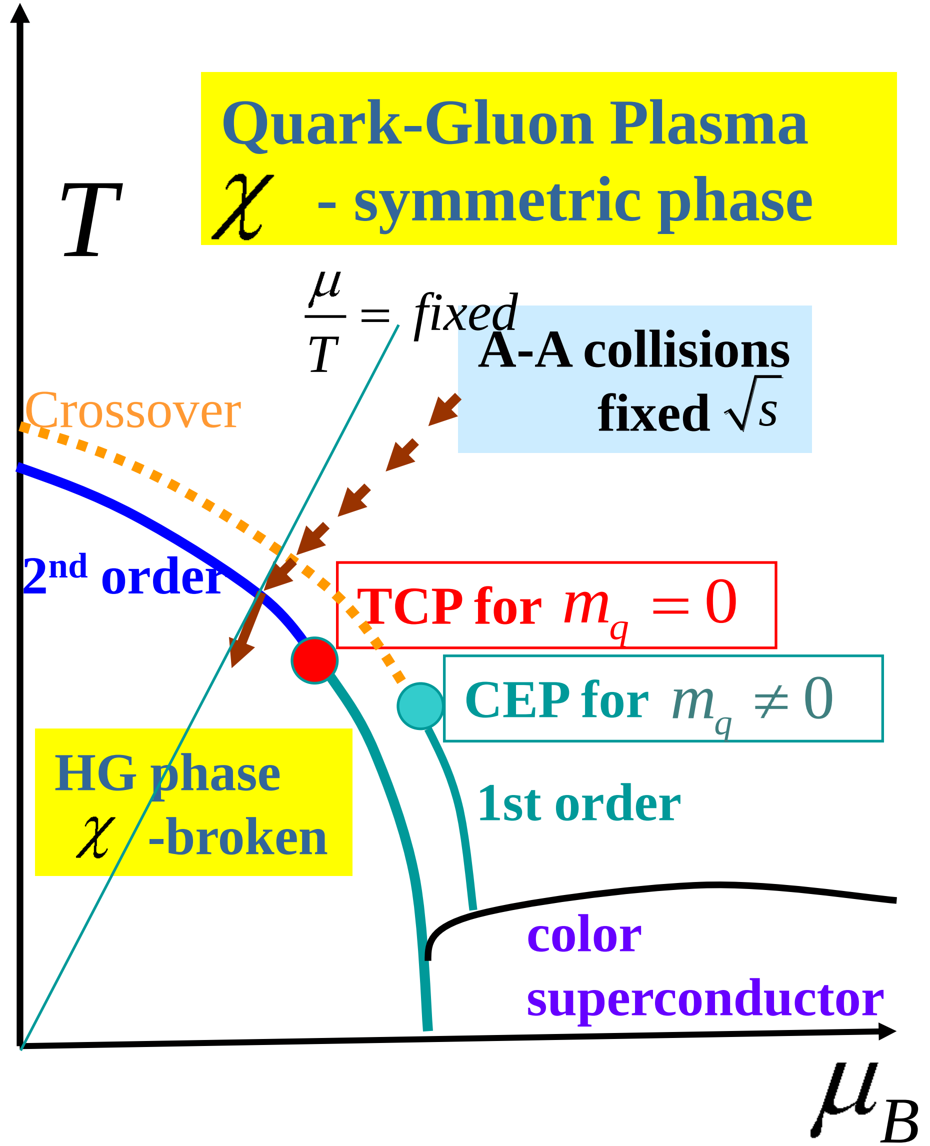
$$V(r) \sim \frac{\exp(-\mu_D r)}{r} \qquad V(r) = \sigma \cdot r$$

Chiral Condensate

Chiral Symmetry restoration

$$<\bar{q}q> \approx 0 \qquad <\bar{q}q> \neq 0$$

- **Effective Chiral Models and QCD phase diagram:**
- **Charge Fluctuations - probe of deconfinement and chiral phase transition :**
  - Focusing of isentropic trajectories near CEP
  - Bulk and shear viscosity near CEP



Extendet PNJL model and its mean field dynamics

$$L_{NJL} = \bar{q}(iD_\mu - m)q + G_S[(\bar{q}q)^2 + (\bar{q}i\vec{\tau}\gamma_5 q)^2] - G_V^{(S)}(\bar{q}\gamma_\mu q)^2$$
$$-G_V^{(V)}(\bar{q}\vec{\tau}\gamma_\mu q)^2 + \mu_q q^+ q + \mu_I q^+ \tau_3 q - U(\Phi[A], \bar{\Phi}[A], T)$$

$$D_\mu = \partial_\mu - iA_\mu \delta_{\mu 0} \qquad \Phi = \frac{1}{N_c} Tr(P \exp[i \int d\tau A_4(\vec{x}, \tau)])$$

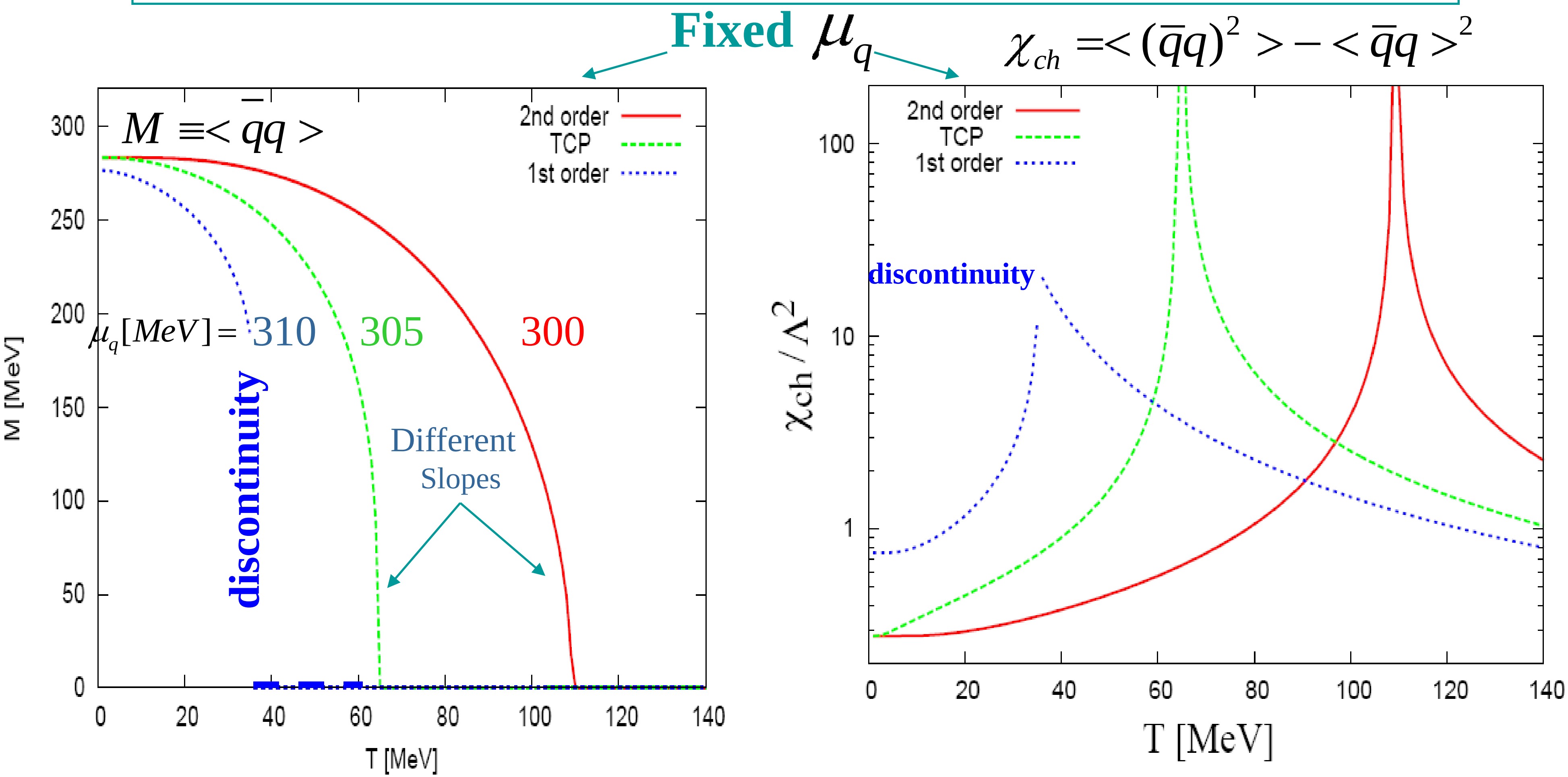
Polyakov loop

- An effective Z(3)-invariant Polyakov loop potential $U(\Phi)$ from A. Dumitru & R. D. Pisarski, fixed so that to reproduce the LGT results obtained in a pure gauge theory at finite T.

- Thermodynamic under mean-field approximation:

$$\Omega(<\bar{q}q>, <\Phi>, \mu, T) \text{ and } \partial\Omega(x_i, T)/\partial x_i = 0$$

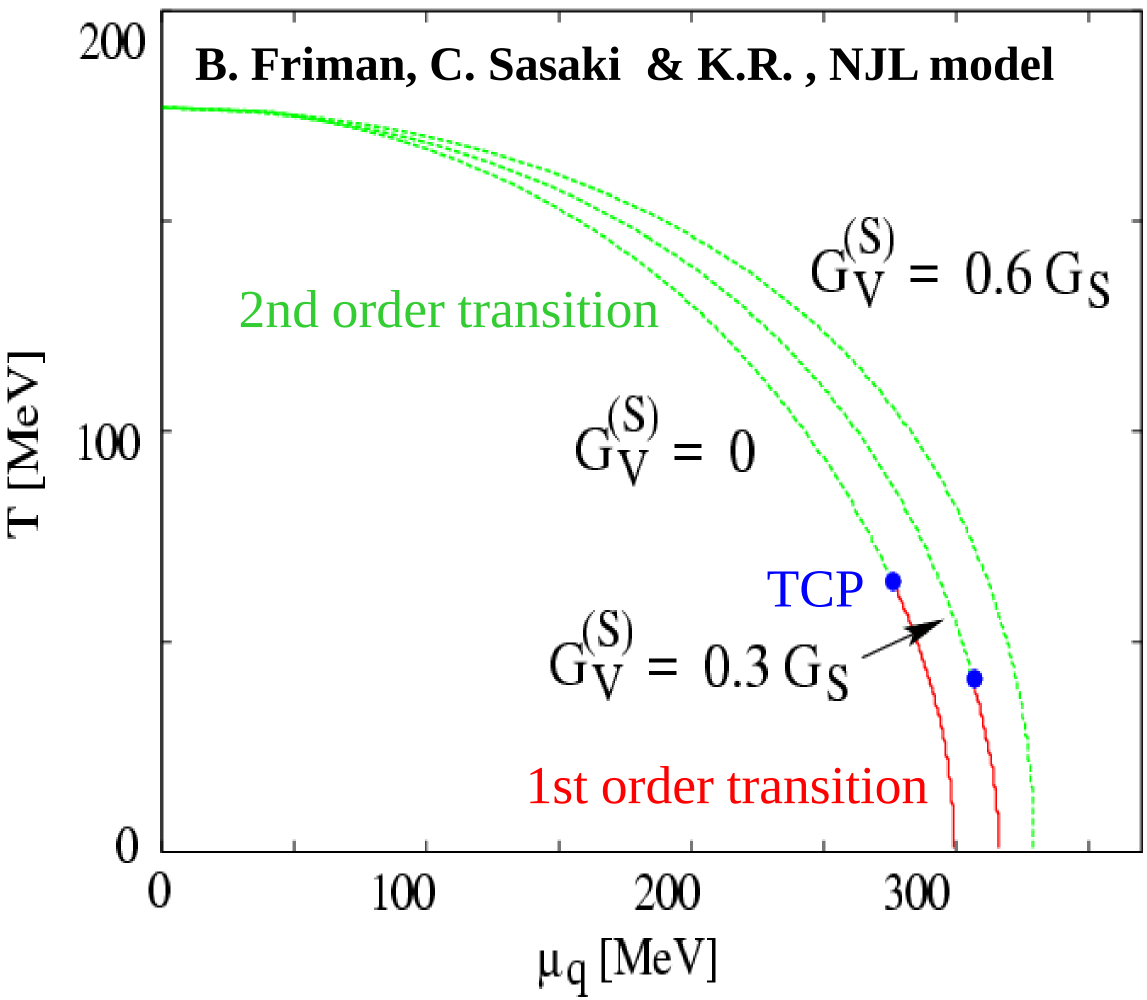
Chiral Symmetry Restoration – Order Parameter



Divergence of the chiral susceptibility at the **2nd** order transition and at the **TCP**

Discontinuity of the chiral susceptibility: at the 1st order transition

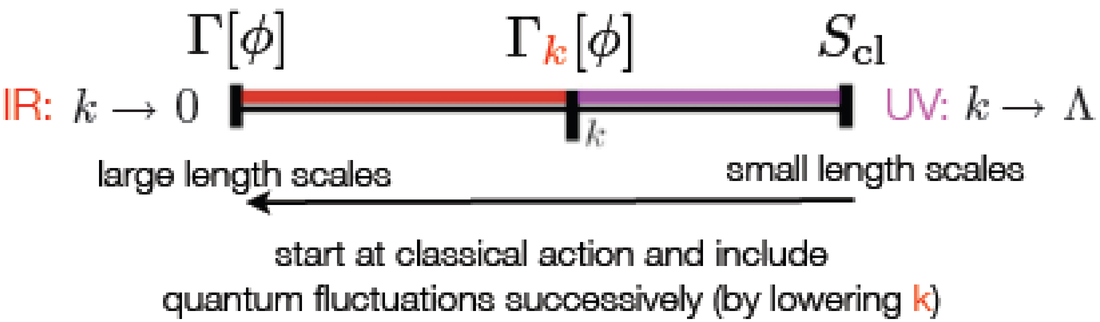
Generic Phase diagram for effective chiral Lagrangians



$G_V^{(S)} \rightarrow$ quantifies repulsive interactions between quarks

- Generic structure of the phase diagram as expected in QCD and in different chiral models see eg.:J. Berges & Rajagopal; M. Alford et al; C.Ratti & W. Weise; B. J. Schaefer & J. Wambach; M. Buballa &D.Blaschke; B. Friman, C. Sasaki at al., M.Stephanov et al.,……
- Quantitative properties of the phase diagram and the position of TCP are strongly model dependent
- Large G_V^S no TCP at finite T
- $m_q \neq 0$ acts as an external magnetic field and destroys the 2nd order transition to the cross-over and moves TCP to CEP

