The earliest phase of relativistic heavy-ion collisions

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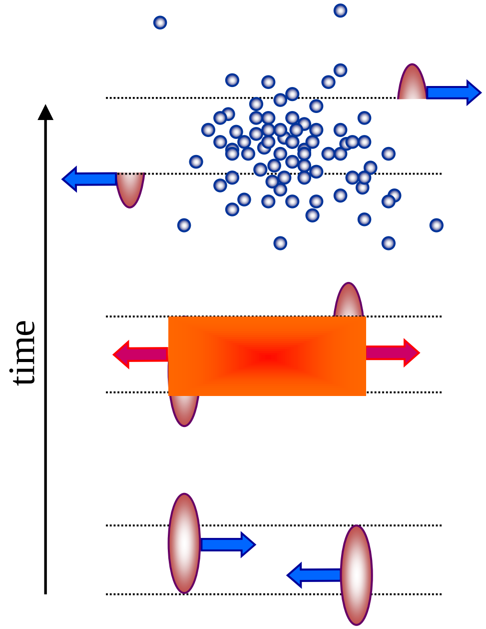
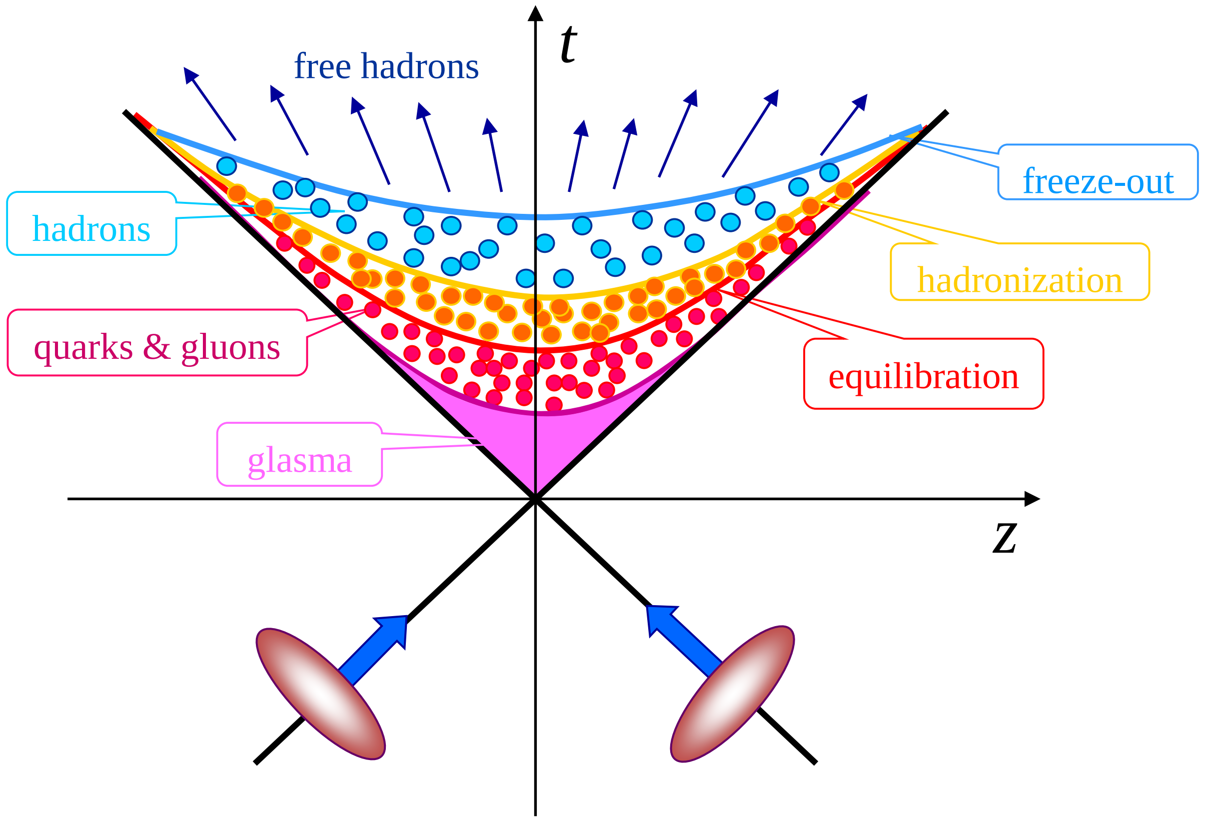
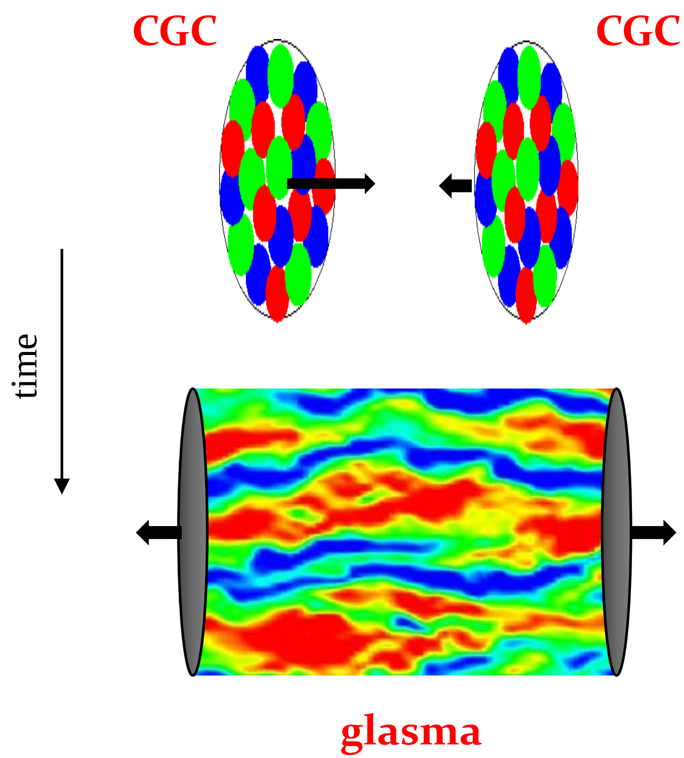
Relativistic heavy-ion collisions





