Strongly Interacting Matter: Phases and Transitions

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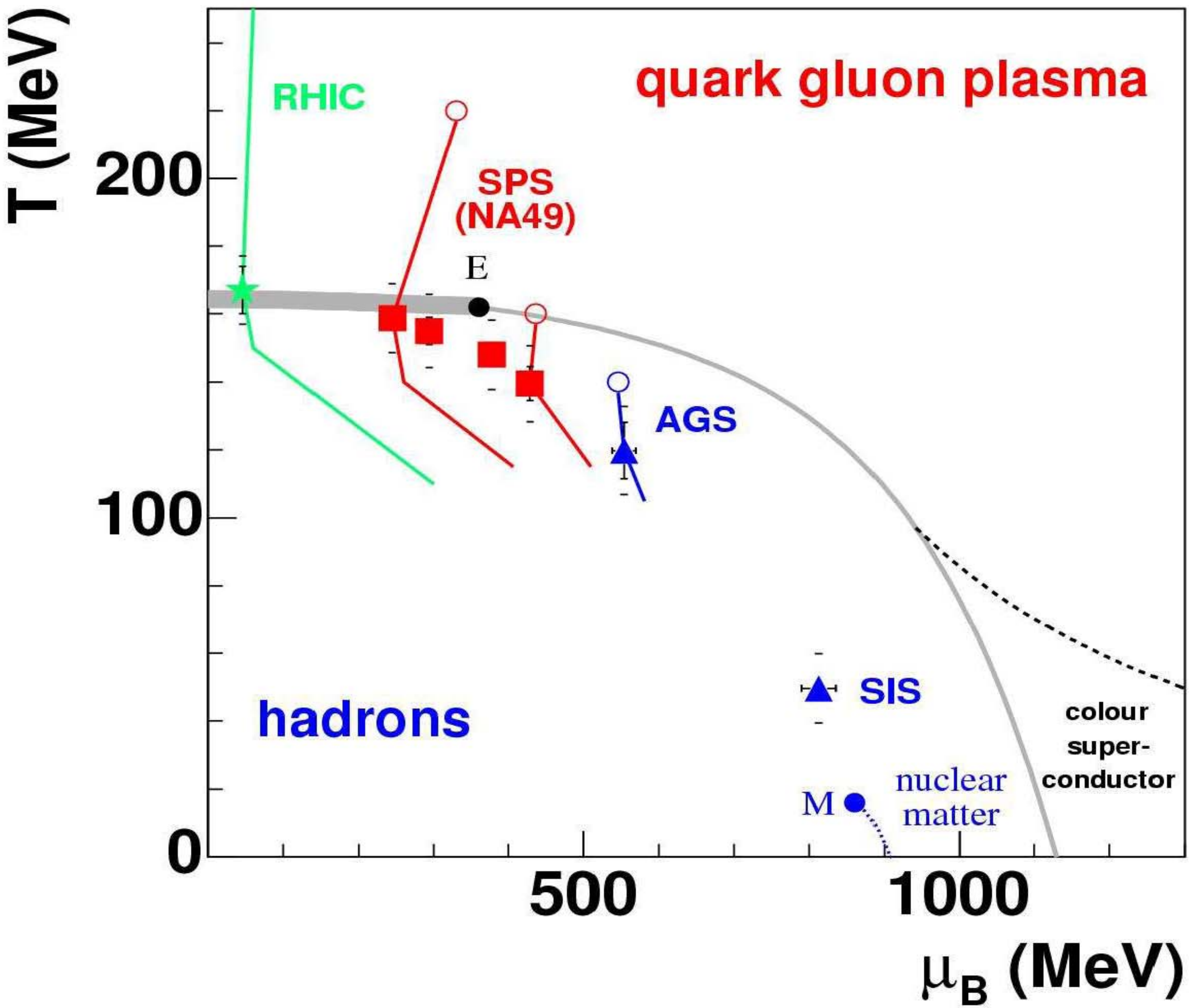
BITP, Kiev

1. Hagedorn Model
2. Phase transitions in the gas of quark-gluon bags
3. 1st order, 2nd order, 3rd order …phase transitions
4. Crossover
5. Phase diagram

M.I.G, Petrov, Zinovjev, Phys. Lett. B 1981

M.I.G., Greiner, Shin Nan Yang, J. Phys. G 1999

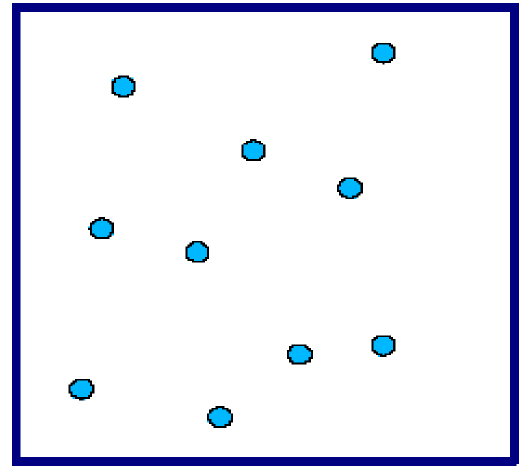
M.I.G., Gazdzicki, Greiner, Phys. Rev. C 2005



T (MeV)
200
100
0
500
1000
μB (MeV)
quark gluon plasma
RHIC
SPS
(NA49)
E
AGS
SIS
hadrons
colour
super-
conductor
nuclear
matter
M