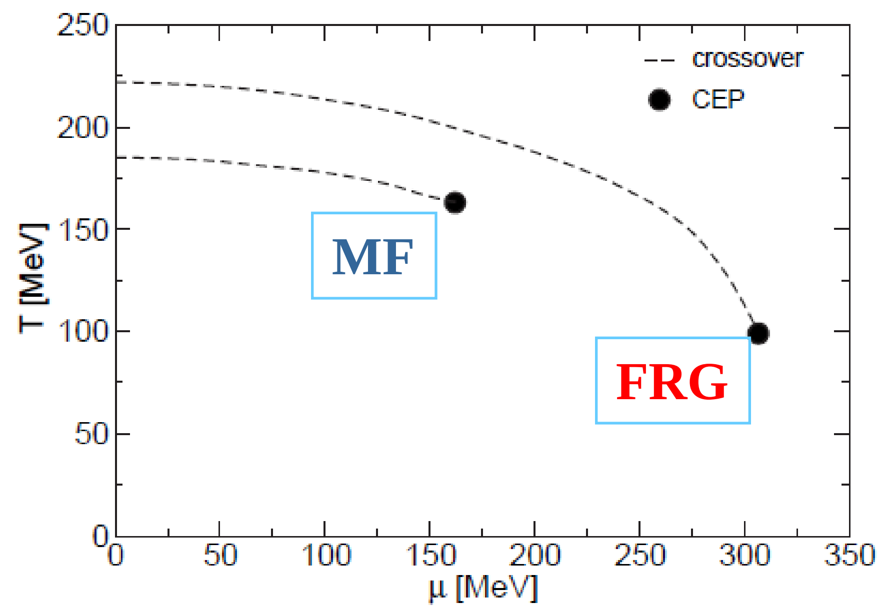
Including quantum fluctuations: FRG approach



B. Friman, B. Stokic & K.R.

FRG flow equation (C. Wetterich 93)

$$k\partial_k \Gamma_k \equiv \partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}$$



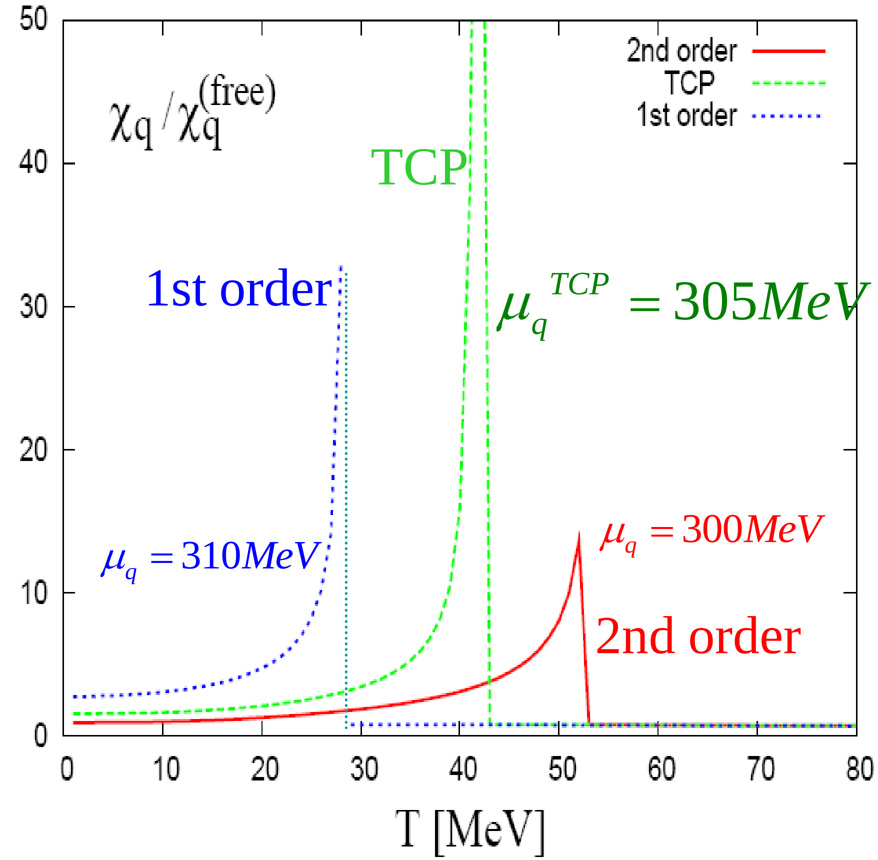
The position of CEP is strongly modify by quantum fluctuations

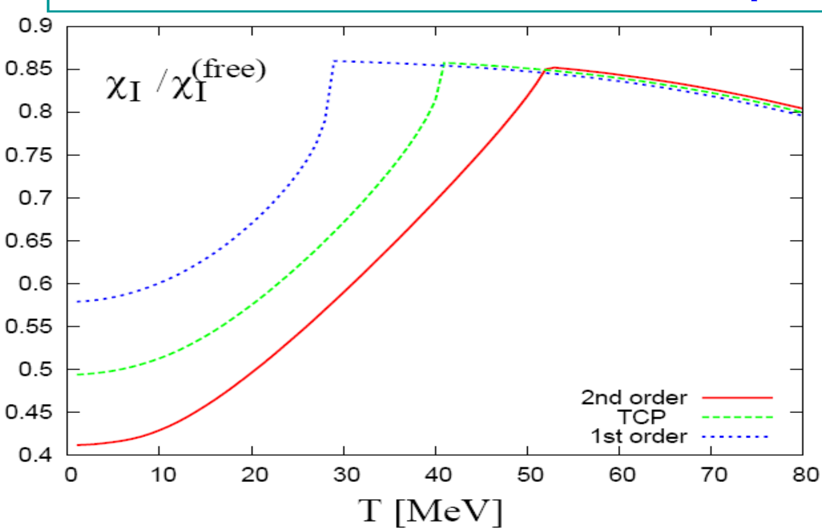
Susceptibilities of conserved charges

- Net quark-number χ_q,isovector χ_I and electric charge χ_Q fluctuations $\chi_A = \langle A^2 \rangle - \langle A \rangle^2$

$$\chi_q = \frac{\partial^2 P}{\partial \mu_q^{\ 2}} \qquad \chi_I = \frac{\partial^2 P}{\partial \mu_I^{\ 2}}$$

$$\chi_Q = \frac{1}{36}\chi_q + \frac{1}{4}\chi_I + \frac{1}{6}\frac{\partial^2 P}{\partial \mu_q \partial \mu_I}$$





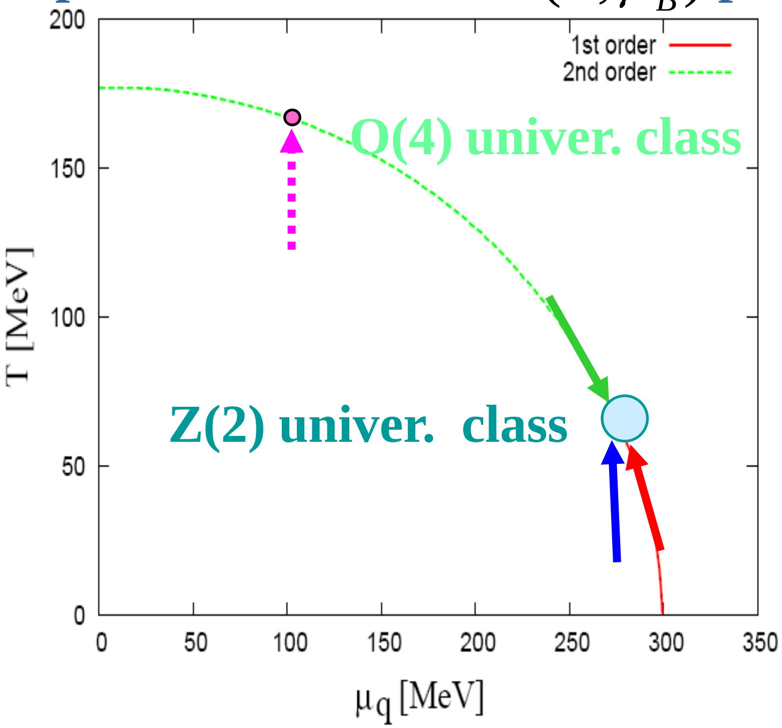
No mixing of isospin density with the sigma field due to isospin conservation

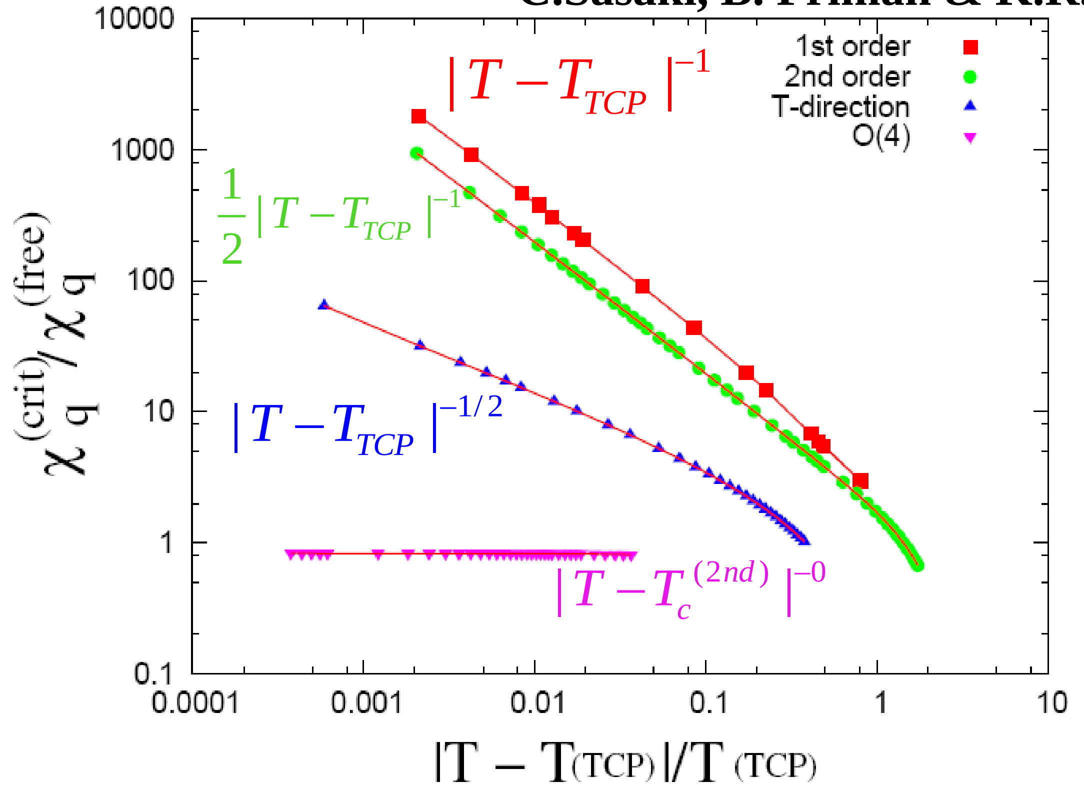
Hatta & Stephanov

Scaling properties: $\chi_q = \partial^2 P / \partial \mu^2 \approx a + b\,|T - T_c|^{-\theta}$

C.Sasaki, B. Friman & K.R.

The strength of the singularity at TCP depends on direction in (T, μ_B) plane





$\chi_q \propto |T - T_{TCP}|^{-1}$ along 1st order line

$\chi_q \propto |T - T_{TCP}|^{-1/2}$ any direction not parallel

$\chi_q \propto \frac{1}{2}|T - T_{TCP}|^{-1}$ along 2nd order line

See also Y. Hatta, T. Ikeda

FRG- beyond the mean field:
B.-J. Schaefer & J. Wambach
$\chi_q \propto |T - T_{TCP}|^{-0.53(m\neq 0 => 0.78)}$

FRG: Stokic, Friman & K.R

$\alpha = (0 -> MF)\ (-0.25 -> O(4))$

$(-0.3 -> FRG)$

Scaling properties: $\chi_q = \partial^2 P / \partial \mu^2 \approx a + b\,|T - T_{TCP}|^{-\theta}$

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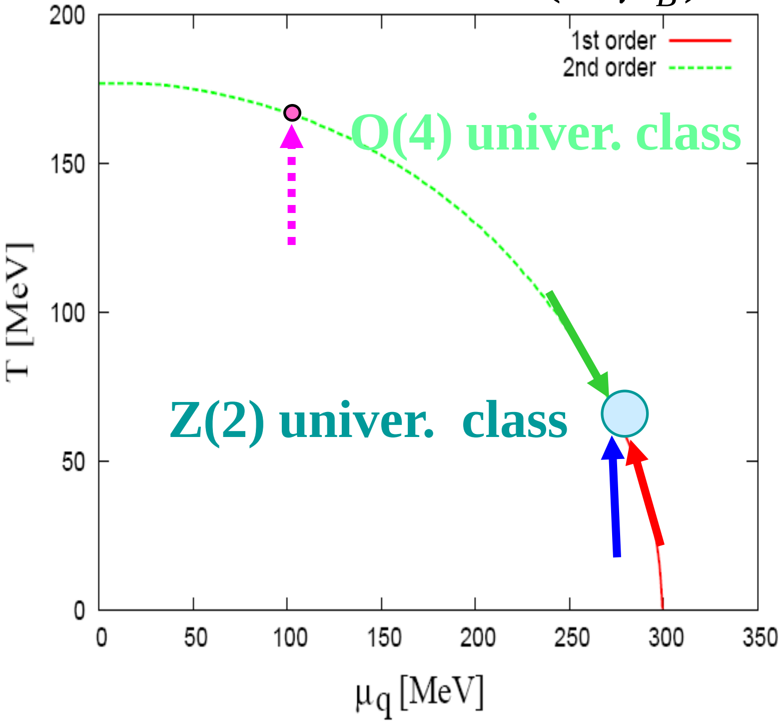

