Jet quenching in glasma

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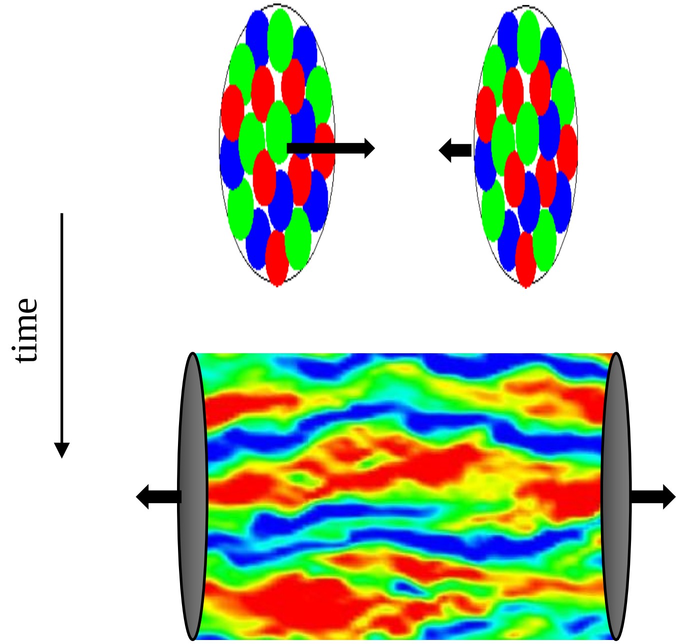
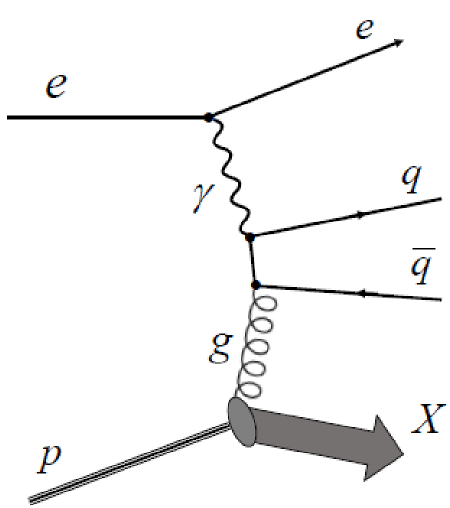
Glasma & Color Glass Condensate

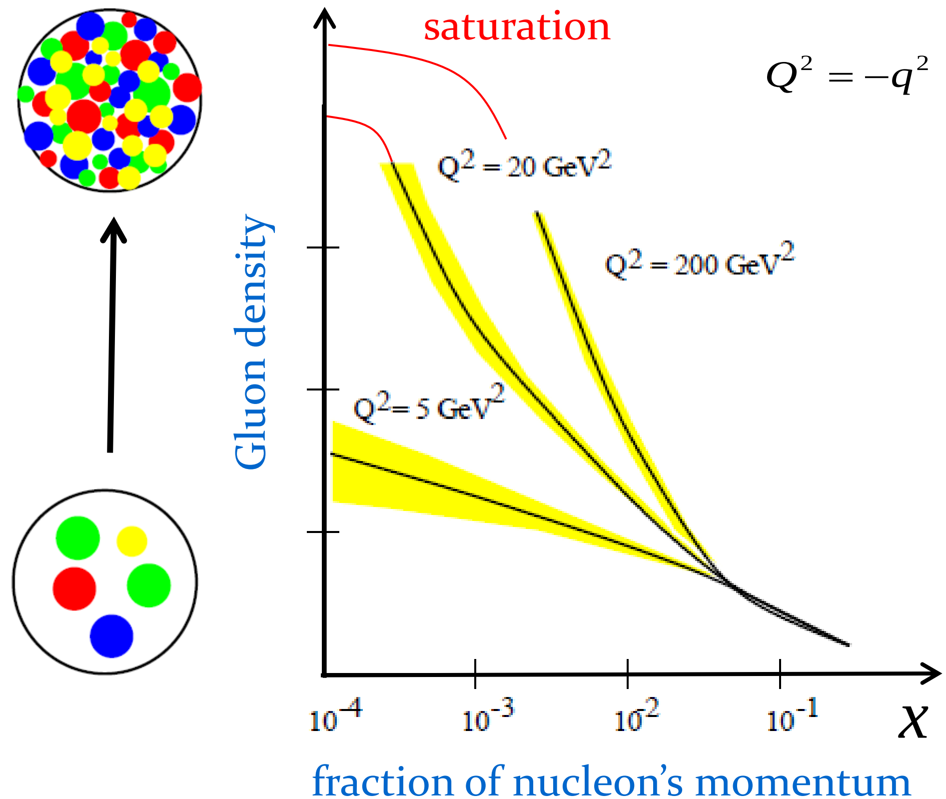
Color charges confined in the colliding nuclei generate **glasma** – a system of strong mostly classical chromodynamic fields which evolve towards equilibrium.



Saturation

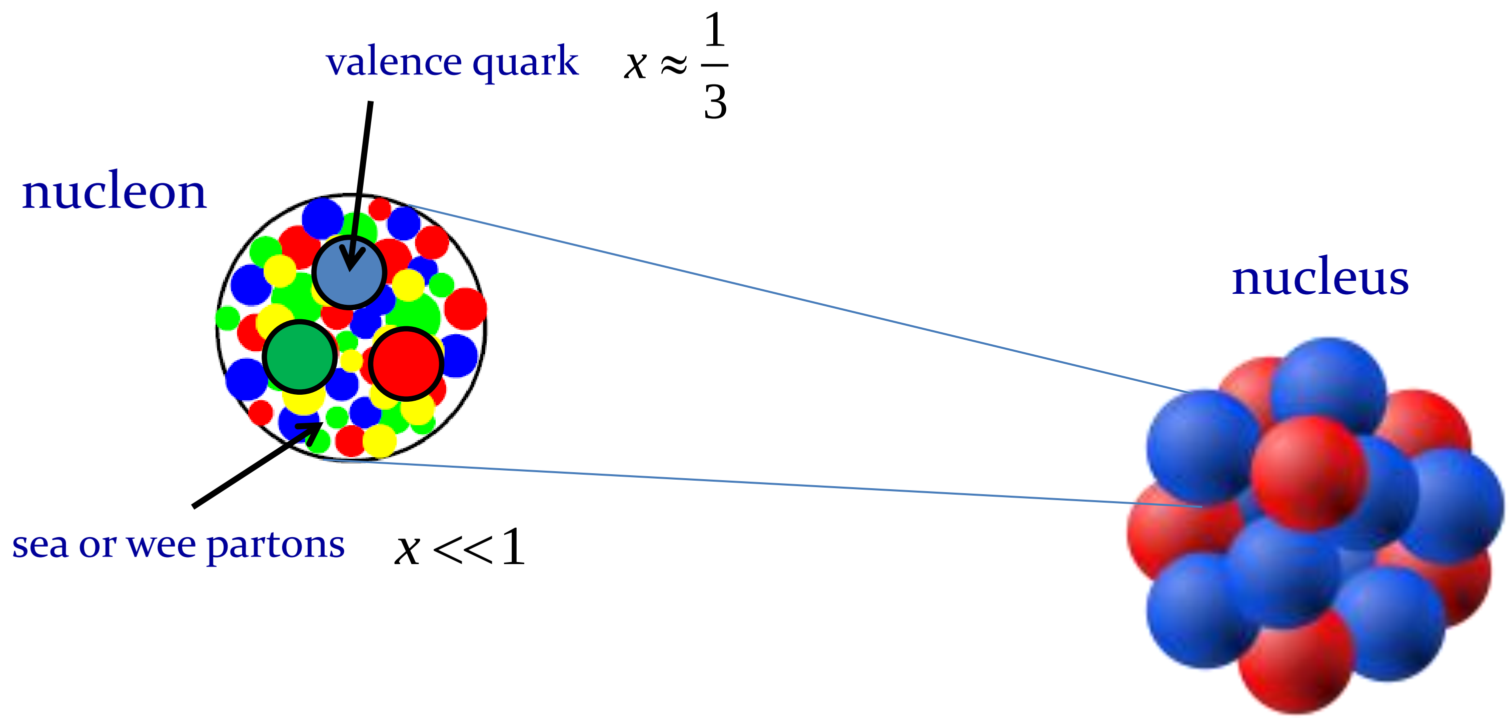
Deep Inelastic Scattering





Saturated gluon system can be described in terms of classical chromodynamic fields.