V, T ; m

Partition Function of the Ideal Gas:



$$Z(V,T) = \sum_{N=0}^{\infty} \frac{1}{N!} \prod_{j=1}^{N} \int \frac{V\, d^3k_j}{(2\pi)^3} \times \exp\left[- \frac{(k_j^2 + m^2)^{1/2}}{T} \right]$$

$$= \sum_{N=0}^{\infty} \frac{[V\ \phi(T,m)]^N}{N!} = \exp[V\phi(T,m)]$$

Particle Number Density:

$$\phi(T,m) \equiv \frac{1}{2\pi^2} \int_0^\infty k^2 dk\ \exp\left[- \frac{(k^2 + m^2)^{1/2}}{T} \right]$$

$$= \frac{m^2 T}{2\pi^2} K_2\left(\frac{m}{T}\right)$$

$$\overline{N}(V,T) = V\ \phi(T,m)\ , \qquad n(T) \equiv \frac{N}{V} = \phi(T,m)$$

Pressure:

$$p(T) \equiv T\,\frac{\ln Z(V,T)}{V} = T\,\phi(T,m)$$

Energy Density:

$$\varepsilon(T) \equiv T\,\frac{dp}{dT} - p(T) = T^2\,\frac{d\phi(T,m)}{dT}$$

$$p(T) = T\sum_i \phi(T,m_i)\ , \qquad \varepsilon(T) = T^2\,\sum_i \frac{d\phi(T,m_i)}{dT}$$

$$p(T) = T \int_0^\infty dm\ \rho(m)\ \phi(T,m)$$

$$\varepsilon(T) = T^2 \int_0^\infty dm\ \rho(m)\ \frac{d\phi(T,m)}{dT}$$

Limiting Temperature

Hagedorn (1965), Frautschi (1971) SBM

$$\rho(m)_{m\to\infty} \simeq C\ m^{-a}\ \exp(bm)\ , \qquad b \equiv \frac{1}{T_H}$$

$$\phi(T,m) \simeq \left(\frac{mT}{2\pi}\right)^{3/2} \exp\left(-\frac{m}{T}\right)$$

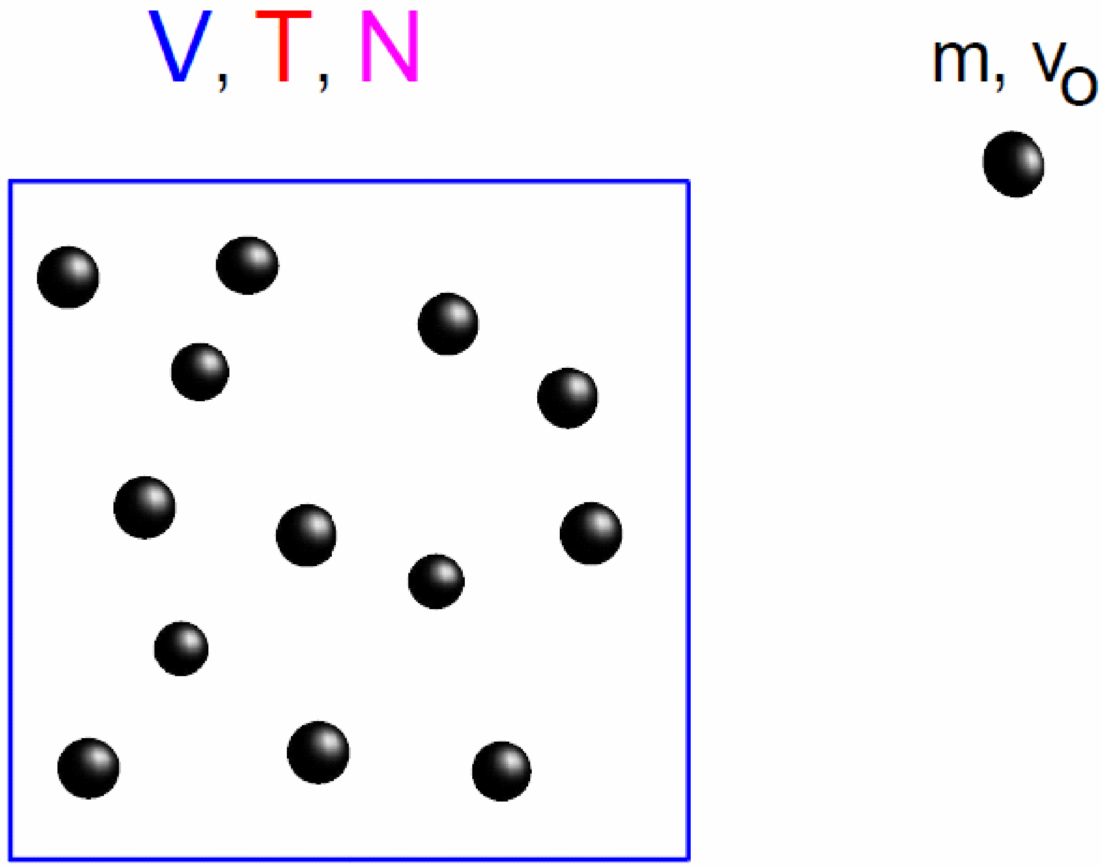
$$T < T_H\ , \qquad T \to T_H :$$

$$p\ ,\ \varepsilon \to \infty\ , \quad a \leq \frac{5}{2}$$

$$p \to const\ ,\ \varepsilon \to \infty\ , \quad \frac{5}{2} \leq a \leq \frac{7}{2}$$

$$p\ ,\ \varepsilon \to const\ , \quad a > \frac{7}{2}$$

van der Waals



$$V \longrightarrow V - N v_o$$

Van der Waals repulsion: $V \to V - v_o N$

$$Z(V,T) = \sum_{N=0}^{\infty} \frac{[(V - v_o N)\ \phi(T,m)]^N}{N!} \theta(V - v_o N)$$