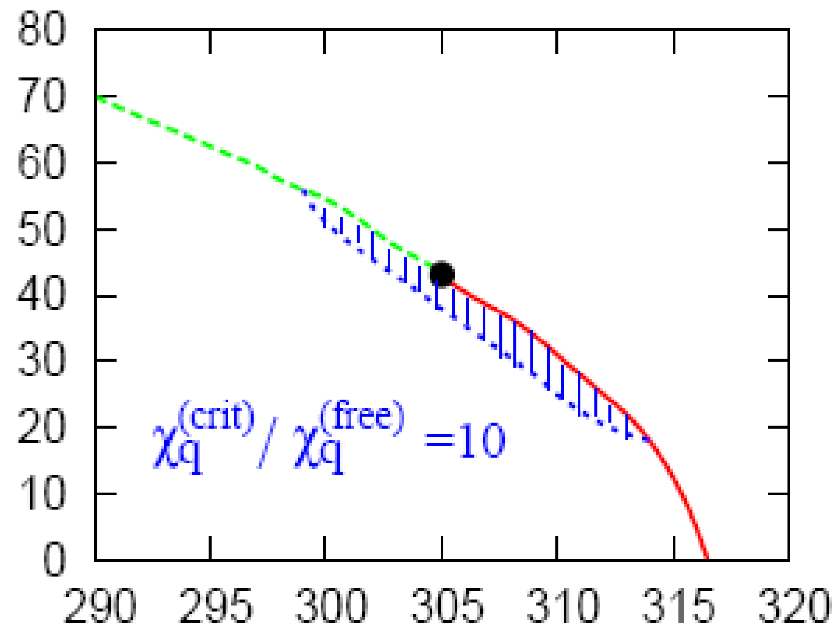

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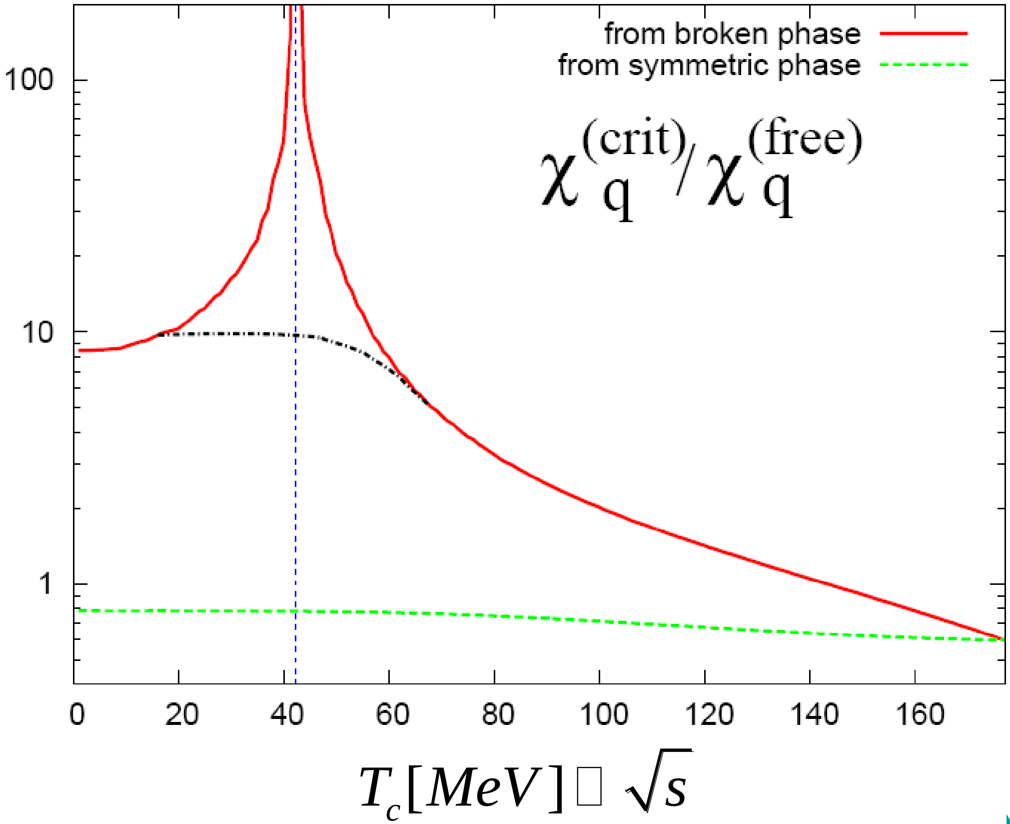
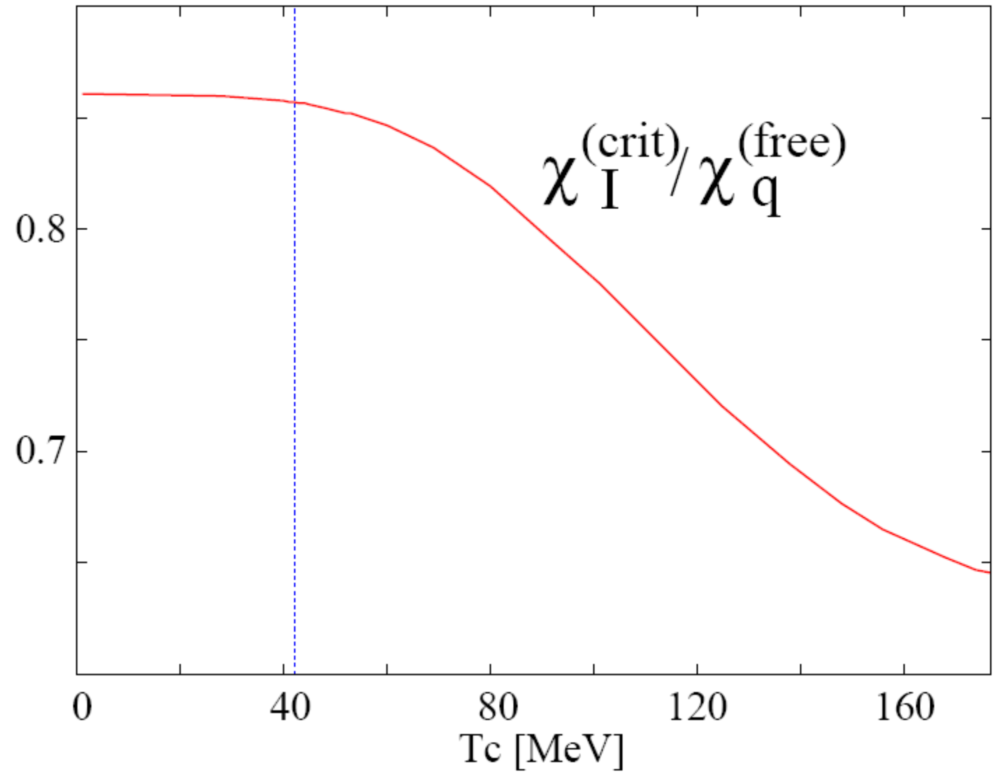
$(-0.3 -> FRG)$

Quark and isovector fluctuations along the critical line

NJL-model results: C. Sasaki, B. Friman, K.R.





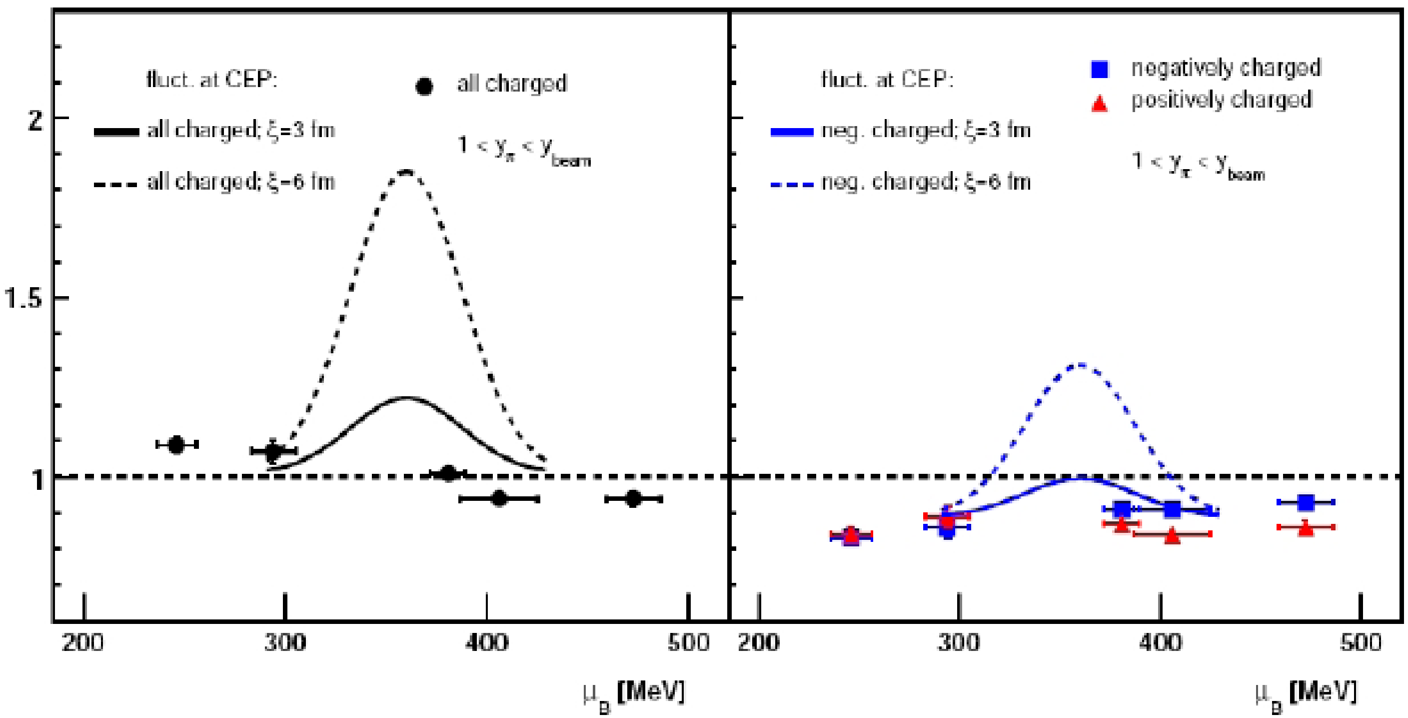
Non-monotonic behavior of the net quark susceptibility as function of (T_c, μ_c) in LGT or $\sqrt{s}$ in HIC

sensitive probes of TCP/CEP

In HIC critical fluctuations are suppressed due to finite size and live time of collision fireball

central Pb+Pb collisions (NA49) multiplicity fluctuations

M. Gazdzicki et al. NA61

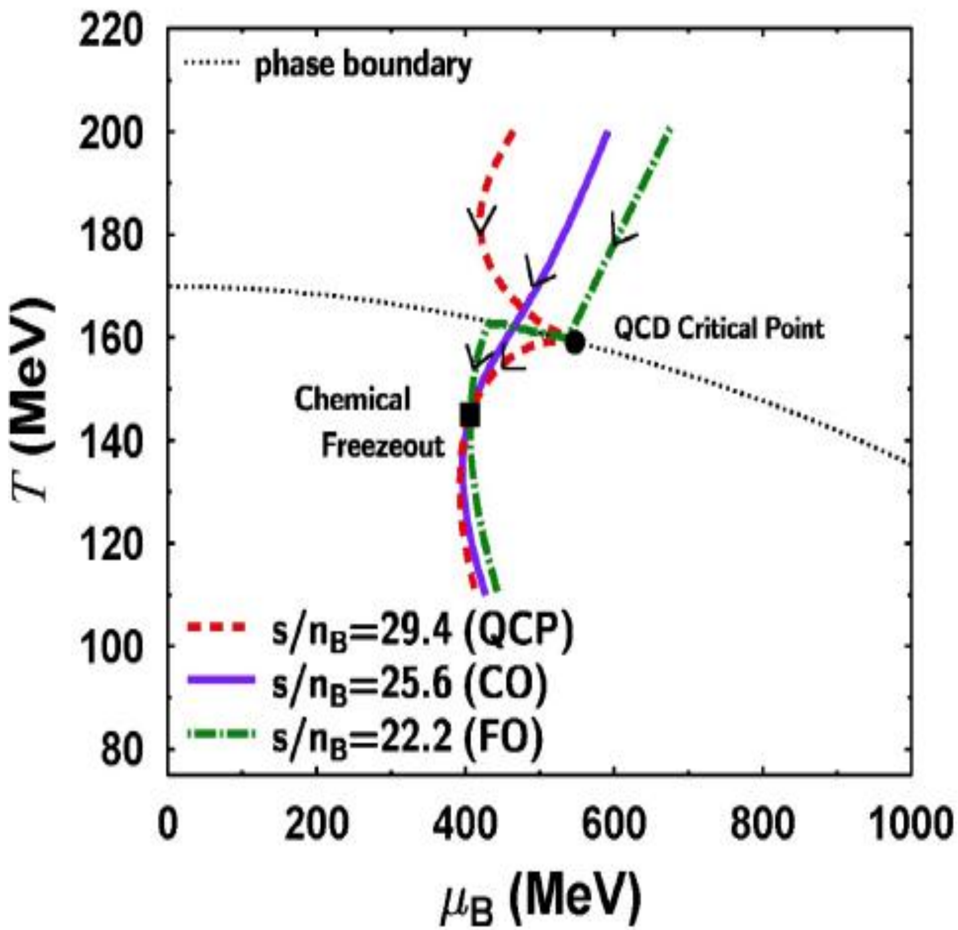


Data: PRC78:034914
CP: PRD60:114028

the predicted CP fluctuations are not observed, freeze-out far from CP?