Wee partons & valence quarks



<u>In relativistic heavy-ion collsions</u>

- Saturated wee partons – classical chromodynamic fields
- Valence quarks – classical sources of chromodynamic fields
- Saturation scale for A-A at LHC: $Q_s \approx 2\,\text{GeV} >> \Lambda_{\text{QCD}} \approx 0.2\,\text{GeV} \Rightarrow \alpha_s(Q_s) << 1$

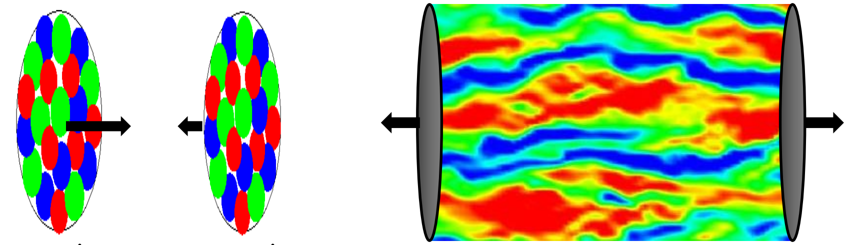
Color Glass Condensate

Classical Yang-Mills equation

$$\boxed{D_\mu F^{\mu\nu}(x) = j^\nu(x)}$$

$$j^\mu(x) = j_1^\mu(x) + j_2^\mu(x)$$

$$j_{1,2}^\mu(x) = \pm\delta^{\mu\mp}\delta(x^\pm)\rho_{1,2}(\mathbf{x}_\perp)$$



β_1^i β_2^i

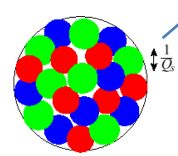
pre-collision potentials

$\alpha, \alpha_\perp^i$

post-collision potentials

Ansatz of gauge potentials

$$\begin{cases} A^+(x) = \Theta(x^+)\Theta(x^-)x^+\alpha(\tau, \mathbf{x}_\perp) \\ A^-(x) = -\Theta(x^+)\Theta(x^-)x^-\alpha(\tau, \mathbf{x}_\perp) \\ A^i(x) = \Theta(x^+)\Theta(x^-)\alpha_\perp^i(\tau, \mathbf{x}_\perp) \\ \qquad + \Theta(-x^+)\Theta(x^-)\beta_1^i(\mathbf{x}_\perp) + \Theta(x^+)\Theta(-x^-)\beta_2^i(\mathbf{x}_\perp) \end{cases}$$



Boundary condition

$$\begin{cases} \alpha(0, \mathbf{x}_\perp) = \beta_1^i(\mathbf{x}_\perp) + \beta_2^i(\mathbf{x}_\perp) \\ \alpha_\perp^i(0, \mathbf{x}_\perp) = -\dfrac{ig}{2}[\beta_1^i(\mathbf{x}_\perp), \beta_2^i(\mathbf{x}_\perp)] \end{cases}$$

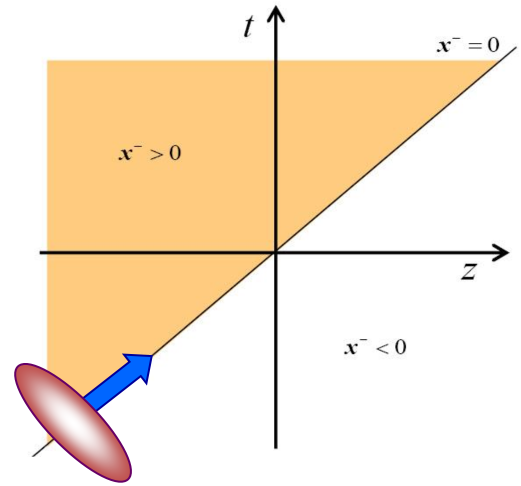
Gauge condition

$$x^+A^- + x^-A^+ = 0$$

E. Iancu, R. Venugopalan, in *Quark-Gluon Plasma 3*,
ed. by R.C. Hwa, X.-N. Wang (World Scientific, Singapore, 2004

Pre-collision potentials

$$\boxed{j^\mu(x^-,\mathbf{x}_\perp) = \delta^{\mu +}\delta(x^-)\rho(\mathbf{x}_\perp)}$$



Gauge condition: $A^i(x^-,\mathbf{x}_\perp) = 0$

$$D_\mu F^{\mu\nu} = j^\nu \quad \Rightarrow \quad \begin{cases} A^-(x^-,\mathbf{x}_\perp) = 0 \\ A^+(x^-,\mathbf{x}_\perp) = \delta(x^-)\Lambda(\mathbf{x}_\perp) \end{cases}$$

$$E^z = B^z = 0$$

$$\mathbf{E}_\perp, \mathbf{B}_\perp \sim \delta(x^-)$$

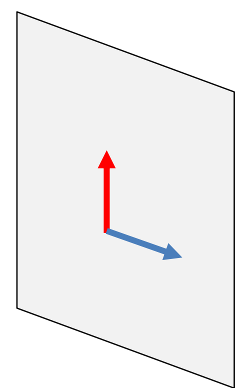
Poisson equation

$$\boxed{\nabla_\perp^2 \Lambda(\mathbf{x}_\perp) = -\rho(\mathbf{x}_\perp)}$$

$$\Lambda(\mathbf{x}_\perp) = \int d^2x'_\perp\, G(\mathbf{x}_\perp - \mathbf{x}'_\perp)\rho(\mathbf{x}'_\perp)$$

$$G(\mathbf{x}_\perp) = \int \frac{d^2k_\perp}{(2\pi)^2}\frac{e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp}}{\mathbf{k}_\perp^2 + m^2} = \frac{1}{2\pi}K_0(m|\mathbf{x}_\perp|)$$

IR regulator $m = \Lambda_{\rm QCD}$



$$x^- = 0$$

Proper time expansion

$$\alpha(\tau,\mathbf{x}_\perp)=\sum_{n=0}^{\infty}\tau^n\alpha_{(n)}(\mathbf{x}_\perp),\qquad \alpha_\perp^i(\tau,\mathbf{x}_\perp)=\sum_{n=0}^{\infty}\tau^n\alpha_{\perp(n)}^i(\mathbf{x}_\perp)$$

Proper time τ is treated as a small parameter $\tau << Q_s^{-1}$

Yang-Mills equations for the expanded potentials are solved recursively

$$\alpha_{(n)}=\alpha_{\perp(n)}^i=0 \quad \text{for} \quad n=1,3,5,\ldots$$

0th order - boundary conditions

$$\begin{cases}\alpha_{(0)}=-\dfrac{ig}{2}[\beta_1^i,\beta_2^i]\\ \alpha_{\perp(0)}^i=\beta_1^i+\beta_2^i\end{cases}$$

Post-collision potentials are expressed through pre-collision potentials

2nd order

$$\begin{cases}\alpha_{(2)}=-\dfrac{ig}{16}[D^j,[D^j,[\beta_1^i,\beta_2^i]]]\\ \alpha_{\perp(2)}^i=\dfrac{ig}{4}\varepsilon^{zij}\varepsilon^{zkl}[D^j,[\beta_1^k,\beta_2^l]]\end{cases}$$

$$D^i\equiv\partial^i-ig(\beta_1^i+\beta_2^i)$$

Fully analytic approach!

R. J. Fries, J. I. Kapusta, and Y. Li, arXiv:nucl-th/0604054
G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Proper time expansion cont.