$$\hat{Z}(s,T) \ \equiv \int_0^{\infty} dV \exp(-sV)\ Z(V,T)$$

$$= \sum_{N=0}^{\infty} \frac{[\phi(T,m)]^N}{N!} \int_{v_o N}^{\infty} dV \exp(-sV)(V \ - \ v_o N)^N$$

$$= \sum_{N=0}^{\infty} \frac{[\phi(T,m)]^N}{N!} \ \cdot \ \frac{\exp(-v_o s N)\ N!}{s^{N+1}}$$

$$= [\ s \ - \ \exp(-v_o s)\phi(T,m)]^{-1}$$

$$\hat{Z}(s,T) \equiv \int_0^\infty dV \exp(-sV)\, Z(V,T)$$

Farthest-Right Singularity of the Laplace Transform:

$$Z(V,T)_{V\to\infty} \simeq \exp\left[\frac{p(T)\,V}{T}\right] \quad \to \quad s^*(T) = \frac{p(T)}{T}$$

$$\hat{Z}(s,T) = [\, s - \exp(-v_o s)\phi(T,m)]^{-1}$$

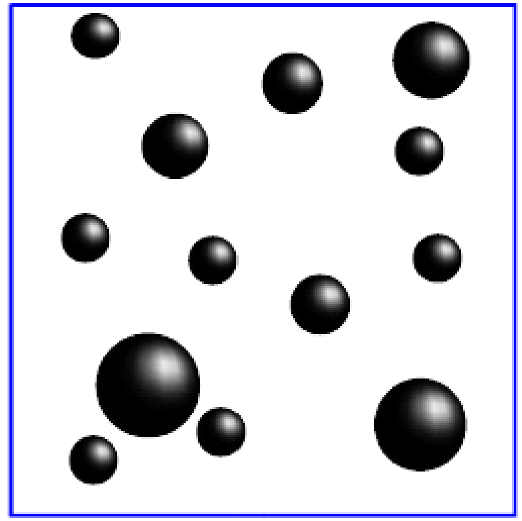
Pole Singularity:

$$s^*(T) = \exp\left[-v_o s^*(T)\right]\, \phi(T,m)$$

$$v_0 = 0$$

$$s^*(T) = \phi(T,m)\,, \qquad p(T) = Ts^*(T) = T\phi(T,m)$$

Multi-Component VdW Gas $m_1, v_1; \ldots; m_n, v_n$



V,T

$$Z(V,T) = \sum_{N_1=0}^{\infty} \ldots \sum_{N_n=0}^{\infty}$$

$$\times \frac{[(V - v_1 N_1 - \ldots - v_n N_n)\,\phi(T, m_1)]^{N_1}}{N_1!} \ldots \times$$

$$\ldots \times \frac{[(V - v_1 N_1 - \ldots - v_n N_n)\,\phi(T, m_n)]^{N_n}}{N_n!}$$

$$\times \; \theta\,(V - v_1 N_1 - \ldots - v_n N_n)$$

$$\sum_{j=1}^{n\to\infty} \ldots \to \int_0^{\infty} dm dv \ldots \rho(m, v)$$

Laplace Transform:

$$\hat{Z}(s,T) \equiv \int_0^\infty dV \exp(-sV)\, Z(V,T)$$

$$= [\, s \; - \; f(T,s)]^{-1}$$

$$f(T,s) \;=\; \int_0^\infty dm dv\; \rho(m,v)\; \exp(-vs)\; \phi(T,m)$$

Pressure: $$p(T) \;=\; T\, s^*(T)$$

Farthest-Right Singularity:

$$s^*(T) \;=\; max\{s_H(T), s_Q(T)\}$$

Pole Singularity:

$$s_H(T) \;=\; f\,(T, s_H(T))$$

Mass-Volume Spectrum of Quark-Gluon Bags

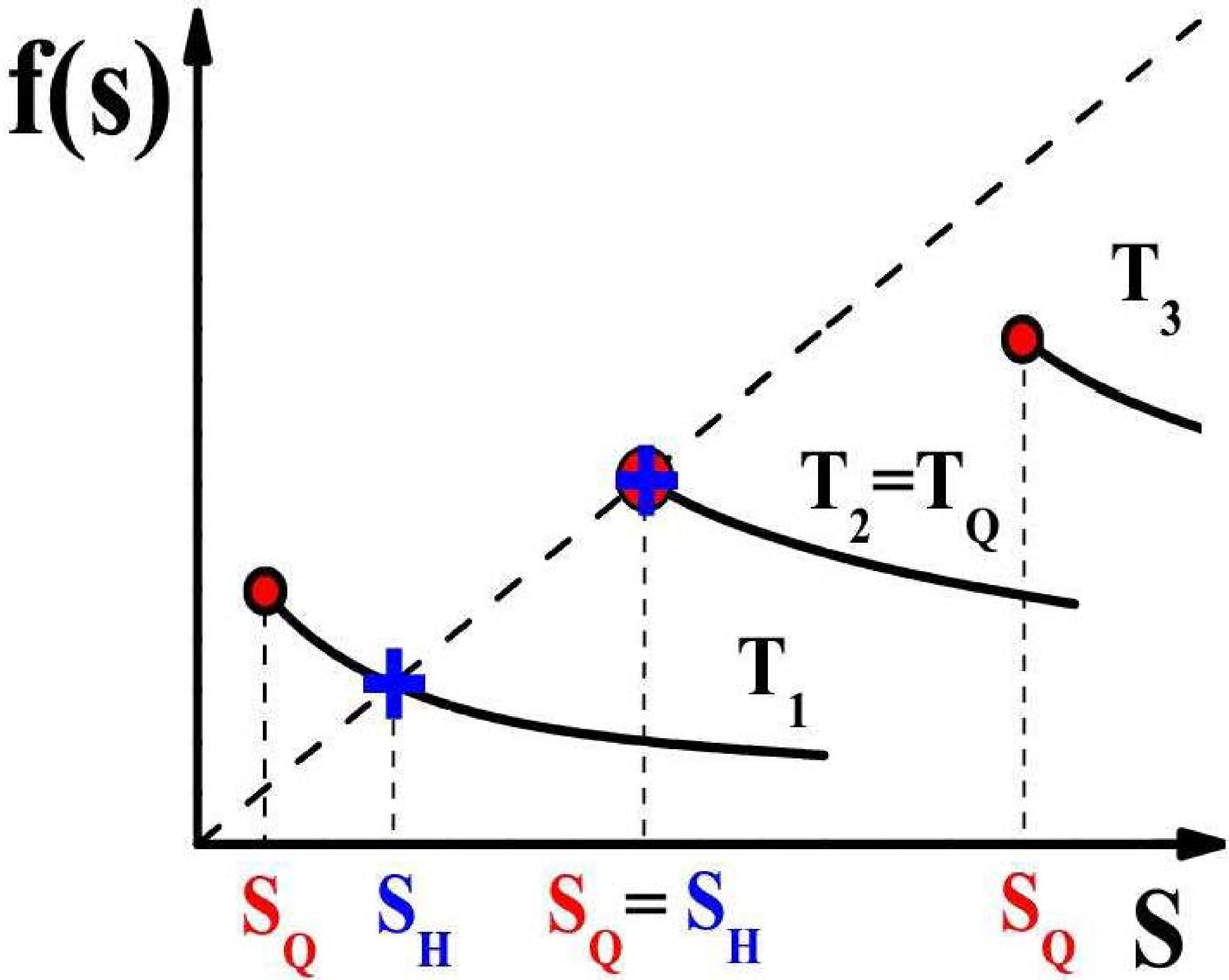
$$\rho(m,v) \simeq C\, v^{\gamma}(m - Bv)^{\delta} \times \exp\left[\frac{4}{3}\,\sigma_Q^{1/4}\, v^{1/4}\,(m - Bv)^{3/4}\right]$$

$$\sigma_Q = \frac{\pi^2}{30}\left(d_g + \frac{7}{8}d_{q\bar{q}}\right) = \frac{\pi^2}{30}\left(2\cdot 8 + \frac{7}{8}\cdot 2\cdot 2\cdot 3\cdot 3\right) = \frac{\pi^2}{30}\,\frac{95}{2}$$

$$f(T,s) \equiv f_H(T,s) + f_Q(T,s) = f_H(T,s)$$

$$+ \int_{V_o}^{\infty} dv \int_{M_o+Bv}^{\infty} dm\; \rho(m,v)\exp(-sv)\phi(T,m)$$

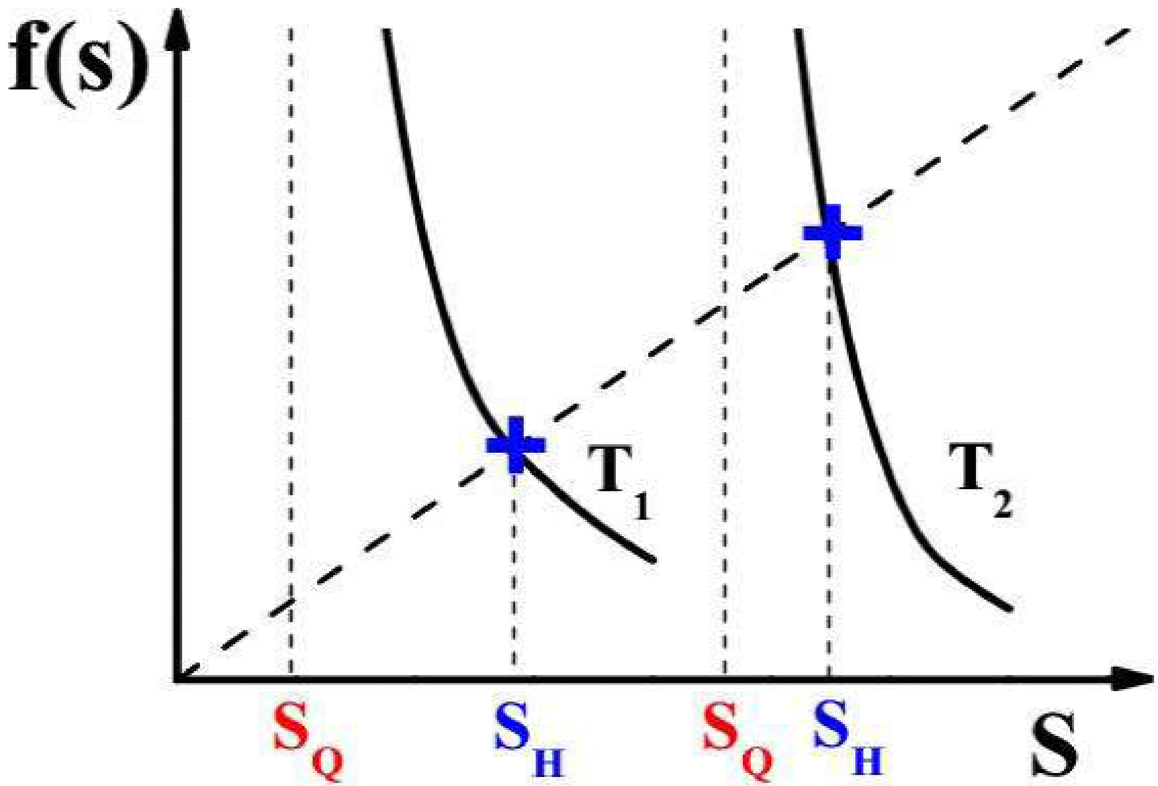
$$\gamma < -\frac{5}{4}, \qquad \delta < -\frac{7}{4}$$