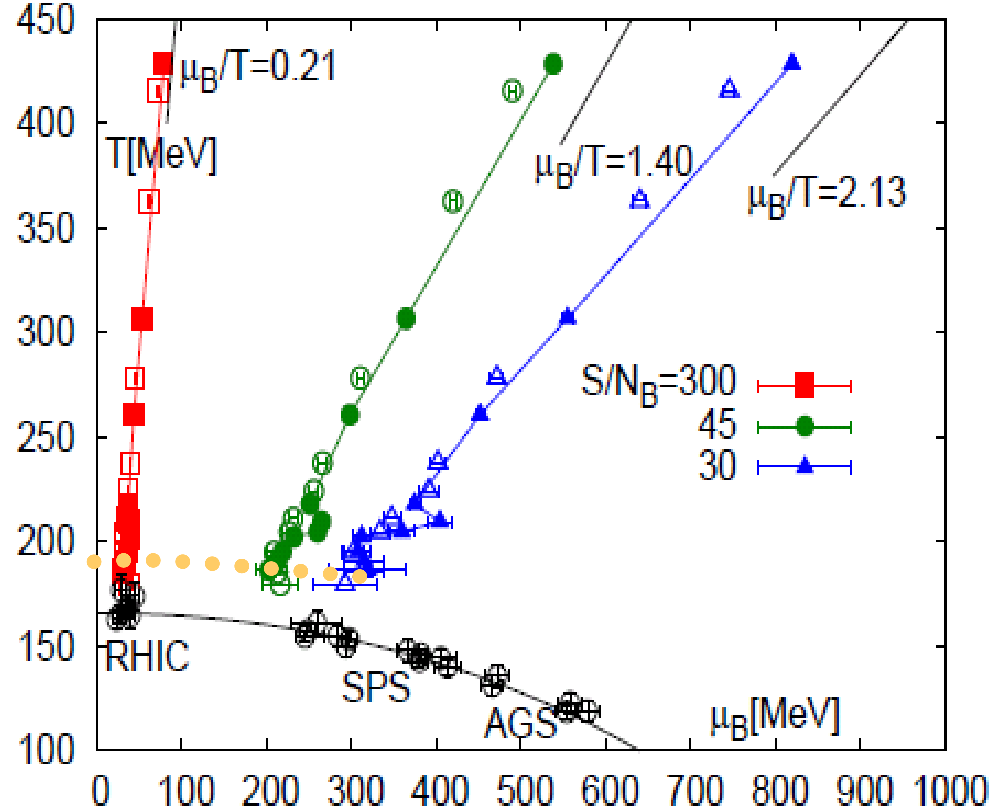
Focusing: CEP attractor for isentrops

For a given chemical freezeout point, prepare three isentropic trajectories: w/ and w/o CP: **Asakawa, Nonaka**



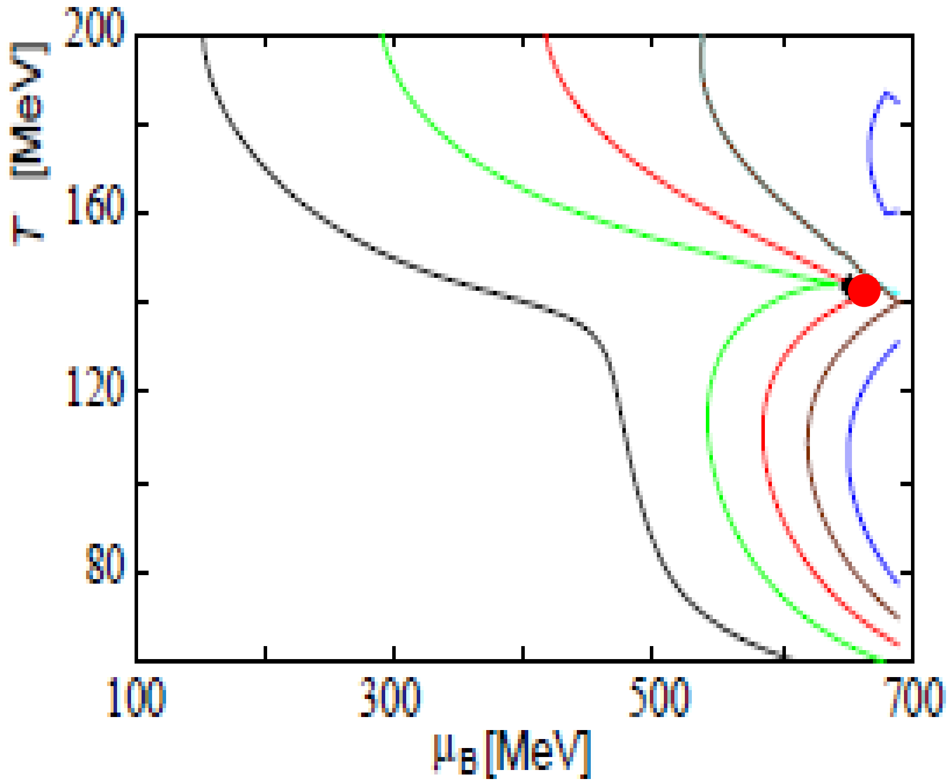
LGT results and isentrops for different S/N

F. Karsch et al.

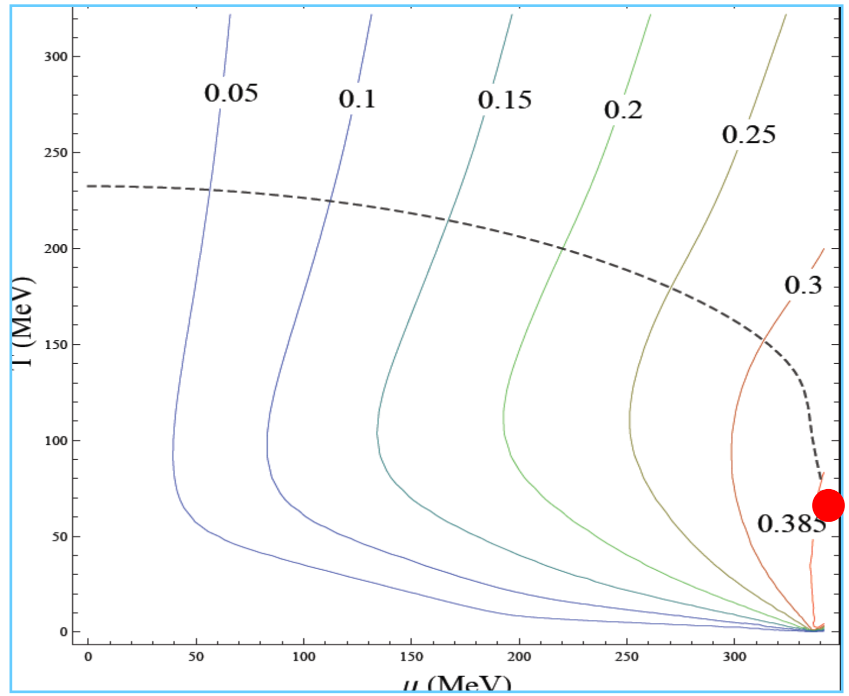


Focusing in the model calculations

M. Asakawa & Ch. Nonanka

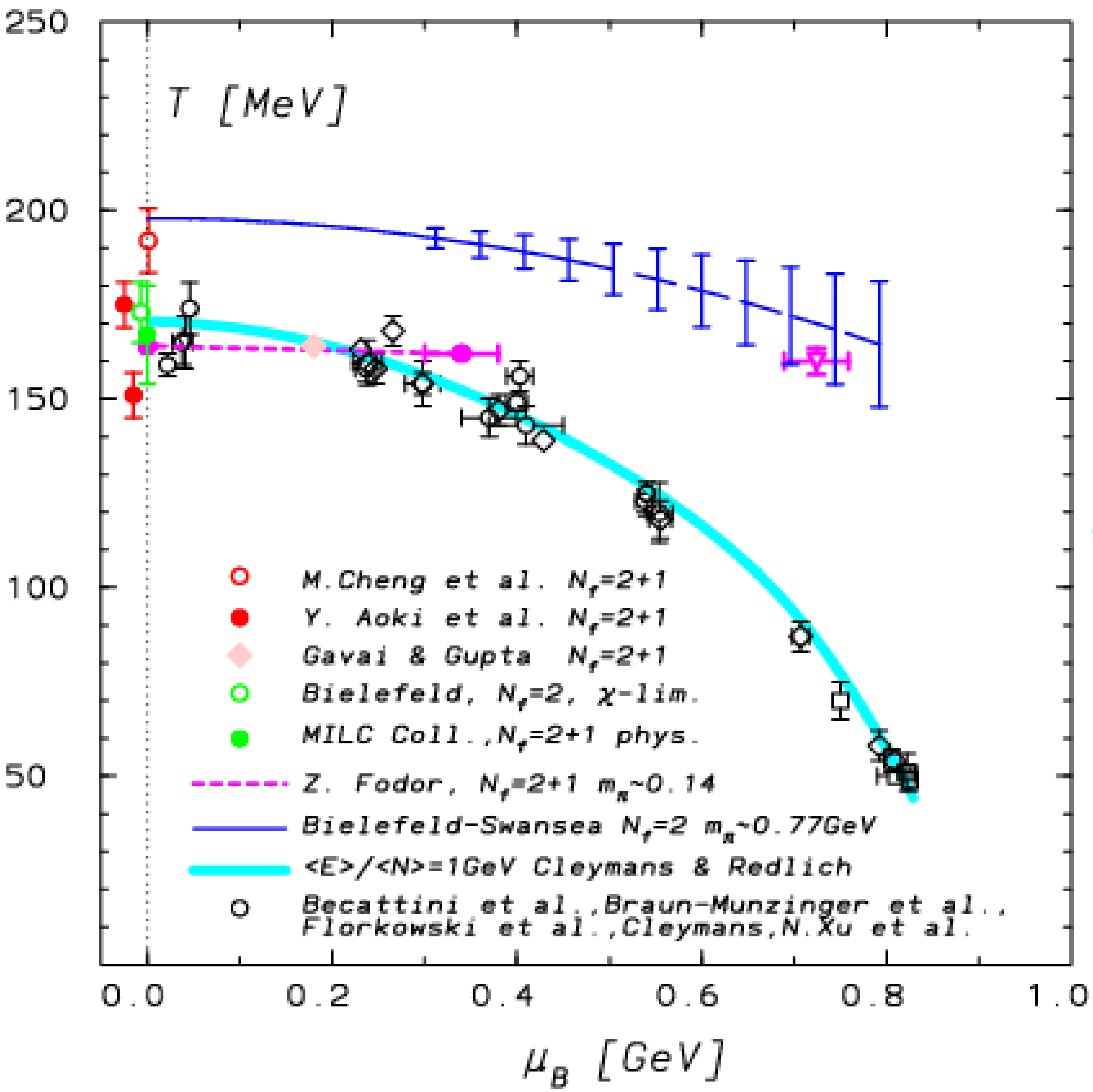


B. Friman, E. Nakano, B. Stokic & K.R.



- Focusing in the model where the EQS is constructed on the basis of Z(2) universality argument:
- No-focusing in the quark-meson model with and without quantum fluctuations

LGT phase boundary and chemical freezeout



- At present, the critical curve and CEP obtained in LGT "coincide" with the chemical freeze-out

However: recent LGT results show that:

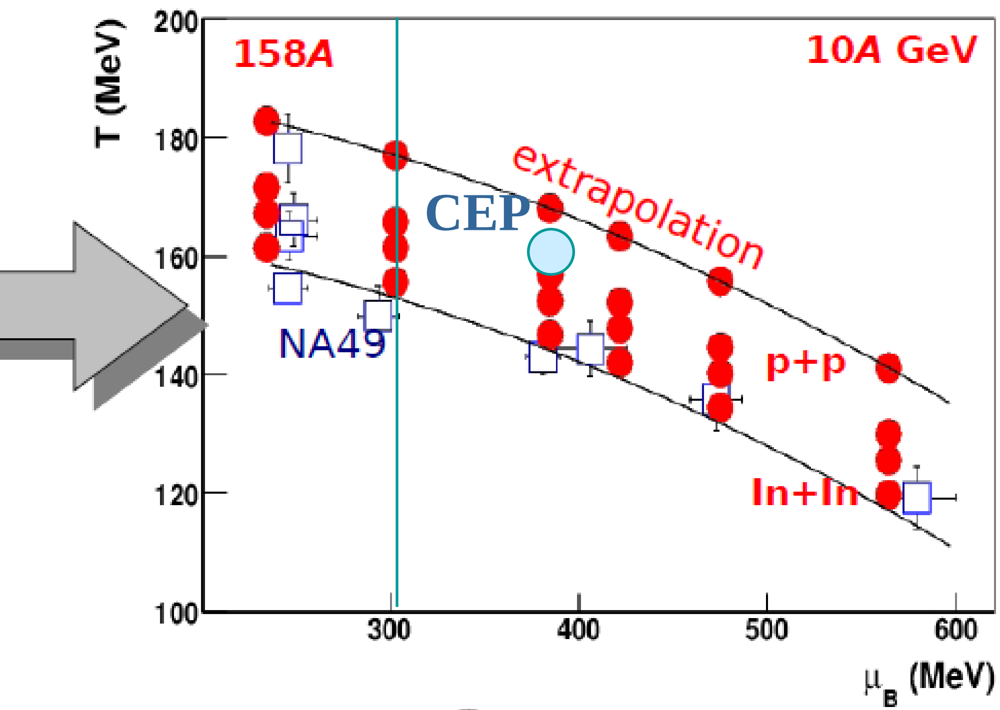
- there is no unique value of T_c at $\mu_q = 0$ (Y.Aoki at.al)
- Critical temperature T_c at $\mu_q = 0$ can be as large as (M. Cheng et al.)