Chromoelectric and chromomagnetic fields

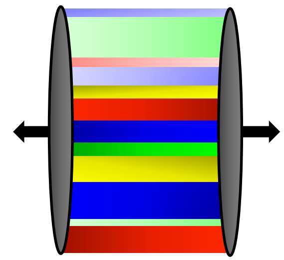
$$E^i = F^{i0}, \quad B^i = \frac{1}{2}\varepsilon^{ijk}F^{kj}$$

oth order

$$\mathbf{E}_{(0)} = (0,0,E), \qquad \mathbf{B}_{(0)} = (0,0,B)$$

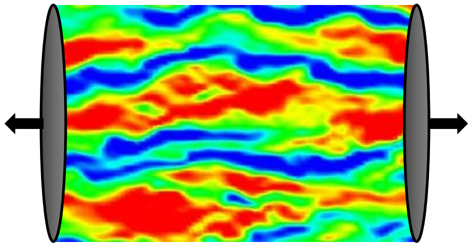
$$E^z_{(0)}(\mathbf{x}_\perp) = ig[\beta^i_1(\mathbf{x}_\perp), \beta^i_2(\mathbf{x}_\perp)]$$

$$B^z_{(0)}(\mathbf{x}_\perp) = ig\varepsilon^{zij}[\beta^i_1(\mathbf{x}_\perp), \beta^j_2(\mathbf{x}_\perp)]$$



E & *B* fields along the axis *z*

At higher orders transverse fields show up



R. J. Fries, J. I. Kapusta, and Y. Li, arXiv:nucl-th/0604054
G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Field correlators

The correlators e.g.

$$\left\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle, \quad \left\langle E_a^i(t,\mathbf{r})B_b^j(t',\mathbf{r}')\right\rangle, \quad \left\langle B_a^i(t,\mathbf{r})B_b^j(t',\mathbf{r}')\right\rangle$$

are expressed through

$$\sum \partial^i \partial^j \left\langle \beta_a^k(\mathbf{x}_\perp)\beta_b^l(\mathbf{y}_\perp)\ldots\beta_c^m(\mathbf{z}_\perp)\right\rangle$$

IR regulator $m = \Lambda_{\mathrm{QCD}}$

In covariant gauge $\partial_\mu \beta^\mu = 0$

$$-\nabla^2 \beta^+(\mathbf{x}_\perp) = \rho(\mathbf{x}_\perp) \quad \Rightarrow \quad \beta^+(\mathbf{x}_\perp) = \frac{1}{2\pi}\int d^2x'_\perp K_0(m|\mathbf{x}_\perp - \mathbf{x}'_\perp|)\rho(\mathbf{x}'_\perp)$$

The potentials are transformed from covariant to light-cone gauge

Wick theorem

$$\left\langle \rho_a^k(\mathbf{x}_\perp)\rho_b^l(\mathbf{y}_\perp)\ldots\rho_c^m(\mathbf{z}_\perp)\right\rangle = \sum\prod\left\langle \rho_a^i(\mathbf{x}_\perp)\rho_b^j(\mathbf{y}_\perp)\right\rangle$$

Glasma graph approximation

$$\left\langle \beta_a^k(\mathbf{x}_\perp)\beta_b^l(\mathbf{y}_\perp)\ldots\beta_c^m(\mathbf{z}_\perp)\right\rangle = \sum\prod\left\langle \beta_a^i(\mathbf{x}_\perp)\beta_b^j(\mathbf{y}_\perp)\right\rangle = \sum\prod B_{ab}^{ij}(\mathbf{x}_\perp - \mathbf{y}_\perp)$$

Basic two-point correlator

$$B_{ab}^{ij}(\mathbf{x}_\perp,\mathbf{y}_\perp) \equiv \left\langle \beta_a^i(\mathbf{x}_\perp)\beta_b^j(\mathbf{y}_\perp)\right\rangle = \int d^2x'_\perp\, d^2y'_\perp \cdots\cdots \left\langle \rho_a^i(\mathbf{x}'_\perp)\rho_b^j(\mathbf{y}'_\perp)\right\rangle$$

$$\left\langle \rho_a^i(\mathbf{x}_\perp)\rho_b^j(\mathbf{y}_\perp)\right\rangle = g^2\mu\,\delta^{ab}\delta^{(2)}(\mathbf{x}_\perp-\mathbf{y}_\perp)$$

color charge surface density

$$\mu = g^{-4}Q_s^2$$

$$\boxed{B_{ab}^{ij}(\mathbf{x}_\perp,\mathbf{y}_\perp) \equiv \delta^{ab}\left(\delta^{ij}C_1(r) - \hat{r}^i\hat{r}^j C_2(r)\right)}$$

$$\mathbf{r} \equiv \mathbf{x}_\perp - \mathbf{y}_\perp, \quad r \equiv |\mathbf{r}|, \quad \hat{r}^i \equiv \frac{r^i}{r}$$

$$\begin{cases} C_1(r) \equiv \dfrac{m^2K_0(mr)}{g^2N_c\left(mrK_1(mr)-1\right)}\left\{\exp\left[\dfrac{g^4N_c\mu\left(mrK_1(mr)-1\right)}{4\pi m^2}\right]-1\right\} \\[2ex] C_2(r) \equiv \dfrac{m^3rK_1(mr)}{g^2N_c\left(mrK_1(mr)-1\right)}\left\{\exp\left[\dfrac{g^4N_c\mu\left(mrK_1(mr)-1\right)}{4\pi m^2}\right]-1\right\} \end{cases}$$

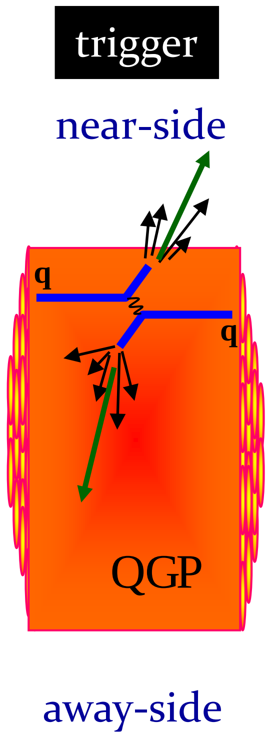
$$\underset{r<<m^{-1}}{\approx} \quad \#\log(mr)$$

UV regularization required

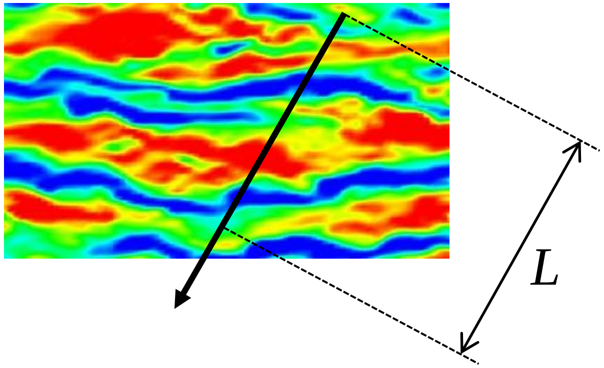
$$r > Q_s^{-1}$$

J. Jalilian-Marian, A. Kovner, L. D. McLerran and H. Weigert, Physical Review D **55**, 5414 (1997)

Jet quenching in glasma



How hard probes propagate through the glasma?



$$\frac{dE}{dx}$$ - collisional energy loss

$$\hat{q}$$ - transverse momentum broadening

$$\frac{dE^{\mathrm{rad}}}{dx} = -\frac{1}{8}\alpha_s N_c \hat{q} L$$ - radiative energy loss

Fokker-Planck Equation

- Transport of hard probes can be described using the Fokker-Planck equation.

$$\underbrace{\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)}_{\text{drift}} n(t,\mathbf{r},\mathbf{p}) = \underbrace{\left(\nabla_p^i X^{ij}(\mathbf{v})\nabla_p^j + \nabla_p^i Y^i(\mathbf{v})\right)}_{\text{collisions}} n(t,\mathbf{r},\mathbf{p})$$

$n(t,\mathbf{r},\mathbf{p})$ - distribution function of hard probes $\quad \mathrm{v}^i \equiv \dfrac{p^i}{E_p}, \quad \nabla_p^i \equiv \dfrac{\partial}{\partial p^i}$

$$X^{ij}(\mathbf{v}),\ Y^i(\mathbf{v}) \Rightarrow \begin{cases} \dfrac{dE}{dx} = -\dfrac{\mathrm{v}^i}{\mathrm{v}} Y^i(\mathbf{v}) & \text{collisional energy loss} \\ \hat{q} = \dfrac{2}{\mathrm{v}}\left(\delta^{ij} - \dfrac{\mathrm{v}^i \mathrm{v}^j}{\mathrm{v}^2}\right) X^{ji}(\mathbf{v}) & \text{momentum broadening} \end{cases}$$

$$n(t,\mathbf{r},\mathbf{p}) = n_{\mathrm{eq}}(\mathbf{p}) \sim e^{-\frac{E_{\mathbf{p}}}{T}} \quad \Leftrightarrow \quad \boxed{Y^j(\mathbf{v}) = \frac{\mathrm{v}^i}{T} X^{ij}(\mathbf{v})}$$

solves FP equation

Origin of Fokker-Planck Equation

Boltzmann equation

Diffusive approximation

$$p >> \Delta p$$

p – momentum of a hard probe

Δp – momentum transfer in a single collision

Fokker-Planck equation

- How to obtain a Fokker-Planck equation for glasma?