To have $s^* = s_Q$ at high T one needs

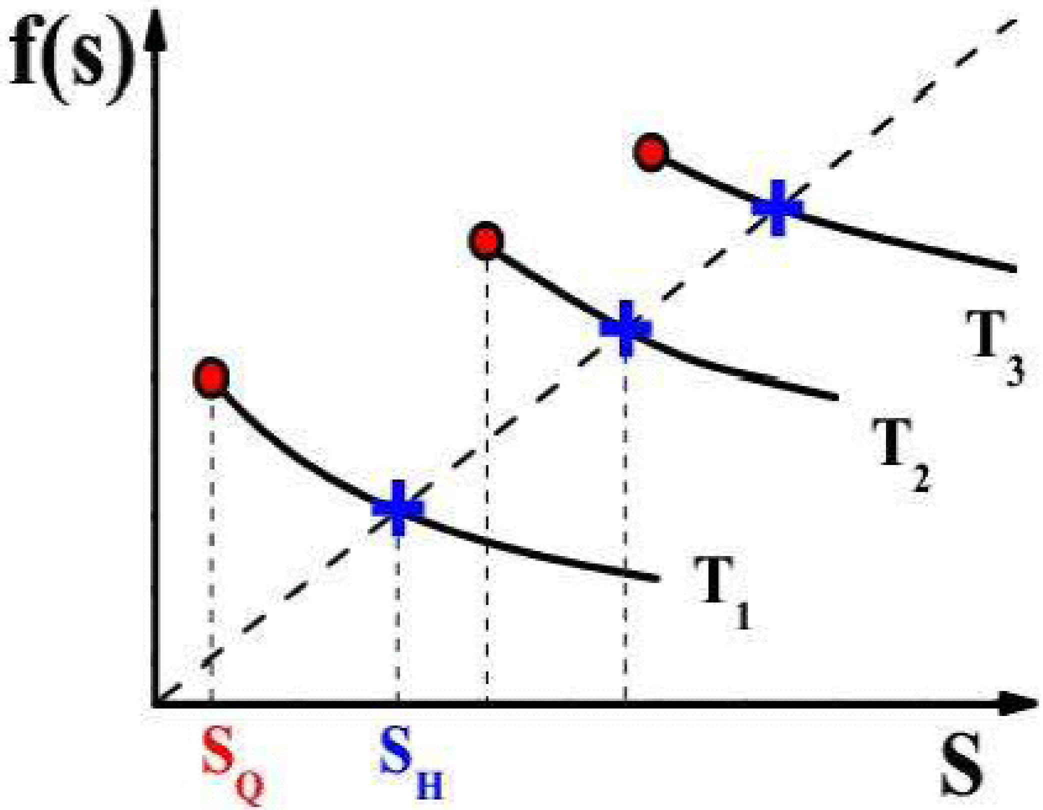
$$\underline{\gamma + \delta < -3},$$

otherwise $f(T, s_Q) = \infty$, and $s_H > s_Q$ for all T:

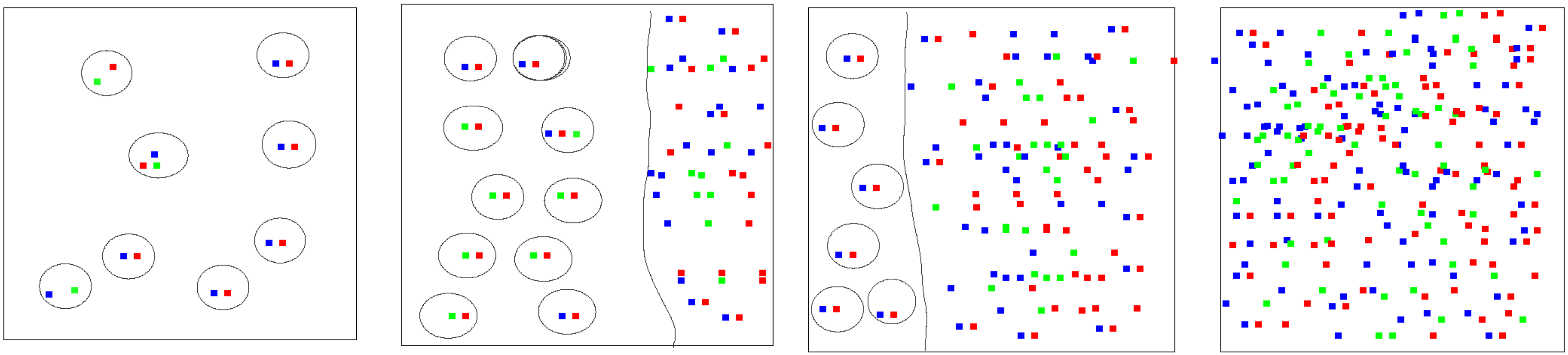


$$\underline{\delta < -\frac{7}{4}},$$

otherwise $f(T, s_Q) > s_Q$, and $s_H > s_Q$ for all T:



T < Tc | T = Tc | T=Tc | T > Tc



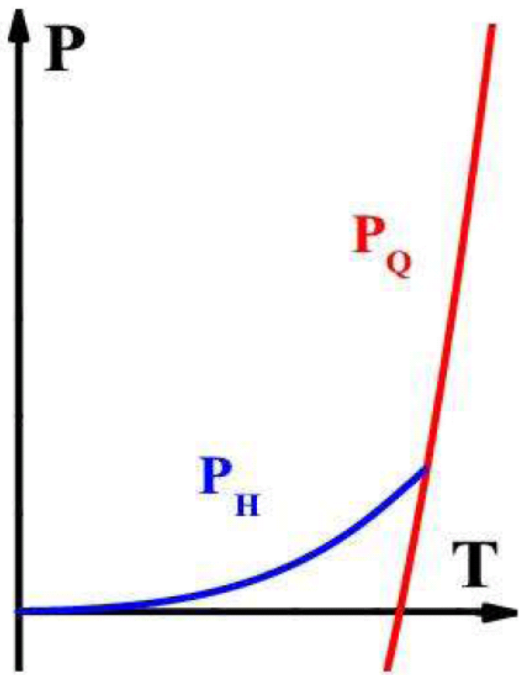
$$\alpha \equiv -(\gamma + \delta + 2)$$