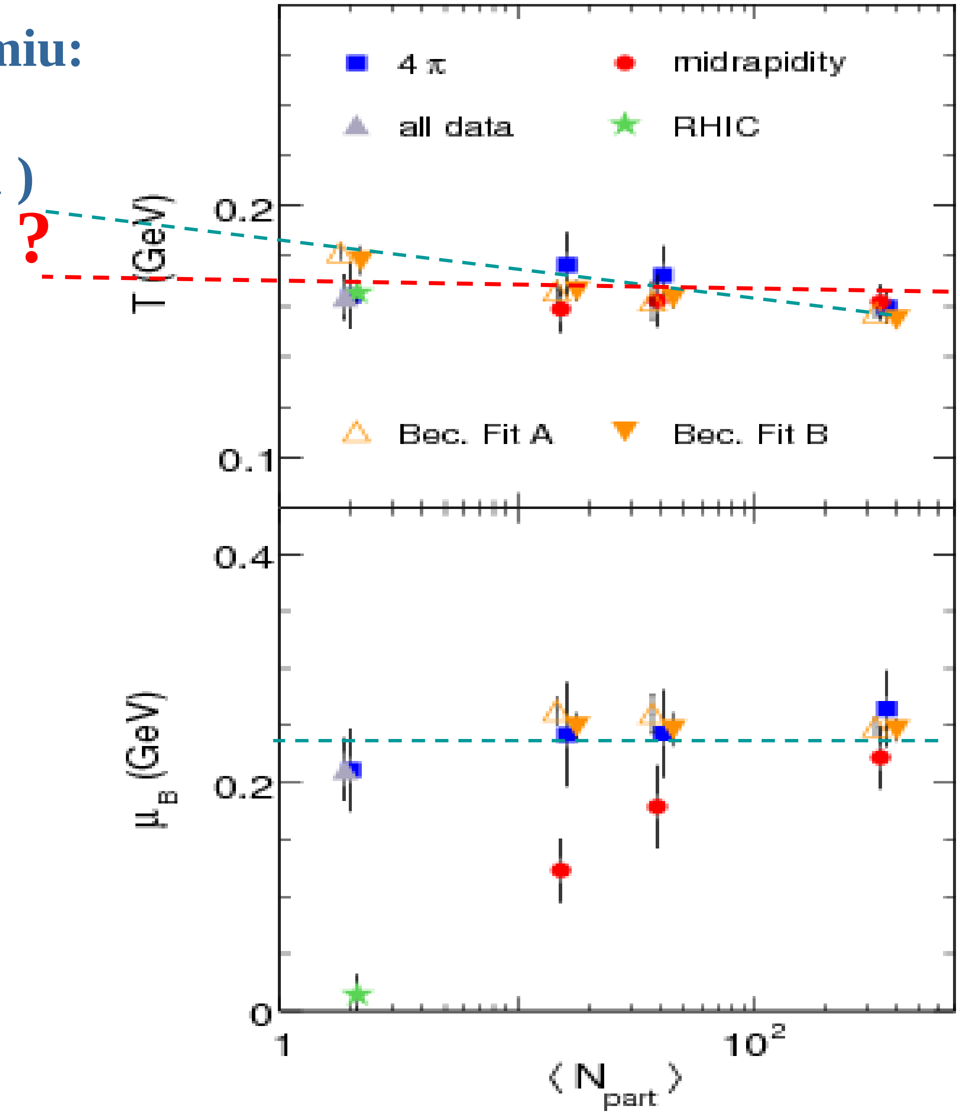
$$T_c = 192(7)(4)\ MeV$$

Changing system size to approach CEP ?

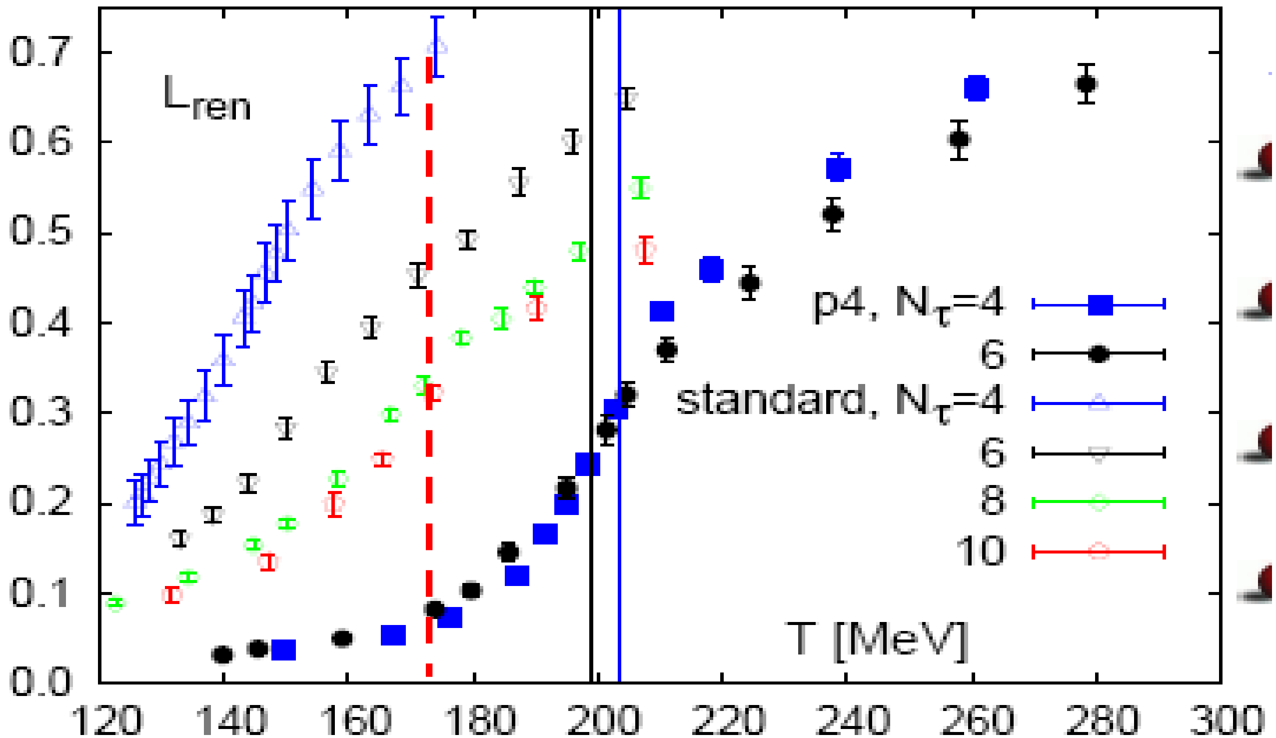
Decreasing system size increases T at fixed miu: approaching towards CEP ?

(M. Gazdzicki et. al. NA61)





LGT results in 2+1 flavor QCD with physical mass spectrum



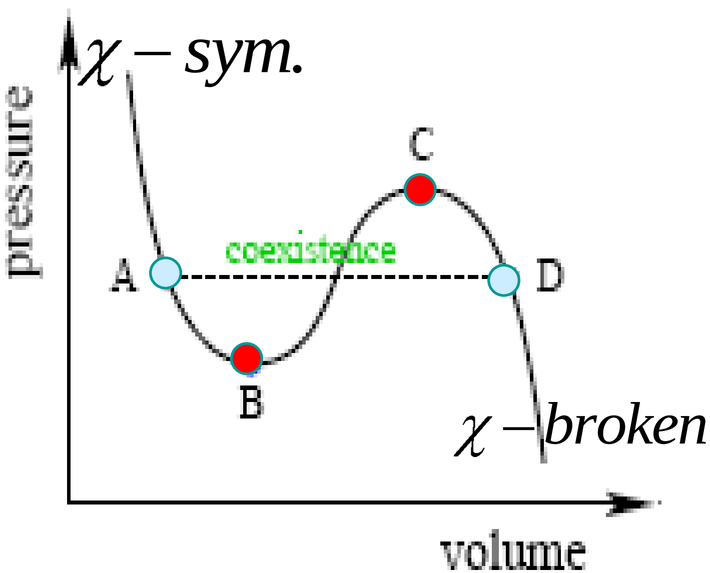
Different critical temperatures: $T_c \approx 175 MeV$ **Z. Fodor et al.**

$T_c \approx 192 MeV$ **M. Cheng et al.**

Different critical temperatures for different observables: Z. Fodor et al.

The nature of the 1st order chiral phase transition

instability of a system:



$\partial P / \partial V < 0$: stable

$\partial P / \partial V > 0$: unstable

$\partial P / \partial V = 0$: spinodal

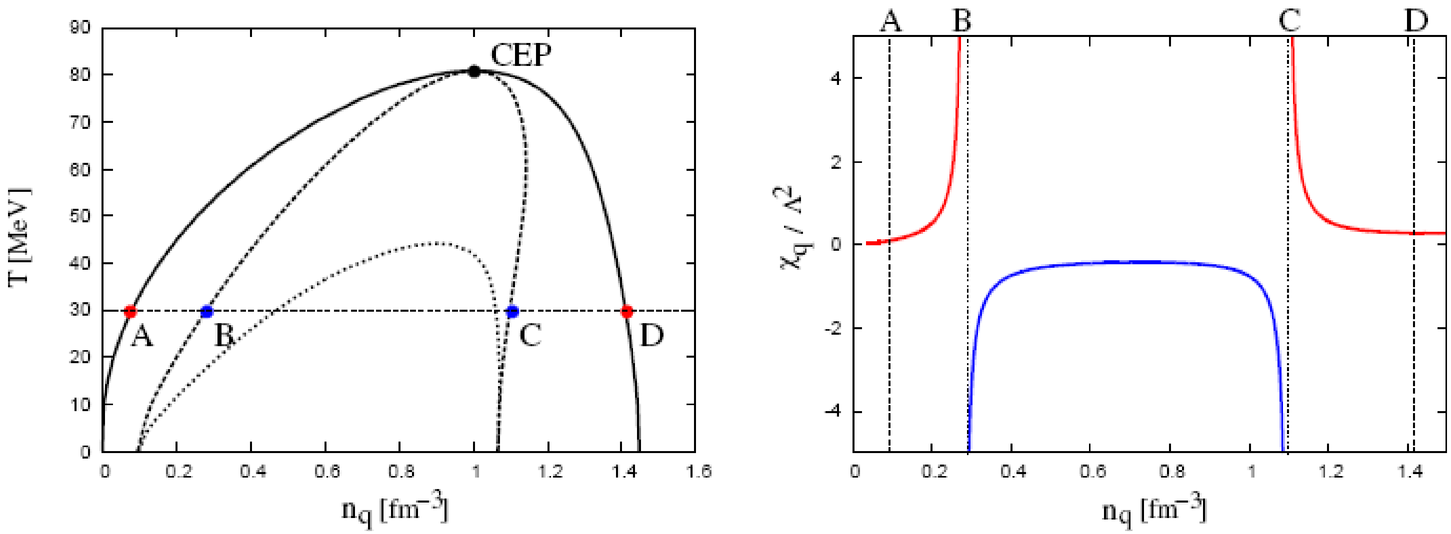
A-B: supercooling (symmetric phase)

B-C: non-equilibrium state

C-D: superheating (broken phase)

Quark number susceptibility

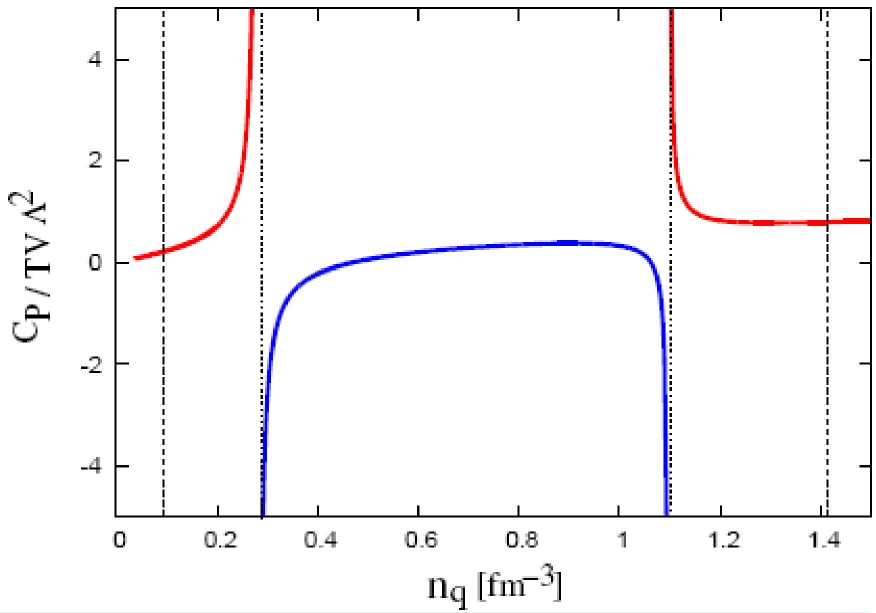
- deviation from equilibrium, large fluctuations induced by instabilities



- at 1st order transition point (A, D) : χ_q is finite
- at isothermal spinodal point (B, C) : χ_q diverges and changes its sign
  $\frac{\partial P}{\partial V} < 0$ for stable/meta-stable state $\Rightarrow$ $\frac{\partial P}{\partial V} > 0$ for unstable state
- in unstable region (B-C) : χ_q is finite and negative

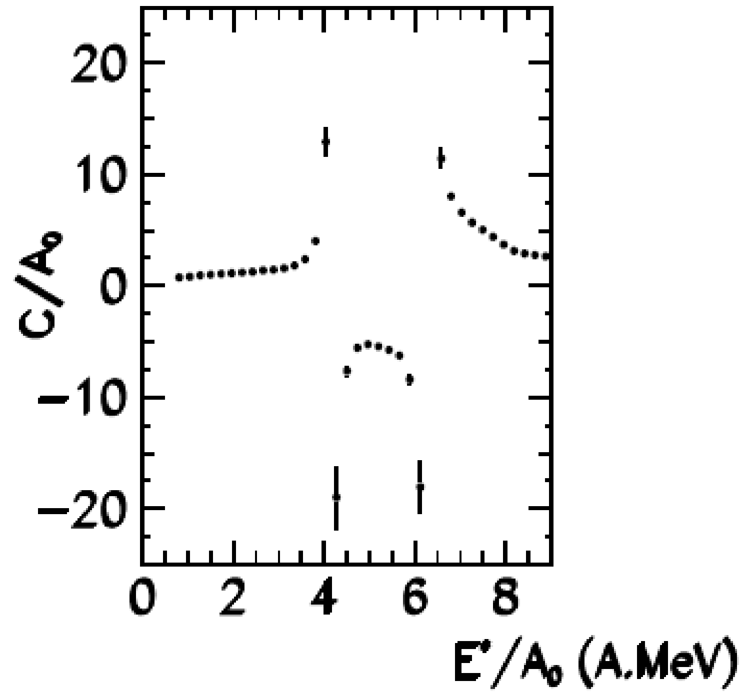
Experimental Evidence for 1st order transition

Specific heat for constant pressure:



$$C_P = T\left(\frac{\partial S}{\partial T}\right)_P = TV\left[\chi_{TT} - \frac{2s}{n_q}\chi_{\mu T} + \frac{s^2}{n_q{}^2}\chi_q\right]$$