  Apply a *quasilinear* method known in plasma physics.

Derivation of Fokker-Planck Equation

The dynamics is assumed to be dominated by strong classical fields.

Liouville (Vlasov) equation

$$\boxed{p_\mu D^\mu Q(t,\mathbf{r},\mathbf{p}) - \frac{g}{2}\, p^\mu \{F_{\mu\nu}(t,\mathbf{r}), \partial_p^\nu Q(t,\mathbf{r},\mathbf{p})\} = 0}$$

free streaming

mean-field force

$Q(t,\mathbf{r},\mathbf{p})$ - exact distribution function of hard probes which is a $N_c \times N_c$ matrix

$$D^\mu \equiv \partial^\mu - ig[A^\mu, ...]$$

chromodynamic strength tensor

$$\{A, B\} \equiv AB + BA, \qquad F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$$

St. Mrówczyński, European Physical Journal A **54**, 43 (2018)

Derivation of Fokker-Planck Equation

Regular and fluctuating quantities

fluctuating part

$$Q(t,\mathbf{r},\mathbf{p}) = \langle Q(t,\mathbf{r},\mathbf{p})\rangle + \delta Q\,(t,\mathbf{r},\mathbf{p})$$

regular colorless part

$$\langle Q(t,\mathbf{r},\mathbf{p})\rangle = n(t,\mathbf{r},\mathbf{p})I$$

$n(t,\mathbf{r},\mathbf{p})$ - averaged distribution function

- $$|\,n\,| >> |\,\delta Q\,|, \quad |\,\nabla_p n\,| >> |\,\nabla_p \delta Q\,|$$
- $$|\frac{\partial n}{\partial t}| << |\frac{\partial \delta Q}{\partial t}|, \qquad |\,\nabla n\,| << |\,\nabla \delta Q\,|$$
- $$\langle \mathbf{E}\rangle = 0, \quad \langle \mathbf{B}\rangle = 0, \quad \mathbf{E}, \mathbf{B}, A^{\mu} \sim \delta Q$$

Derivation of Fokker-Planck Equation

$$Q(t,\mathbf{r},\mathbf{p}) = n(t,\mathbf{r},\mathbf{p})I + \delta Q\,(t,\mathbf{r},\mathbf{p})$$

Lorentz force

$$\mathbf{F} \equiv g\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right)$$

Liouville (Vlasov) equation

$$\left(D^0 + \mathbf{v}\cdot\mathbf{D}\right)Q - \frac{1}{2}\nabla_p\left\{\mathbf{F},Q\right\} = 0$$

ensemble averaging $\Big\Downarrow$ $\mathrm{Tr}\langle\ldots\rangle$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)n(t,\mathbf{r},\mathbf{p}) = \overbrace{\frac{1}{N_c}\mathrm{Tr}\left\langle\mathbf{F}(t,\mathbf{r})\cdot\nabla_p\delta Q(t,\mathbf{r},\mathbf{p})\right\rangle}^{\text{collision term}}$$

Fluctuations provide a collision term.

Derivation of Fokker-Planck Equation

How to compute the collision term?

$$C \equiv \frac{1}{N_c} \mathrm{Tr} \left\langle \mathbf{F} \cdot \nabla_p \delta Q \right\rangle = ?$$

$$Q(t, \mathbf{r}, \mathbf{p}) = n(t, \mathbf{r}, \mathbf{p}) I + \delta Q\,(t, \mathbf{r}, \mathbf{p})$$

Liouville (Vlasov) equation

$$\left(D^0 + \mathbf{v} \cdot \mathbf{D} \right) Q - \frac{1}{2} \nabla_p \left\{ \mathbf{F}, Q \right\} = 0$$

$$n \;\; >> \;\; |\delta Q|$$
$$|\nabla_p n| >> |\nabla_p \delta Q|$$

linearization

$$\left|\frac{\partial n}{\partial t}\right| << \left|\frac{\partial \delta Q}{\partial t}\right|$$
$$|\nabla n| << |\nabla \delta Q|$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \delta Q = \mathbf{F} \cdot \nabla_p n$$

Derivation of Fokker-Planck Equation

Solution of the linearized transport equation

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)\delta Q(t,\mathbf{r},\mathbf{p})=\mathbf{F}(t,\mathbf{r})\cdot\nabla_p n(\mathbf{p})$$

initial value

$$\delta Q(t,\mathbf{r},\mathbf{p})=\int_0^t dt'\,\mathbf{F}(t',\mathbf{r}-\mathbf{v}(t-t'))\cdot\nabla_p n(\mathbf{p})+\delta Q_0(\mathbf{r}-\mathbf{v}t,\mathbf{p})$$

$$\Downarrow$$

$$C\equiv\frac{1}{N_c}\mathrm{Tr}\left\langle\mathbf{F}(t,\mathbf{r})\cdot\nabla_p\delta Q(t,\mathbf{r},\mathbf{p})\right\rangle=\left(\nabla_p^i X^{ij}(\mathbf{v})\nabla_p^j+\nabla_p^i Y^i(\mathbf{v})\right)n(\mathbf{p})$$

$$\begin{cases} X^{ij}(\mathbf{v})=\dfrac{1}{N_c}\displaystyle\int_0^t dt'\left\langle F^i(t,\mathbf{r})F^j(t',\mathbf{r}-\mathbf{v}(t-t'))\right\rangle \\ Y^i(\mathbf{v})=\dfrac{1}{N_c}\left\langle F^i(t,\mathbf{r})\delta Q_0(\mathbf{r}-\mathbf{v}t,\mathbf{p})\right\rangle\dfrac{1}{n(\mathbf{p})} \end{cases}$$

Fokker-Planck Equation

$$\boxed{\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)n(t,\mathbf{r},\mathbf{p})=\left(\nabla_p^i X^{ij}(\mathbf{v})\nabla_p^j+\nabla_p^i Y^i(\mathbf{v})\right)n(t,\mathbf{r},\mathbf{p})}$$

$$\begin{cases} X^{ij}(\mathbf{v})=\dfrac{1}{N_c}\displaystyle\int_0^t dt'\left\langle F^i(t,\mathbf{r})F^j(t',\mathbf{r}-\mathbf{v}(t-t'))\right\rangle \\ Y^j(\mathbf{v})=\dfrac{\mathrm{v}^i}{T}X^{ij}(\mathbf{v}) \end{cases} \qquad \mathbf{F}\equiv g\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right)$$

- The collision term is given by field correlators $\left\langle E^iE^j\right\rangle,\left\langle B^iE^j\right\rangle,\left\langle B^iB^j\right\rangle$
- Gauge covariance is lost due to the linearization!