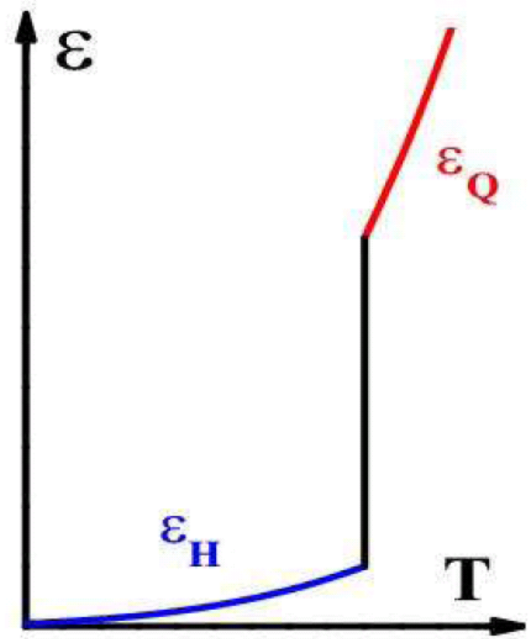
$$\alpha > 2$$

1^{st} Order PT

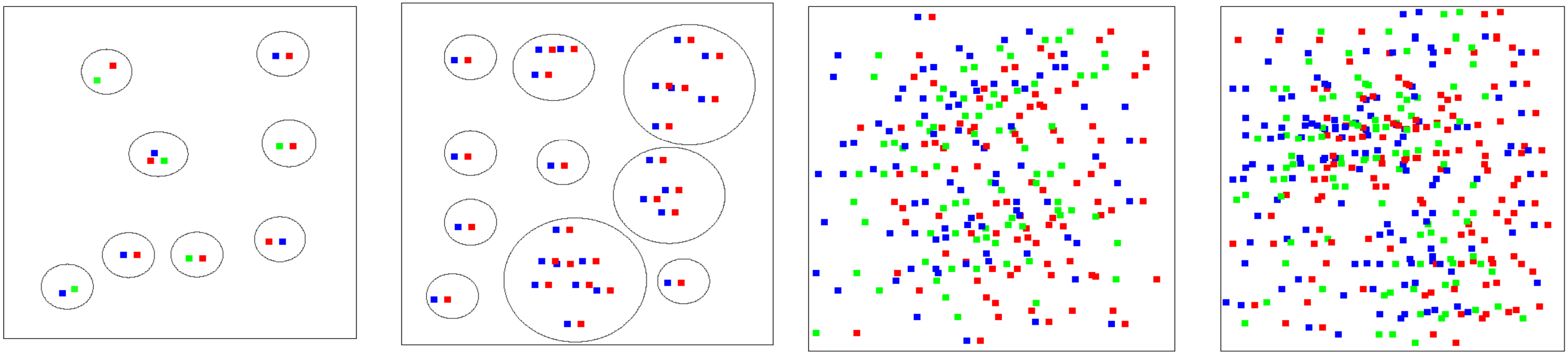
$$s_H(T_C) = s_Q(T_C)$$

$$s_H'(T_C) < s_Q'(T_C)$$





T < Tc | T ---> Tc | T = Tc | T > Tc



$$\alpha \equiv -(\gamma + \delta + 2)$$

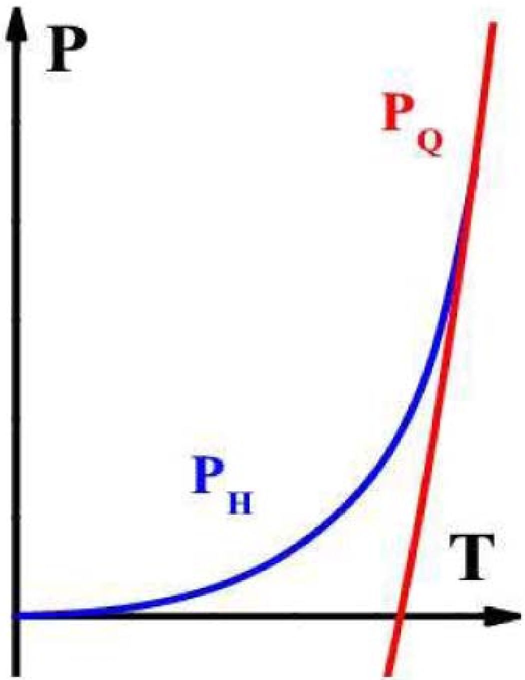
$$\frac{3}{2} < \alpha \leq 2$$

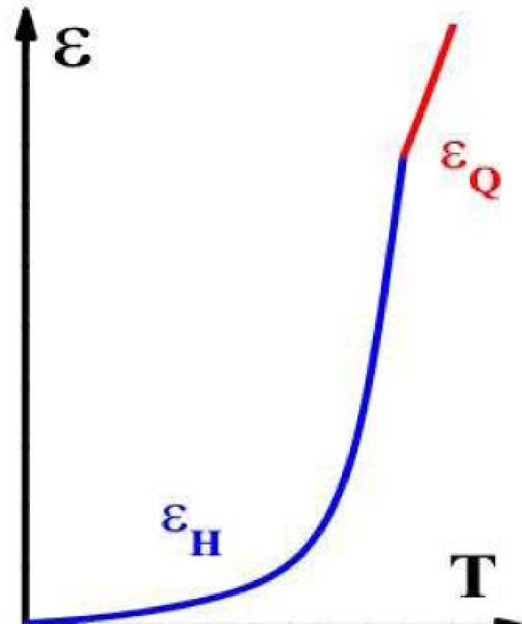
2^{nd} Order PT

$$s_H(T_C) = s_Q(T_C)$$

$$s_H'(T_C) = s_Q'(T_C)$$

$$s_H''(T_C) > s_Q''(T_C)$$

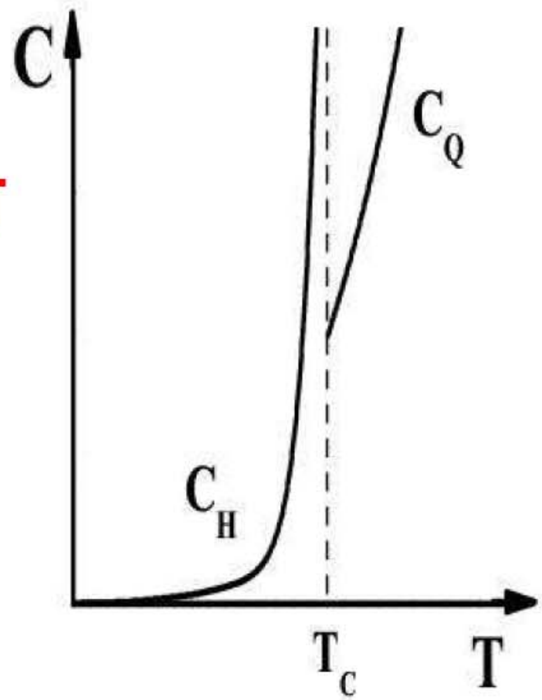




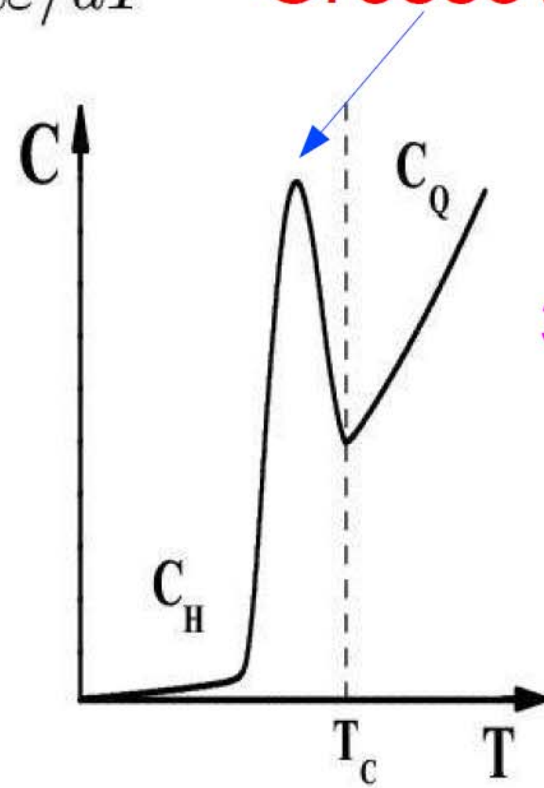
For $\underline{4/3 \leq \alpha < 3/2}$ there is the 3^{rd} order PT