Low energy nuclear collisions

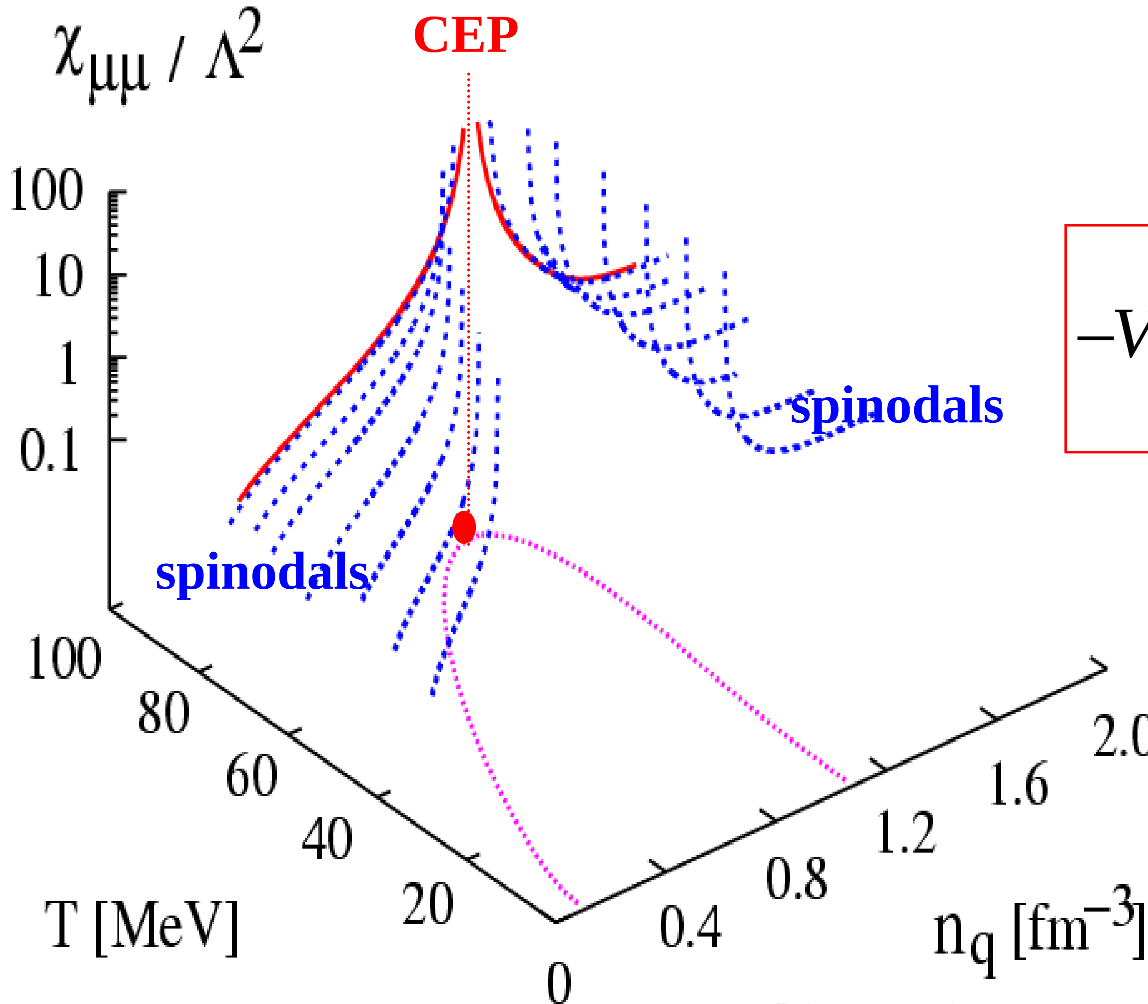


M. D'Agostino *et al.*, Phys. Lett. B 473, 219 (2000)

negative heat capacity : anomalously large fluctuations

$\Rightarrow$ an evidence of the 1st order liquid-gas phase transition

Net-quark fluctuations on spinodals



at any spinodal points:

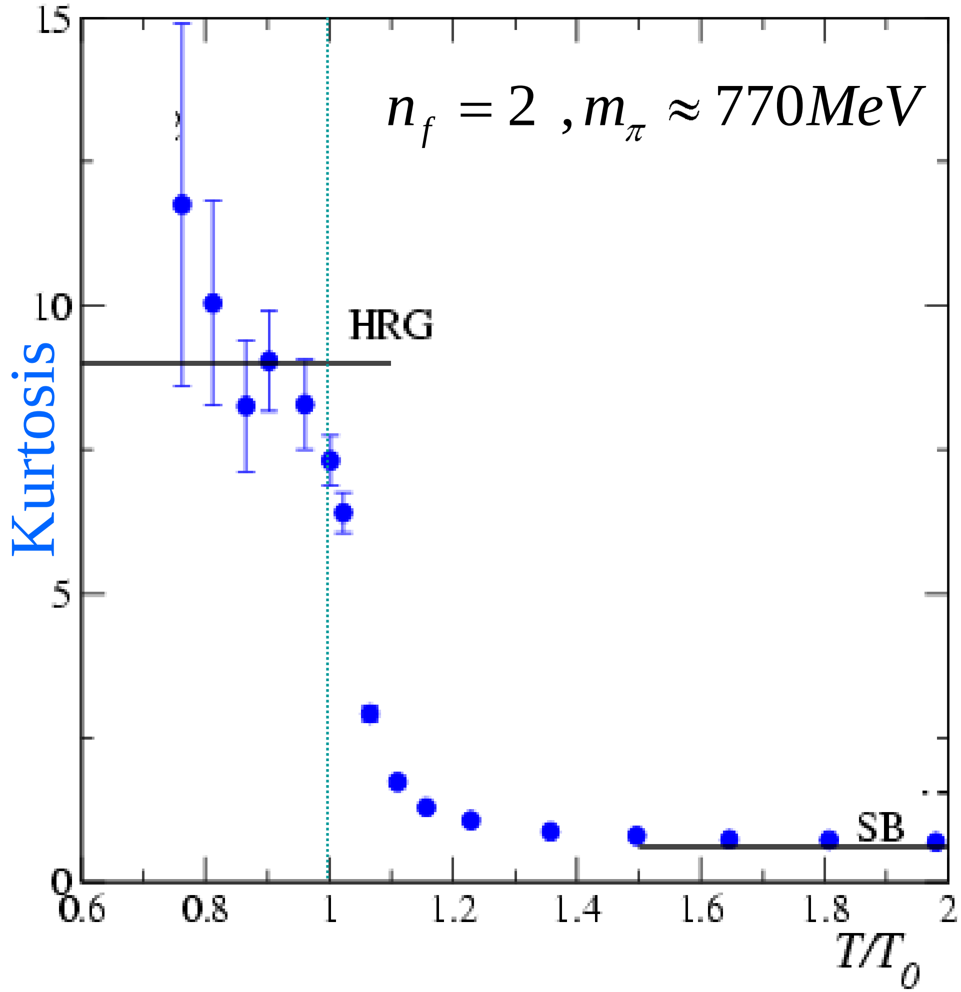
$$-V \frac{\partial P}{\partial V}\Big|_T = \frac{n_q^2}{\chi_q} = \frac{1}{compres.}$$

Singularity at CEP are the remnant of that along the spinodals

C. Sasaki, B. Friman & K.R., Phys.Rev.Lett.99:232301,2007.

Probe of Deconfinement: Kurtosis $\frac{\partial^4 (P/T^4)}{\partial(\mu/T)^4} / \frac{\partial^2 (P/T^4)}{\partial(\mu/T)^2}$

S. Ejiri, F. Karsch & K.R.



- HRG factorization of pressure:

$$P^B(T, \mu_q) = F(T)\cosh(3\mu_q / T)$$

consequently: $d_4^q / d_2^q = 9$ in HRG

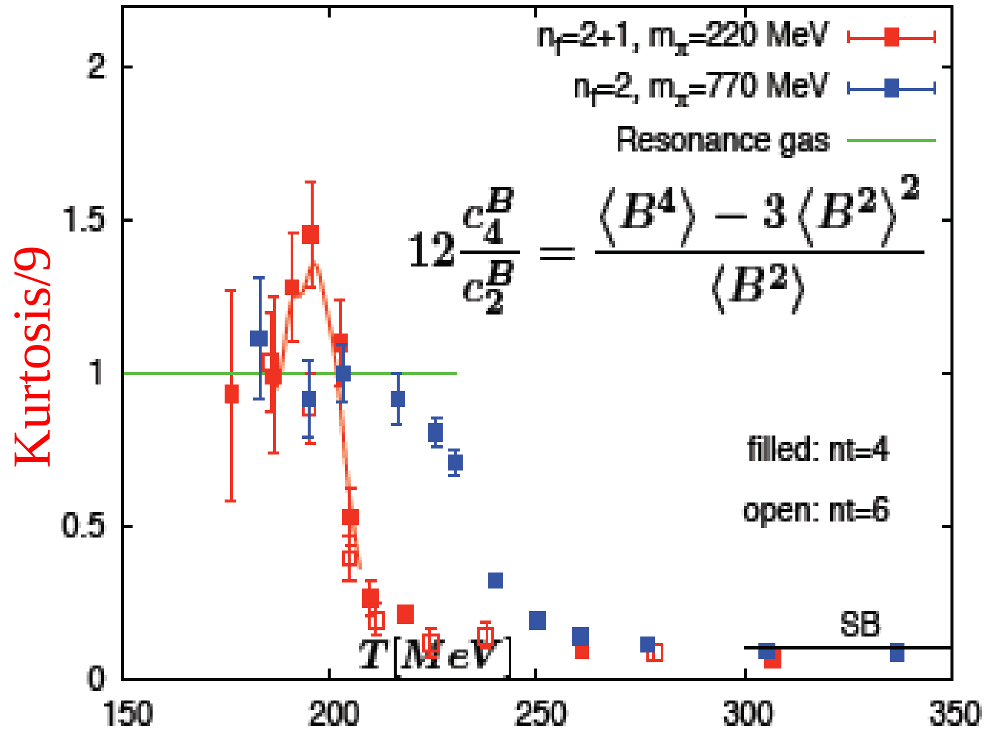
- In ideal QGP, $d_q^4 / d_q^2 = 6/\pi^2$

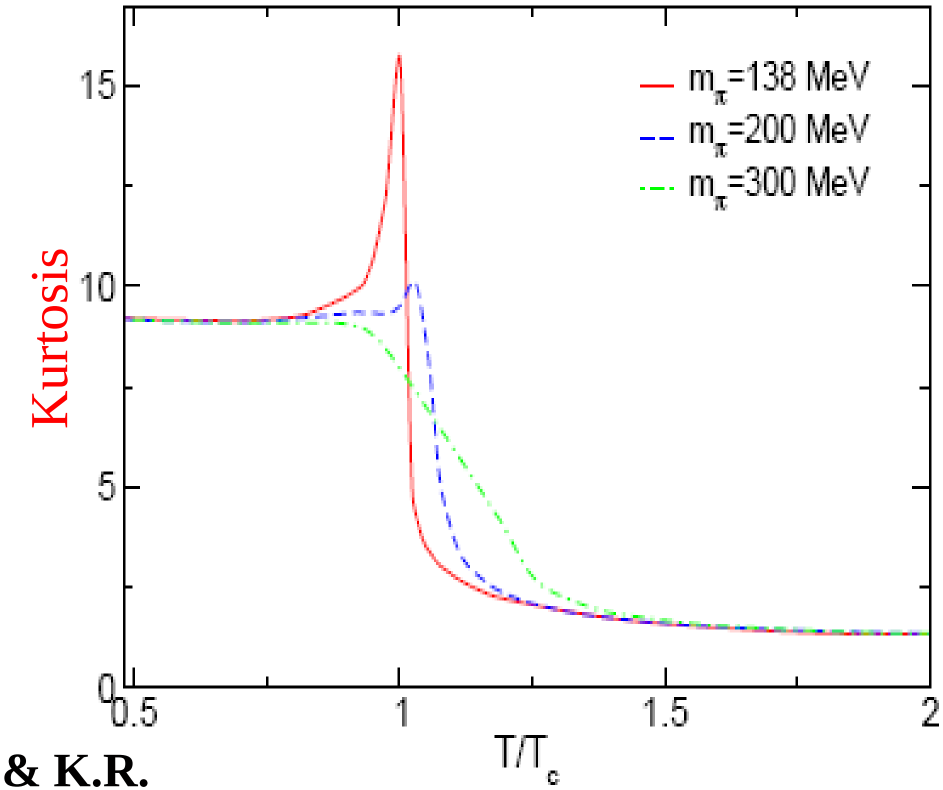
Kurtosis=Ratio of “4/2” cumulants

$$d_4^q / d_2^q = \frac{<(\delta N_q)^4>}{<(\delta N_q)^2>} - 3 <(\delta N_q)^2>$$

excellent probe of deconfinement

Chiral dynamics, Kurtosis and inverse compressibility, model calculations





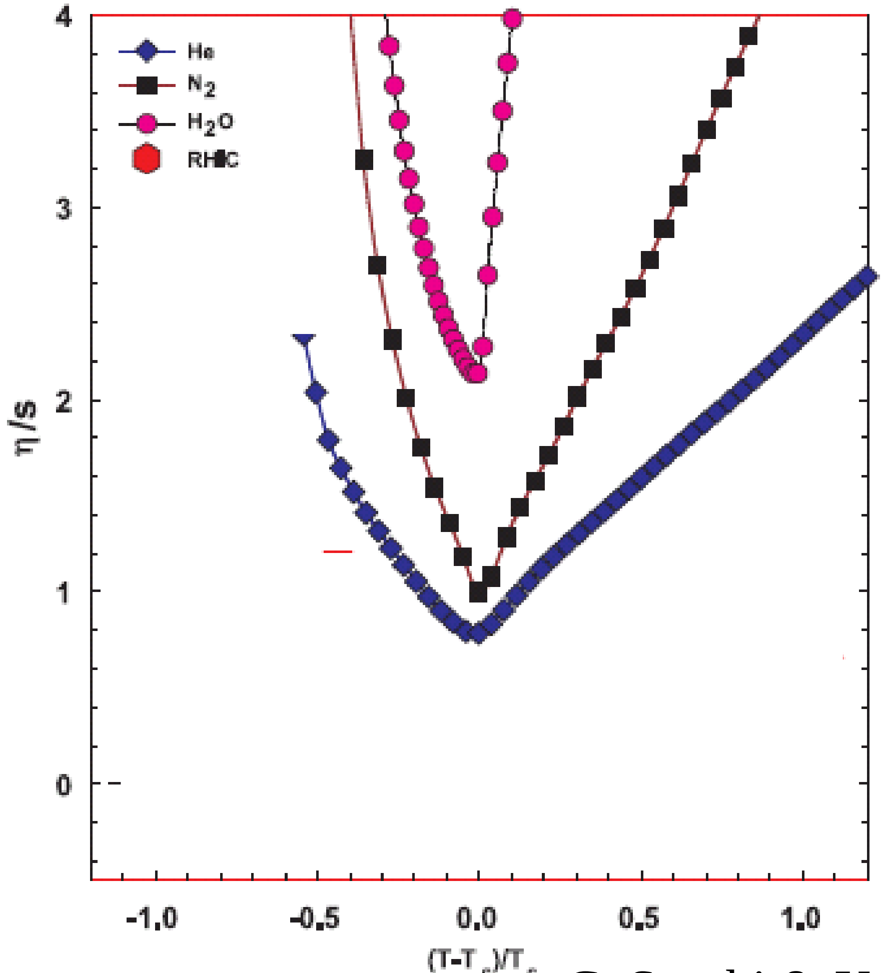
B. Friman, B. Stokic & K.R.