To restore gauge covariance: $\left\langle F_a^i(t,\mathbf{r})F_a^j(t',\mathbf{r}')\right\rangle \rightarrow \left\langle F_a^i(t,\mathbf{r})\Omega_{ab}(t,\mathbf{r}\,|\,t',\mathbf{r}')F_b^j(t',\mathbf{r}')\right\rangle$

$$\Omega(t,\mathbf{r}\,|\,t',\mathbf{r}')\equiv P\exp\left[ig\int_{(t',\mathbf{r}')}^{(t,\mathbf{r})}ds_\mu A^\mu(s)\right]$$

Field correlators in Equilibrium QGP

space-time translational invariance

$$\boxed{\left\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int\frac{d^3k}{(2\pi)^3}\, e^{-i\left(\omega(t-t')-\mathbf{k}(\mathbf{r}-\mathbf{r}')\right)}\overbrace{\left\langle E_a^i E_b^j\right\rangle_{\omega,\mathbf{k}}}^{\text{flucuation spectrum}}}$$

- $$\left\langle E_a^i E_b^j\right\rangle_{\omega,\mathbf{k}} = 2\delta^{ab}\frac{\omega^4}{e^{\beta|\omega|}-1}\left[\frac{k^ik^j}{\mathbf{k}^2}\frac{\mathrm{Im}\,\varepsilon_L(\omega,\mathbf{k})}{|\omega^2\varepsilon_L(\omega,\mathbf{k})|^2}+\left(\delta^{ij}-\frac{k^ik^j}{\mathbf{k}^2}\right)\frac{\mathrm{Im}\,\varepsilon_T(\omega,\mathbf{k})}{|\omega^2\varepsilon_T(\omega,\mathbf{k})-\mathbf{k}^2|^2}\right]$$
- $$\left\langle B_a^i B_b^j\right\rangle_{\omega,\mathbf{k}} = 2\delta^{ab}\frac{\omega^2\mathbf{k}^2}{e^{\beta|\omega|}-1}\left(\delta^{ij}-\frac{k^ik^j}{\mathbf{k}^2}\right)\frac{\mathrm{Im}\,\varepsilon_T(\omega,\mathbf{k})}{|\omega^2\varepsilon_T(\omega,\mathbf{k})-\mathbf{k}^2|^2}$$
- $$\left\langle B_a^i E_b^j\right\rangle_{\omega,\mathbf{k}} = \left\langle E_a^j B_b^i\right\rangle_{\omega,\mathbf{k}} = 2\delta^{ab}\frac{\omega^3}{e^{\beta|\omega|}-1}\varepsilon^{imj}k^m\frac{\mathrm{Im}\,\varepsilon_T(\omega,\mathbf{k})}{|\omega^2\varepsilon_T(\omega,\mathbf{k})-\mathbf{k}^2|^2}$$

$\varepsilon_{L,T}(\omega,\mathbf{k})$ - chromodielectric functions

St. Mrówczyński, Physical Review D **77**, 105022 (2008)

Fokker-Planck Equation of Equilibrium QGP

$$\boxed{\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)n(t,\mathbf{r},\mathbf{p})=\left(\nabla_p^i X^{ij}(\mathbf{v})\nabla_p^j+\nabla_p^i Y^i(\mathbf{v})\right)n(t,\mathbf{r},\mathbf{p})}$$

Isotropy

$$X^{ij}(\mathbf{v})\equiv X_L(\mathrm{v})\frac{\mathrm{v}^i\mathrm{v}^j}{\mathbf{v}^2}+X_T(\mathrm{v})\left(\delta^{ij}-\frac{\mathrm{v}^i\mathrm{v}^j}{\mathbf{v}^2}\right),\qquad Y^j(\mathbf{v})=\frac{\mathrm{v}^i}{T}X^{ij}(\mathbf{v})=\frac{\mathrm{v}^i}{T}X_L(v)$$

$$\begin{cases} X_L(\mathrm{v})=\ldots \\ X_T(\mathrm{v})=\ldots \end{cases}$$

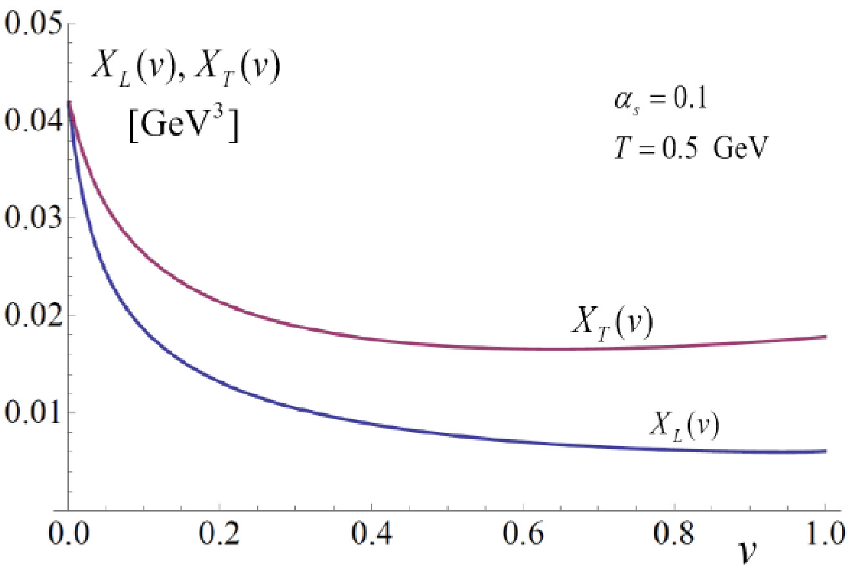
For heavy quarks

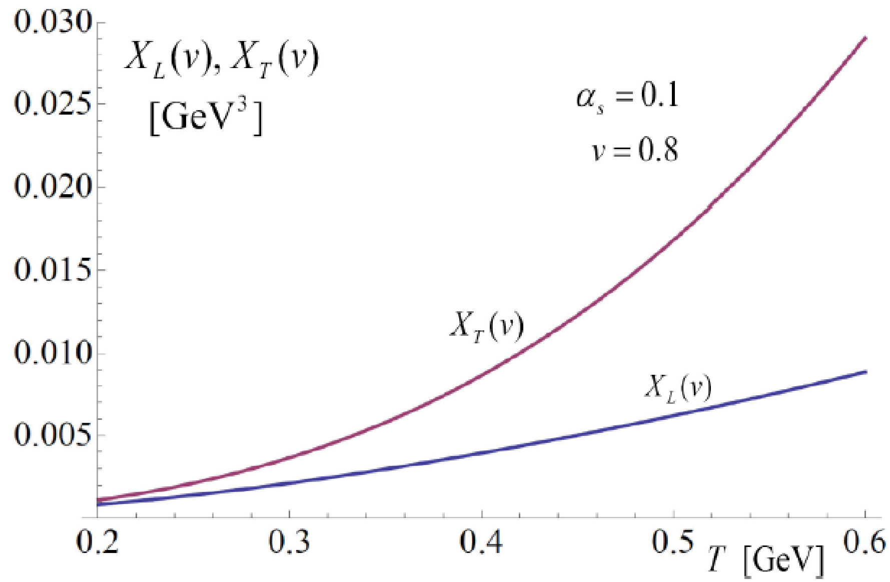
$$\boxed{\mathrm{v}<<1,\quad g<<1}$$

$$\boxed{X_L(\mathrm{v})=X_T(\mathrm{v})\approx\frac{g^2C_F}{12\pi}m_D^2T\log\left(\frac{T}{m_D}\right)}\qquad C_F\equiv\frac{N_c^2-1}{2N_c}$$

G. D. Moore and D. Teaney, Physical Review D **71**, 064904 (2005)

Fokker-Planck Equation of Equilibrium QGP





Quantitative agreement with $X_L(\mathrm{v})$ & $X_T(\mathrm{v})$ obtained from the Boltzmann collision term by means of the diffusive approximation.

The standard FP equation is reproduced!