The specific heat $C \equiv d\varepsilon/dT$

2nd Order PT



Crossover



3rd Order PT

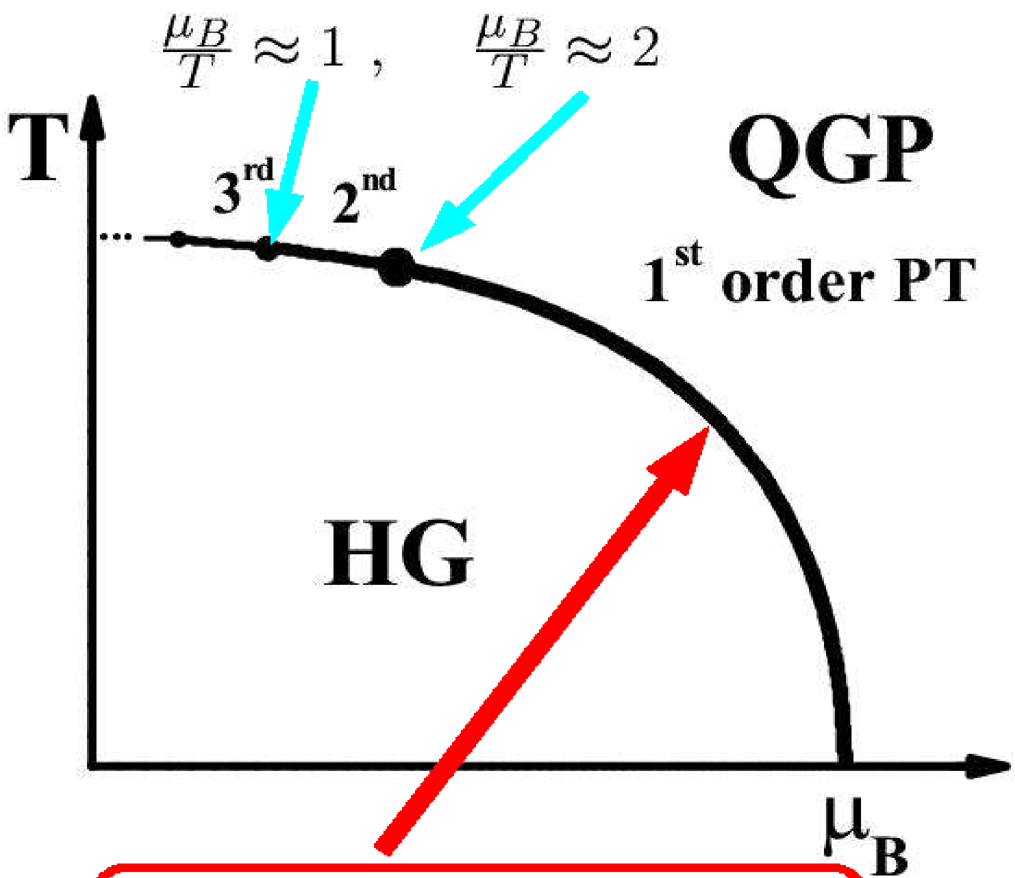
$$\frac{n+1}{n} < \alpha < \frac{n}{n-1}$$

n-th Order PT

$$\alpha > 2\,, \quad \frac{3}{2} \le \alpha \le 2\,, \quad 1 < \alpha \le \frac{3}{2}$$

1st Order PT   2nd Order PT   3rd and Higher Order PTs

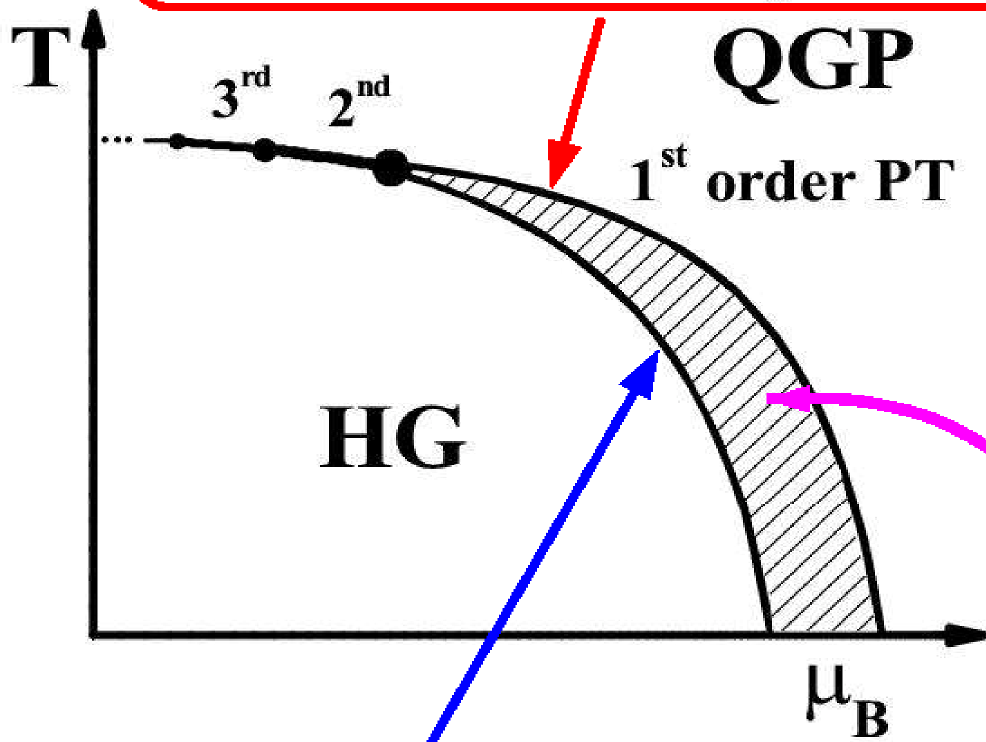
$$\alpha = \alpha_0 + \alpha_1 \frac{\mu_B}{T}\,, \quad \alpha_0 = 1 + \epsilon\,, \quad \alpha_1 \approx 0.5$$



$$s_H(T, \mu_B) = s_Q(T, \mu_B)$$

Strangeness

$$s_Q(T,\mu_B,\mu_S^Q) = s_H(T,\mu_B,\mu_S^Q)$$
$$\mu_S^Q = \mu_B/3$$



Strangeness Separation in the Mixed Phase

$$s_H(T,\mu_B,\mu_S^H) = s_Q(T,\mu_B,\mu_S^H)$$

$$s_H(T,\mu_B,\mu_S) = s_Q(T,\mu_B,\mu_S)$$
$$n_S^{mix} \equiv \delta \cdot n_S^Q + (1-\delta) \cdot n_S^H$$

Carsten Greiner et al. (1987)

Summary

1. Phase Transitions in the gas of quark-gluon bags

2. 1st Order PT, 2nd Order PT, 3rd Order PT,… -- different contributions of massive Q-G bags at T=Tc

3. No phase transitions – ‘cluster’ structure of the QGP

4. ‘Critical line’ of the Phase Transitions in the T-mu_B plane

5. Massive Q-G bags can be observed by measuring the event-by-event fluctuations