A peek in kurtosis appear as remnant of chiral dynamics and O(4) universality!
The 2+1 flavor QCD sensitive to O(4) dynamics expected in 2-flavor QCD

Transport Coefficient near phase transition

L. Csernay, J. Kapusta & L. McLerran 06

R. Lacey et al. 07: data for shear viscosity



- Minimum of shear viscosity may indicate the location of : L.
- Large bulk viscosity at CEP

Kharzeev & Tuchin 07

Indeed: due to dynamic scaling

(Hohenberg and Halperin, 1977)

relaxation time $\tau \sim \xi_T^{z}$ $\quad z \approx 3$

shear viscosity $\eta \sim \xi_T^{z_\eta}$ $\quad z_\eta \approx \frac{1}{19}$

bulk viscosity $\varsigma \sim \xi_T^{z_\varsigma}$ $\quad z_\varsigma \approx 3$

$$\xi_T \sim (T - T_c)^{-\nu}$$

C. Sasaki & K.R. **Hep-ph 0806.4745**

Transport coefficients from kinetic theory

- Energy momentum tensor

Consider M as dynamical quark mass: order parameter

$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3} p^\mu p^\nu \frac{1}{E}[f + \overline{f}] \qquad E^2 = \vec{p}^{\,2} + M^2(T,\mu)$$

- Assume small deviations from equilibrium $\delta f = f - f_0$ with $f^{-1} = \exp(E - \vec{p}\vec{u} \mp \mu) \pm 1$, consequently

from Boltzmann equation

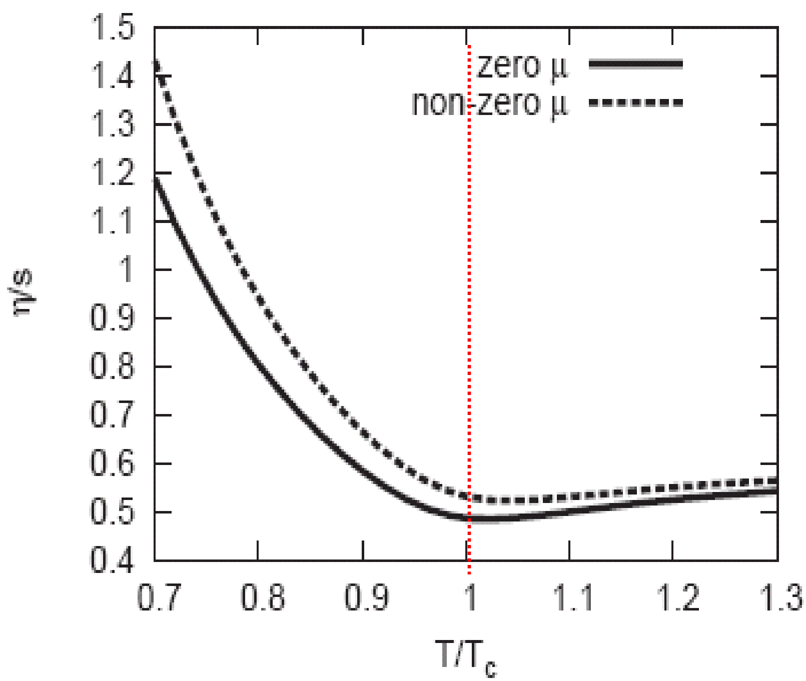
$$\delta T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3} p^\mu p^\nu \frac{1}{E}[\delta f + \delta \overline{f}] \approx -\varsigma \delta_{ij} \partial_k u^k - \eta W_{ij}$$

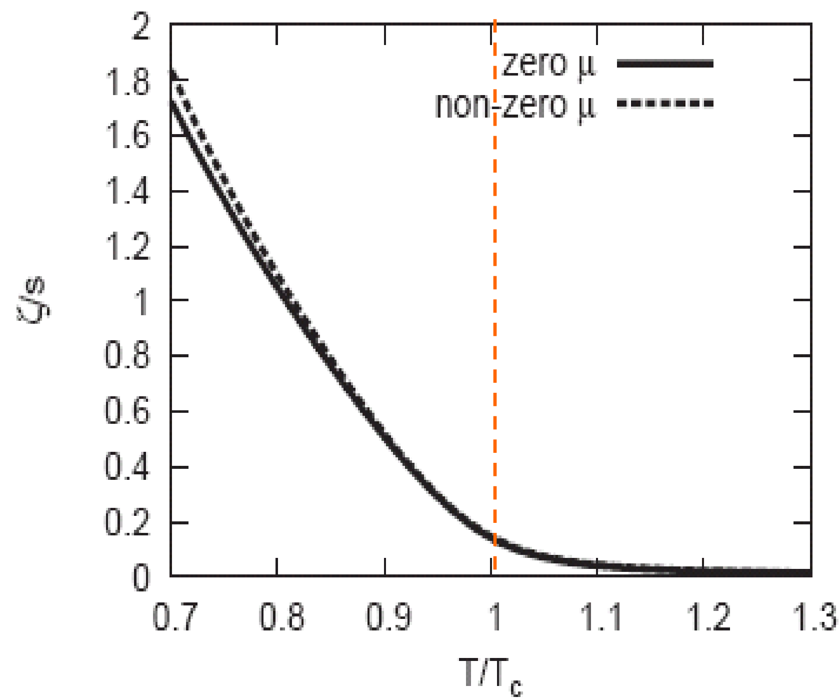
bulk viscosity

shear viscosity

Shear and bulk viscosity per entropy

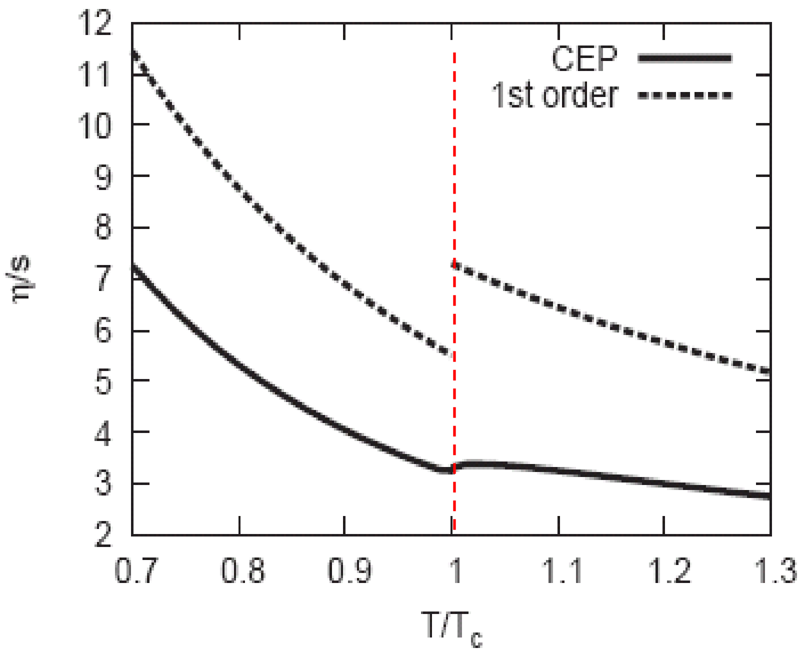
- Crossing the crosover transition line



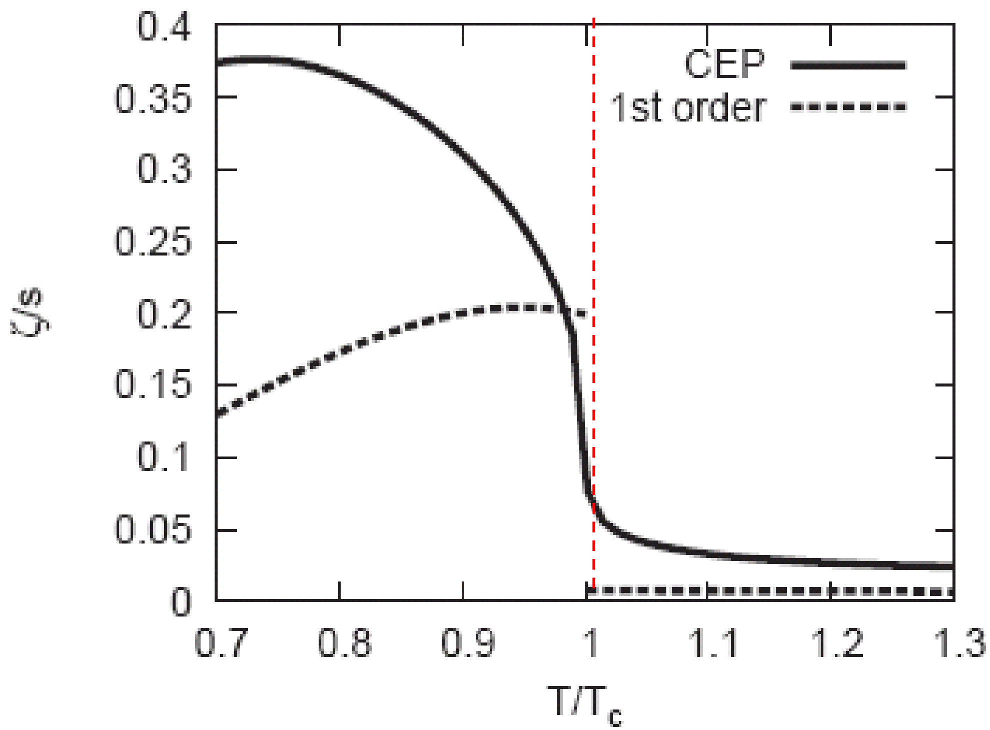


Viscosity to entropy ratio

- Shear



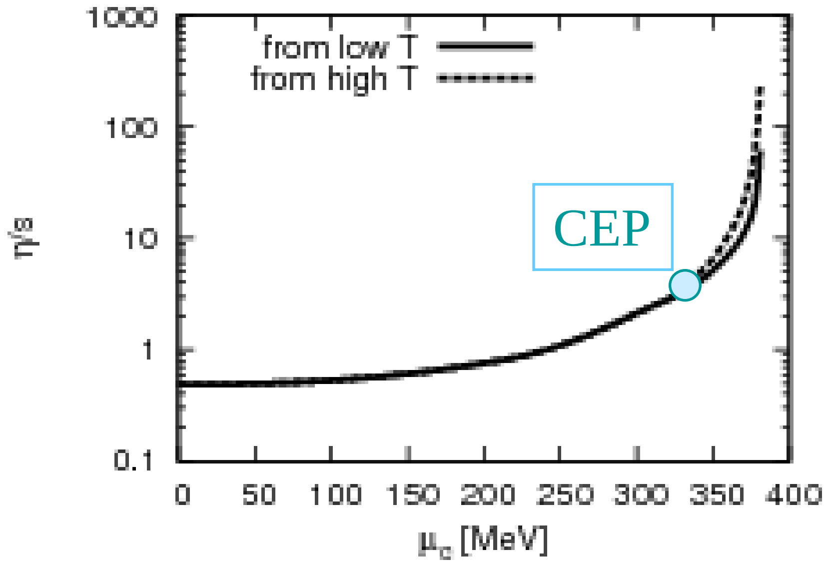
- Bulk

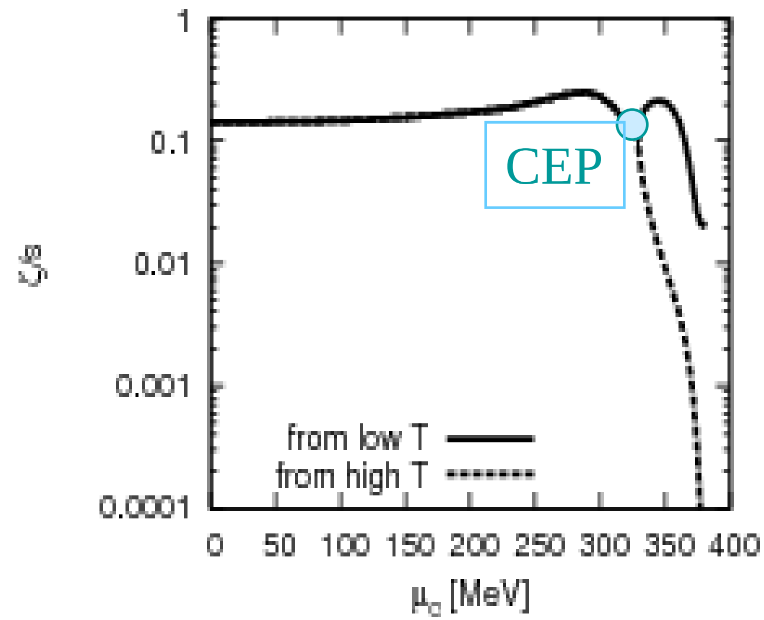


Bulk and shear viscosity per entropy along the chiral phase boundary

- shear viscosity
- bulk viscosity

C. Sasaki & K.R.





Conclusions

- A non-monotonic change of the net-quark susceptibility probes the existence of CEP However in non-equilibrium: due to spinodal instabilitie the charge fluctuations diverge at 1st order critical line

  => Large fluctuations signals 1st order transition

- Kurtosis is an excellent probe of deconfinement and O(4) chiral dynamics

- No-focusing of isentrops near CEP in the chiral quark-meson model solved within RG approach