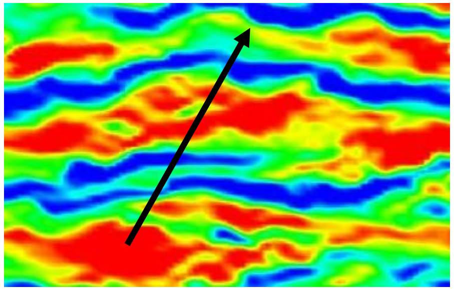
B. Svetitsky, Physical Review D **37**, 2484 (1988)

Transport of hard probes in glasma

$$\boxed{\begin{aligned} X^{ij}(\mathbf{v}) = \frac{g}{N_c}\int_0^t dt'\Big\{ &\left\langle E^i(t,\mathbf{r})E^j(t',\mathbf{r}')\right\rangle + \varepsilon^{jkl}\mathrm{v}^k\left\langle E^i(t,\mathbf{r})B^l(t',\mathbf{r}')\right\rangle \\ &+ \varepsilon^{ikl}\mathrm{v}^k\left\langle B^l(t,\mathbf{r})E^j(t',\mathbf{r}')\right\rangle + \varepsilon^{ikl}\varepsilon^{jmn}\mathrm{v}^k\mathrm{v}^m\left\langle B^l(t,\mathbf{r})B^n(t',\mathbf{r}')\right\rangle\Big\} \end{aligned}}$$

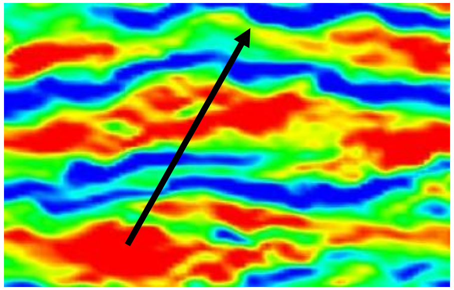
$$\mathbf{r}' \equiv \mathbf{r} - \mathbf{v}(t-t')$$

$$\begin{cases} \hat{q} = \dfrac{2}{\mathrm{v}}\left(\delta^{ij} - \dfrac{\mathrm{v}^i\mathrm{v}^j}{\mathrm{v}^2}\right)X^{ji}(\mathbf{v}) \\[2ex] \dfrac{dE}{dx} = -\dfrac{\mathrm{v}^i\mathrm{v}^j}{\mathrm{v}T}X^{ij}(\mathbf{v}) \end{cases}$$



$\hat{q}$ from Wong equations

$$\hat{q} = \frac{1}{\mathrm{v}}\left(\delta^{ij} - \frac{\mathrm{v}^i \mathrm{v}^j}{\mathrm{v}^2}\right)\frac{\left\langle \Delta p^i \Delta p^j \right\rangle}{\Delta t}\bigg|_{\Delta t \to 0}$$



Wong equations

$$\begin{cases} \dfrac{d\mathbf{r}(t)}{dt} = \mathbf{v}(t) \\[2ex] \dfrac{d\mathbf{p}(t)}{dt} = q_a \mathbf{F}_a(t,\mathbf{r}(t)) \end{cases}$$

Lorentz force

$$\mathbf{F}_a(t,\mathbf{r}) = g\left(\mathbf{E}_a(t,\mathbf{r}) + \mathbf{v}\times\mathbf{B}_a(t,\mathbf{r})\right)$$

$$\Delta p^i = \int_0^t dt' q_a F_a^i\left(t',\mathbf{r}(t')\right)$$

$$\left\langle \Delta p^i \Delta p^j \right\rangle = \int_0^t dt' \int_0^t dt'' \underbrace{\int Dq\, q_a q_b}_{= \delta^{ab} \begin{cases} \frac{1}{2} & \text{quark} \\ N_c & \text{gluon} \end{cases}} \left\langle F_a^i\left(t',\mathbf{r}(t')\right) F_b^j\left(t'',\mathbf{r}(t'')\right)\right\rangle$$

$$\boxed{\hat{q} = \frac{2}{\mathrm{v}}\left(\delta^{ij} - \frac{\mathrm{v}^i \mathrm{v}^j}{\mathrm{v}^2}\right) X^{ji}(\mathbf{v})}$$

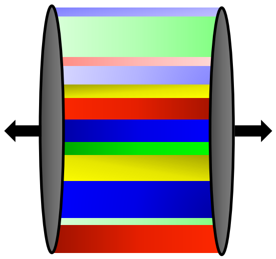
M. Carrington, B. Friesen & St. Mrówczyński, arXiv:2604.02050, Phys. Rev. C in the press

Rough estimate

Density of energy accumulated in the fields

$$\varepsilon_{\text{field}} = \frac{1}{2}\Big(\big\langle E_a^i E_a^i \big\rangle + \big\langle B_a^i B_a^i \big\rangle\Big)$$

E & *B* fields along the axis *z*



Density of energy released in a central collision

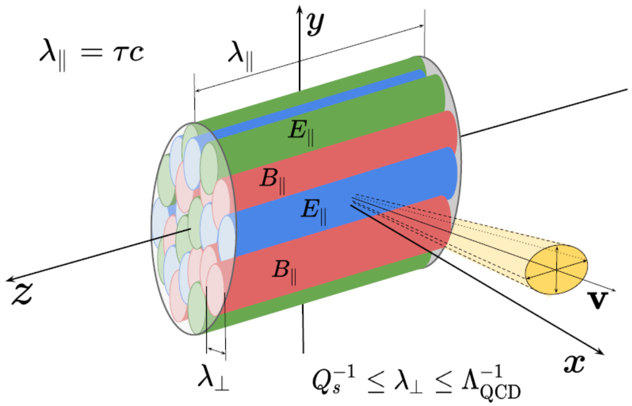
$$\varepsilon_{\text{coll}} = \frac{c_{\text{inel}} A \sqrt{s}}{\pi R_A^2 l}$$

$$\varepsilon_{\text{coll}} = \varepsilon_{\text{field}} \quad \Longrightarrow \quad \begin{cases} -\dfrac{dE}{dx} \sim (0 \div 10) \quad \left[\dfrac{\text{GeV}}{\text{fm}}\right] \\ \hat{q} \sim 10 \quad \left[\dfrac{\text{GeV}^2}{\text{fm}}\right] \end{cases}$$

$c_{\text{inel}} = 0.5, \quad A = 200, \quad \sqrt{s} = 5\,\text{TeV}, \quad l = 1\,\text{fm}$

St. Mrówczyński, European Physical Journal A **54**, 43 (2018)

Hard probes in glasma - $\hat{q}$



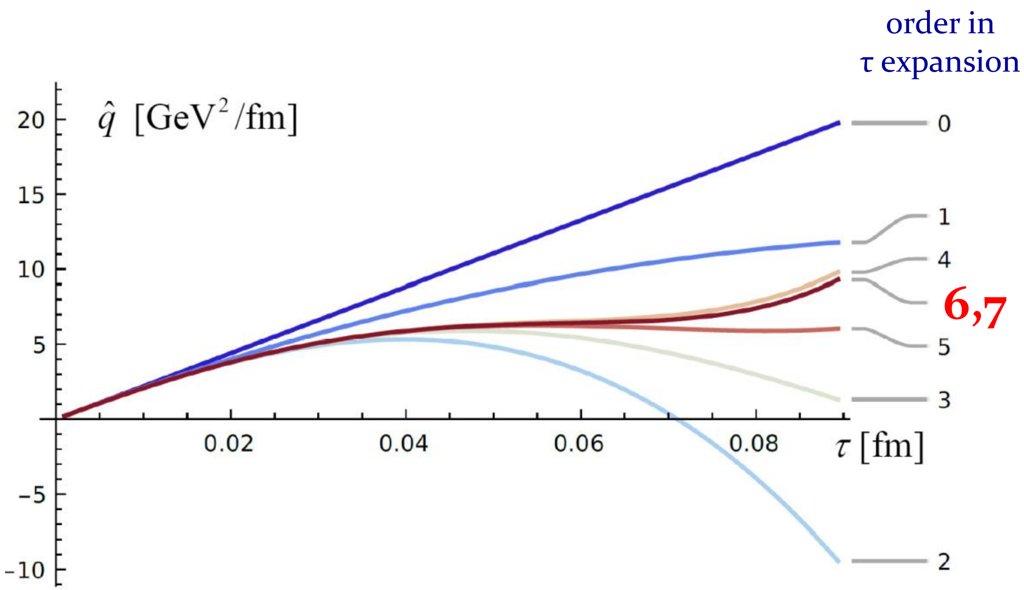
$$\hat{q} = \frac{2}{\mathrm{v}}\left(\delta^{ij} - \frac{\mathrm{v}^i \mathrm{v}^j}{\mathrm{v}^2}\right) X^{ji}(\mathbf{v})$$

$N_c = 3, \quad g = 1$

$Q_s = 2\ \mathrm{GeV}$

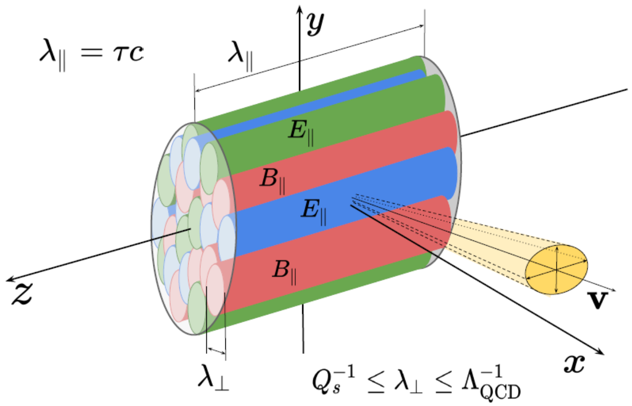
$m = 0.2\ \mathrm{GeV}$

$\mathrm{v} = \mathrm{v}_{\perp} = 1$

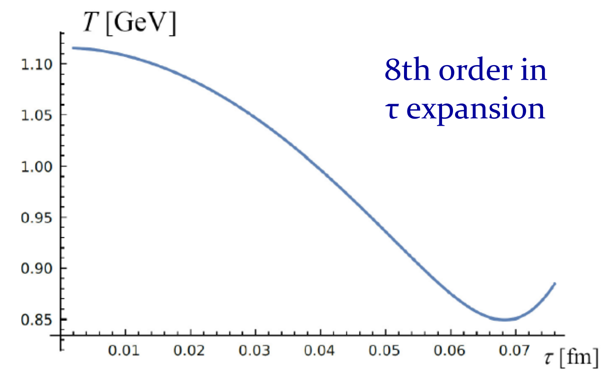


M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Hard probes in glasma - $\frac{dE}{dx}$



$$\frac{dE}{dx} = -\frac{1}{T}\frac{v^i v^i}{v} X^{ij}(\mathbf{v})$$

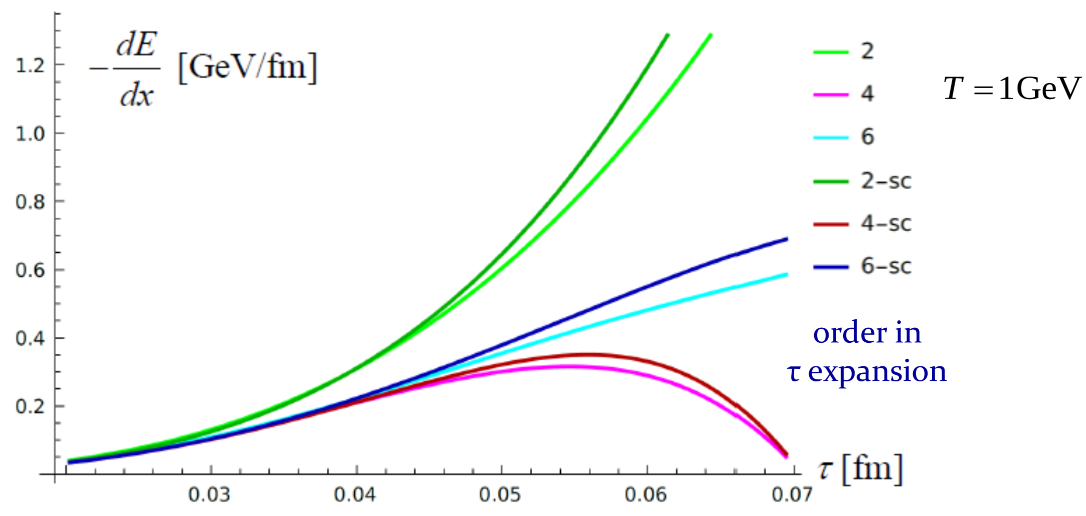


$N_c = 3, \quad g = 1$

$Q_s = 2$ GeV

$m = 0.2$ GeV

$v = v_{\perp} = 1$



M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Gauge dependence

$$\boxed{X^{ij}(\mathbf{v}) = \frac{1}{2N_c}\int_0^t dt' \left\langle F_a^i(t,\mathbf{r}) F_a^j(t',\mathbf{r}')) \right\rangle}$$

$$\mathbf{F}_a(t,\mathbf{r}) = g\left(\mathbf{E}_a(t,\mathbf{r}) + \mathbf{v}\times\mathbf{B}_a(t,\mathbf{r})\right)$$

Gauge transformation (in adjoint representation)

$x \equiv (t,\mathbf{r})$

$$F_a^i(x) \to U_{ab}(x) F_b^i(x) = F_b^i(x) U_{ba}^\dagger(x)$$

$$U_{ca}^\dagger(x) U_{ab}(x) = \delta^{bc}$$

$$\begin{cases} F_a^i(x) F_a^j(x) \to U_{ab}(x) F_b^i(x) F_c^j(x) U_{ca}^\dagger(x) = F_b^i(x) F_b^j(x) & \text{gauge invariant} \\ F_a^i(x) F_a^j(y) \to U_{ab}(x) F_b^i(x) F_c^j(y) U_{ca}^\dagger(y) = U_{ca}^\dagger(x) U_{ab}(y) F_b^i(x) F_c^j(y) & \text{gauge dependent} \end{cases}$$

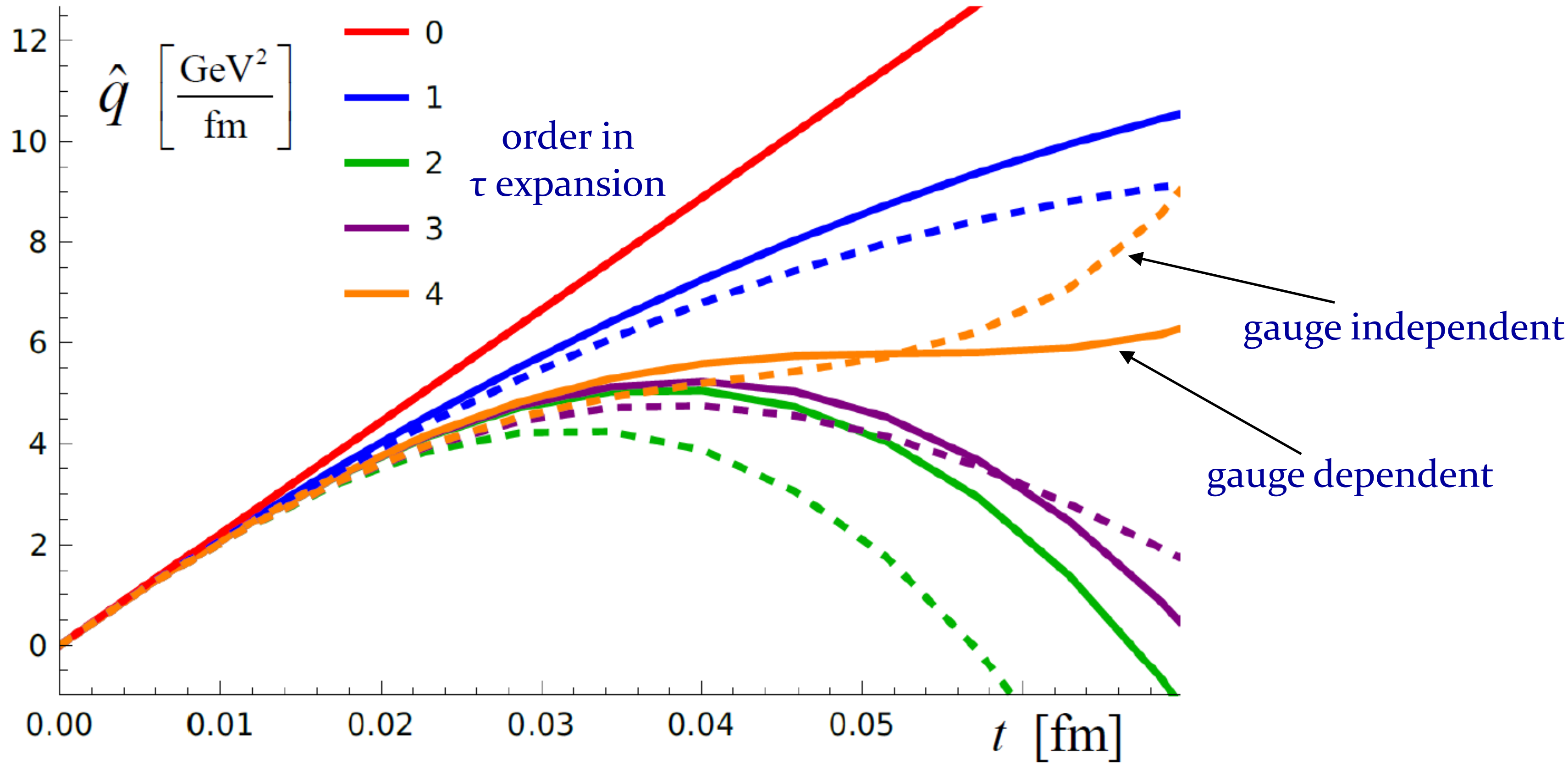
$$\boxed{X^{ij}(\mathbf{v}) = \frac{1}{2N_c}\int_0^t dt' \left\langle F_a^i(t,\mathbf{r}) \Omega_{ab}(t,\mathbf{r}\,|\,t',\mathbf{r}') F_b^j(t',\mathbf{r}')) \right\rangle}$$