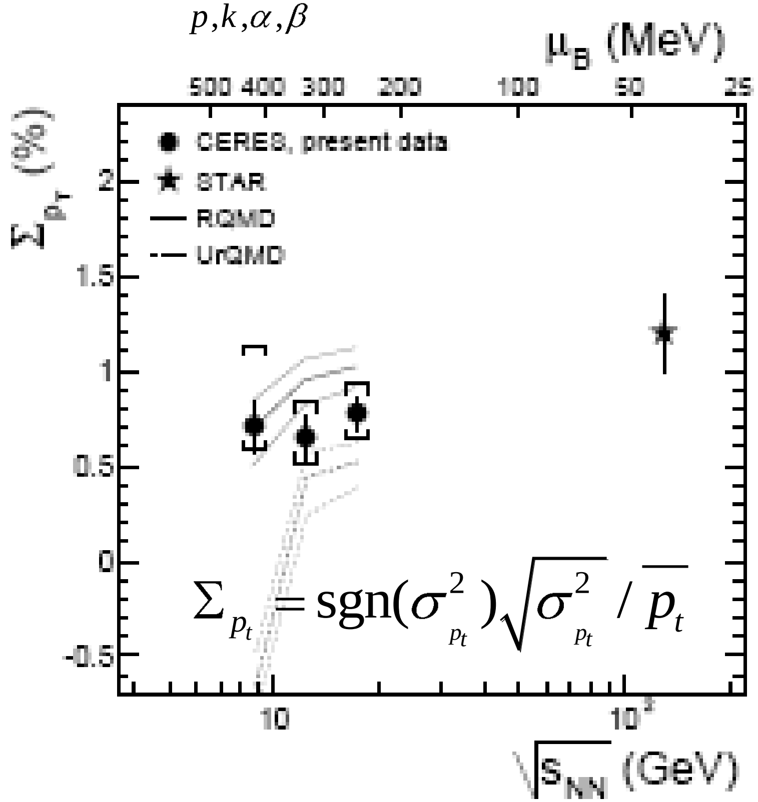
- Under relaxation time approximation the bulk viscosity is finite at CEP and O(4) line

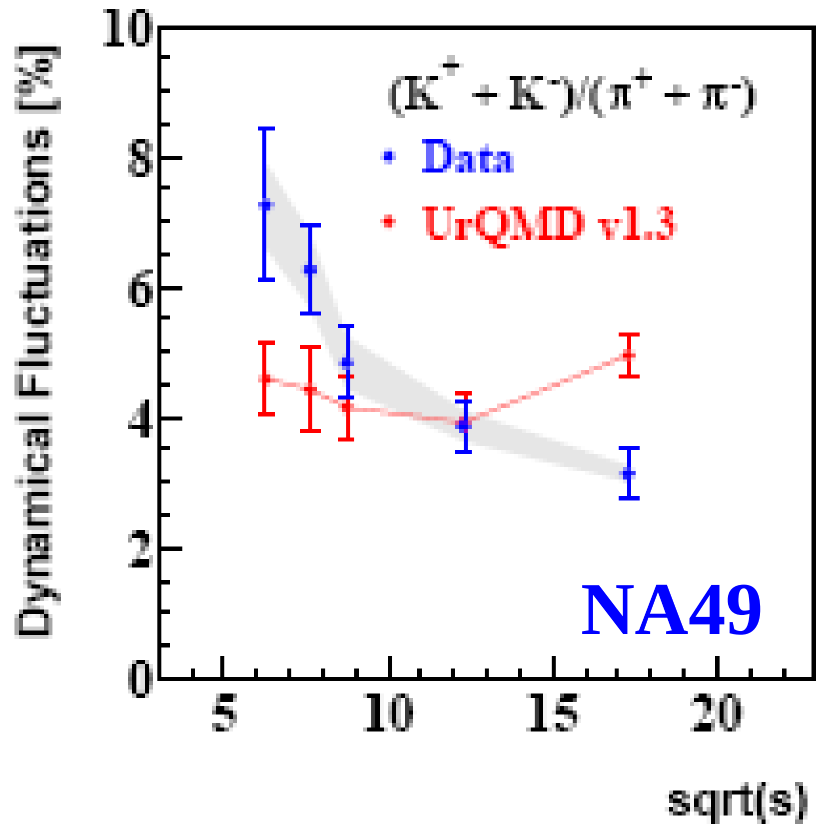
  => Divergence of bulk viscosity controlled by the dynamical, rather then static critical exponents

Energy dependent fluctuations & CEP

$$\sigma_Q^2 = \sum_{p,k,\alpha,\beta} q^\alpha q^\beta < \delta n_p^\alpha \delta n_k^\beta >$$

$$\sigma_{p_t}^2 = \sum_{p,k,\alpha,\beta} \delta p_t{}^\alpha \delta q_t{}^\beta < \delta n_p^\alpha \delta n_k^\beta >$$



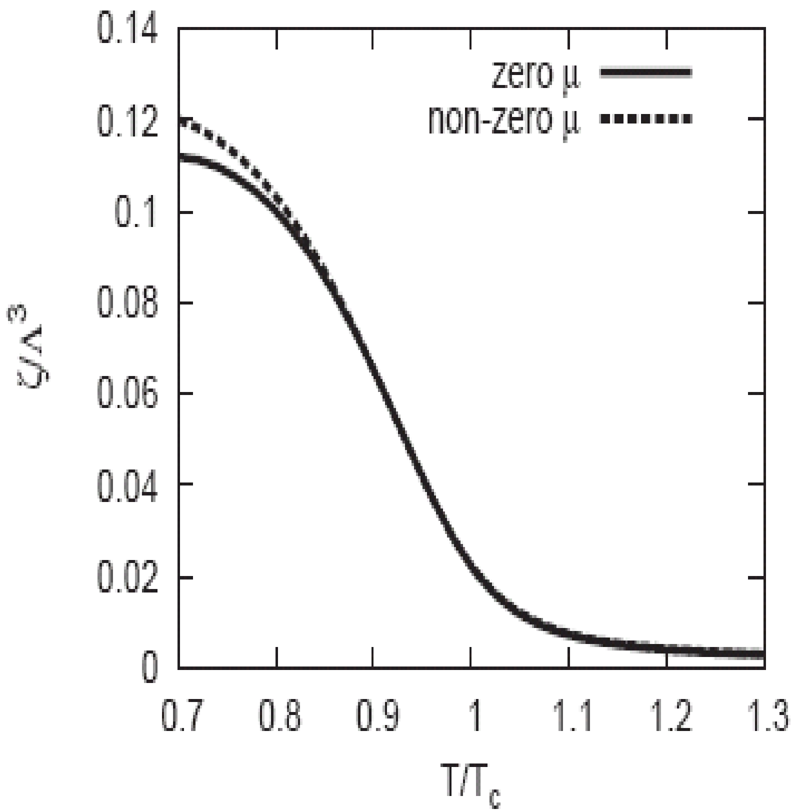


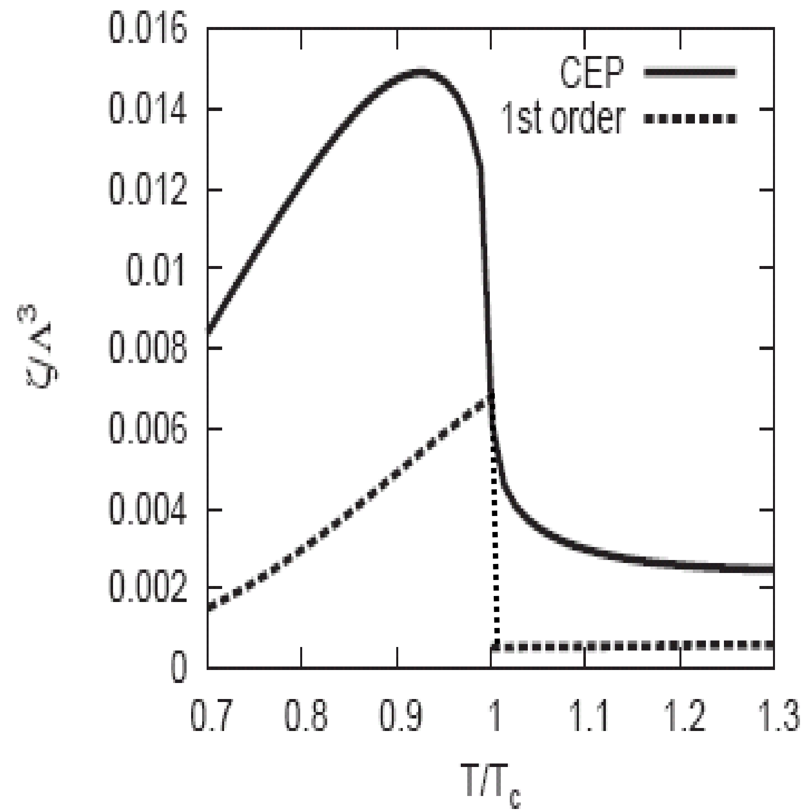
Smooth change of fluctuations with collision energy: no sign of CEP

Bulk Viscosity across the phase transition in NJL model

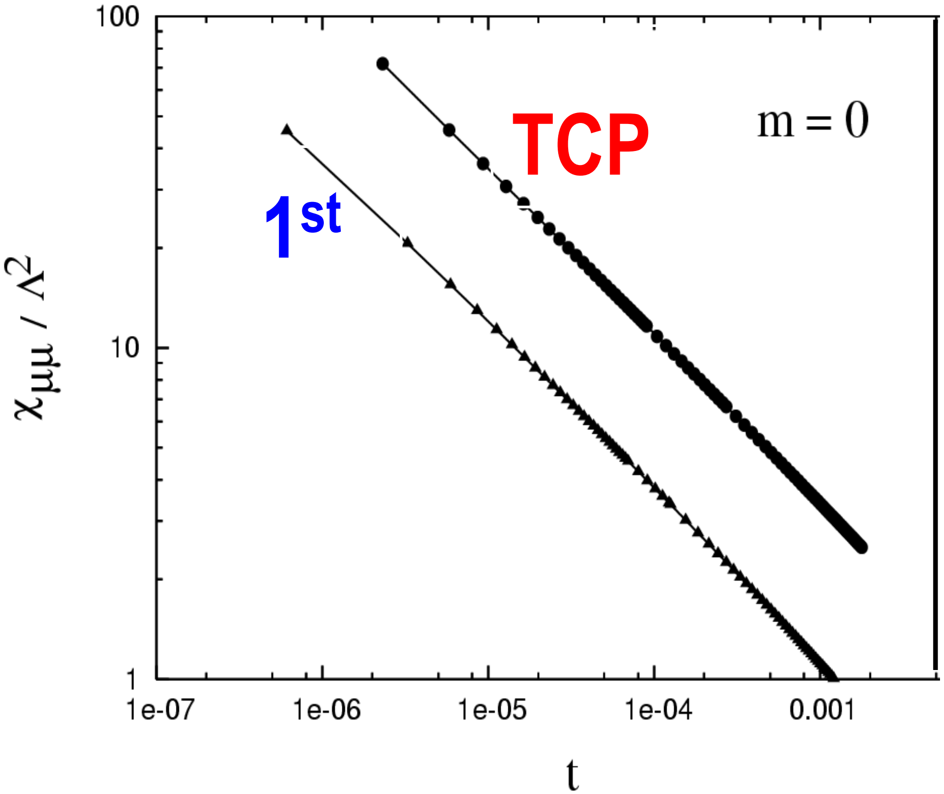
- Cross over region
- CEP and 1st order line

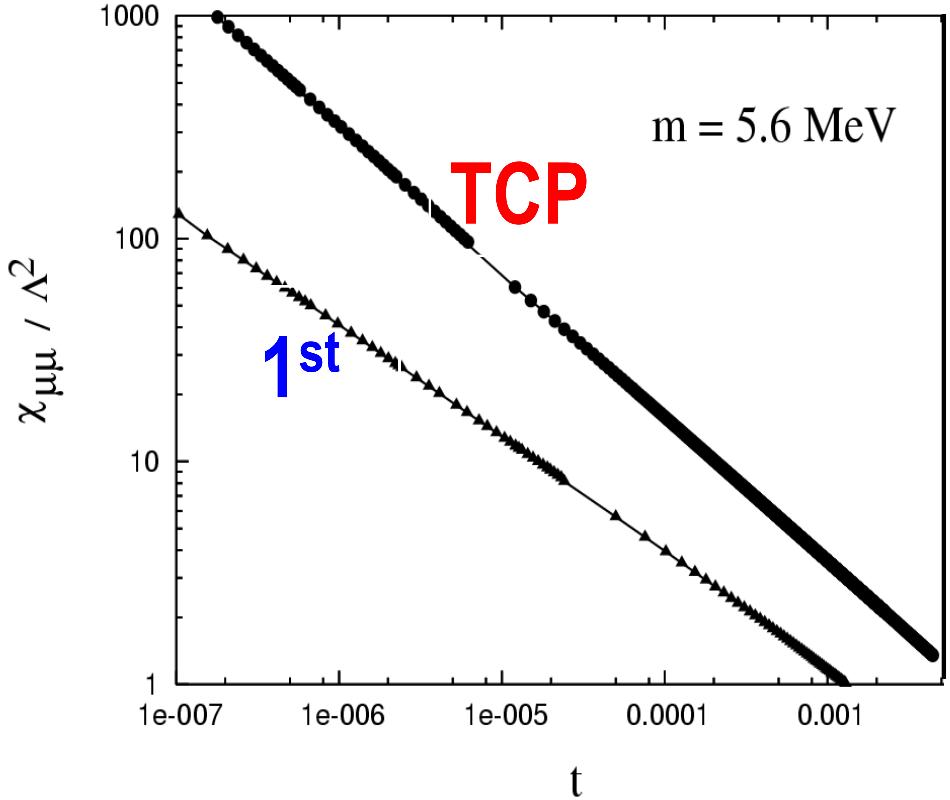
C. Sasaki & K.R.





Critical exponents at 1st order line and CEP





$$\chi_q \sim \left| \frac{\mu - \mu_c}{\mu_c} \right|^{-\gamma}$$ **with** $$\gamma_{m_q=0} = \begin{cases} 1/2 \ \ (0.53) & TCP \\ 1/2 & 1st \end{cases}, \quad \gamma_{m_q \neq 0} = \begin{cases} 2/3 \ \ (0.78) & CEP \\ 1/2 & 1st \end{cases}$$

B. Friman, C. Sasaki & K.R., Phys.Rev.D77:034024,2008.