gauge invariant

$$\Omega(t,\mathbf{r}\,|\,t',\mathbf{r}') = P\exp\left(ig \int_{(t',\mathbf{r}')}^{(t,\mathbf{r})} ds_\mu A_c^\mu(s) T^c \right)$$

$$\Omega(x\,|\,y) \to U(x)\Omega(x\,|\,y)U^\dagger(y)$$

Gauge independent $\hat{q}$



M. Carrington, B. Friesen & St. Mrówczyński, arXiv:2604.02050, Phys. Rev. C in the press

Glasma impact on jet quenching

Glasma

$\hat{q}_{\max} = 6 \ \mathrm{GeV}^2 / \mathrm{fm}$

$t_{\max} = 0.06 \ \mathrm{fm}$

Equilibrium QGP

$\hat{q} = 3T^3$

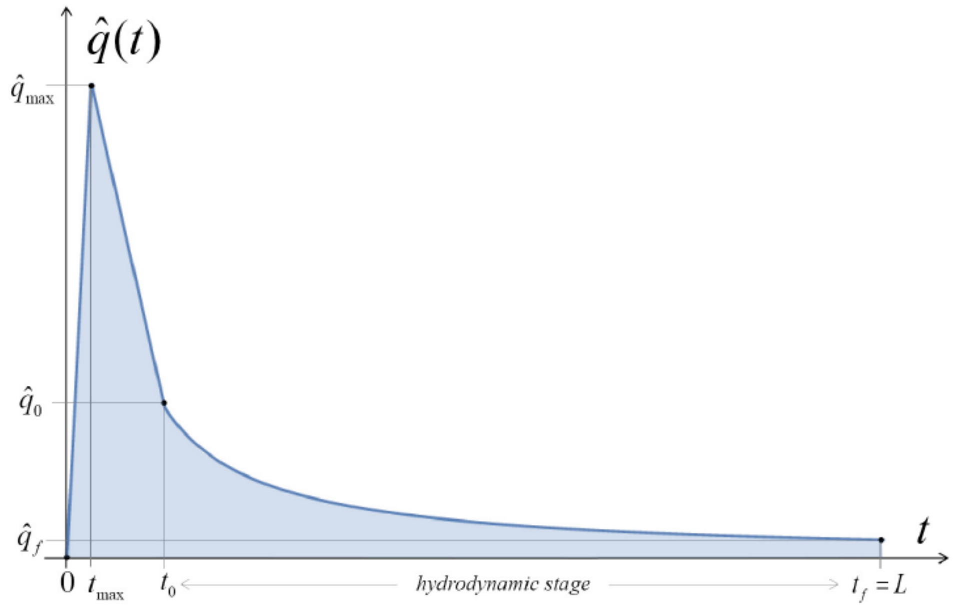
$t_0 = 0.6 \ \mathrm{fm}$

$T_0 = 450 \ \mathrm{MeV}$

$\hat{q}_0 = 1.4 \ \mathrm{GeV}^2 / \mathrm{fm}$

$T = T_0 \left(\frac{t_0}{t} \right)^{1/3}$

$L = 10 \ \mathrm{fm}$



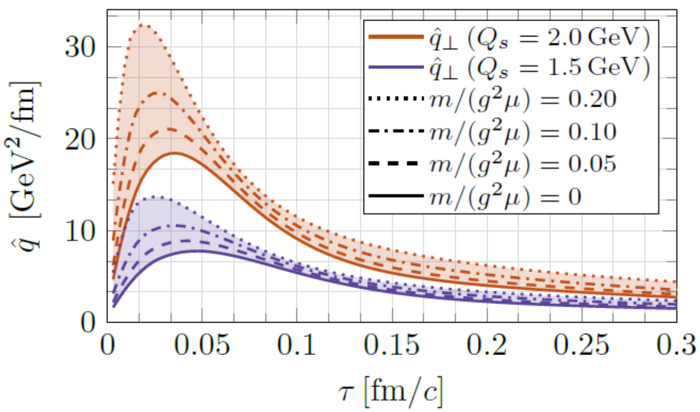
$$\Delta p_T^2 \Big|^{\text{non-eq}} = \int_0^{t_0} dt \, \hat{q}(t)$$

$$\Delta p_T^2 \Big|^{\text{eq}} = \int_{t_0}^{L} dt \, \hat{q}(t)$$

$$\boxed{\frac{\Delta p_T^2 \Big|^{\text{non-eq}}}{\Delta p_T^2 \Big|^{\text{eq}}} = 0.93}$$

S. Cao et al. [JETSCAPE], Physical Review C **104**, 024905 (2021),
C. Shen, U. Heinz, P. Huovinen and H. Song, Physical Review C **84**, 044903 (2011).

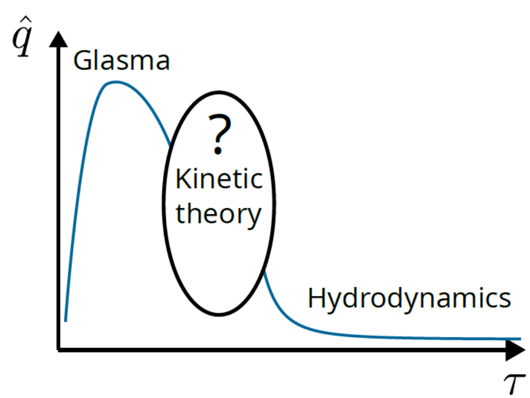
Glasma impact on jet quenching cont.



Full simulations of glasma

A. Ipp, D.I. Műller and D. Schuh, Phys. Lett. B **810**, 135810 (2020)

D. Avramescu, V. Băran, V. Greco, A. Ipp, D.I. Műller & M.Ruggieri, Phys. Rev. D **107**, 114021 (2023)



Kinetic theory interpolates between glasma and equilibrium QGP

K. Boguslavski, A. Kurkela, T.Lappi, F. Lindenbauer & J.Peuron, Phys. Lett. B **850**, 138525 (2024)

Summary & Conclusions

- The Fokker-Planck equation of hard probes interacting with classical chromodynamic fields rather than with plasma constituents is derived.
- The known case of equilibrium plasma is reproduced.
- The field correlators are computed up to τ^7 or τ^8 depending on the considered quantity.
- The gauge dependence of $\hat{q}$: the effect of including the Wilson line changes the result by about 9%.
- The momentum broadening and energy loss in the glasma are significantly bigger than in equilibrated QGP.
- In spite of its short lifetime the glasma provides a significant contribution to the jet quenching.