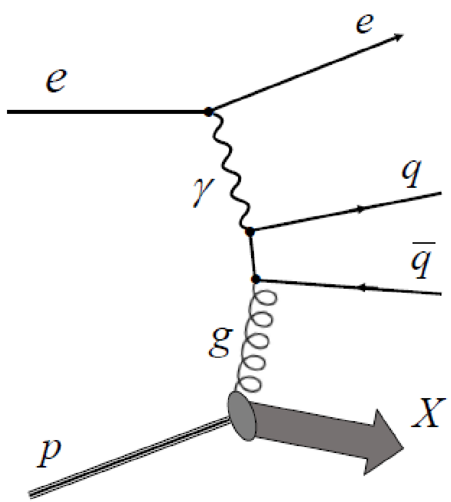
Color Glass Condensate & glasma

Color charges confined in the colliding nuclei generate **CGC** which in turn produce **glasma** – the system of strong mostly classical chromodynamic fields which evolves towards equilibrium.

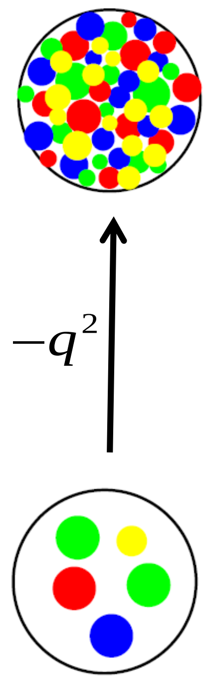


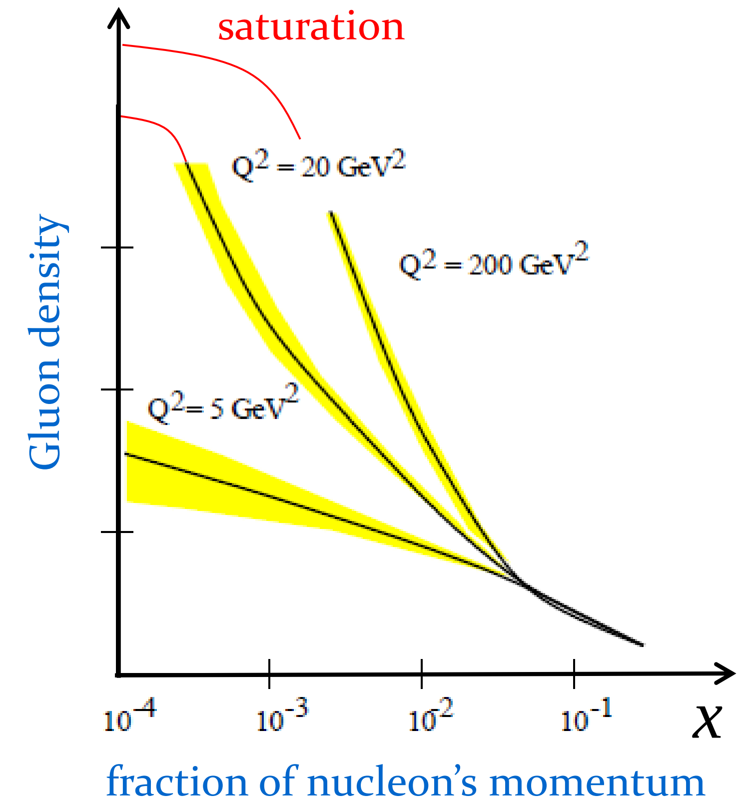
Saturation

Deep Inelastic Scattering



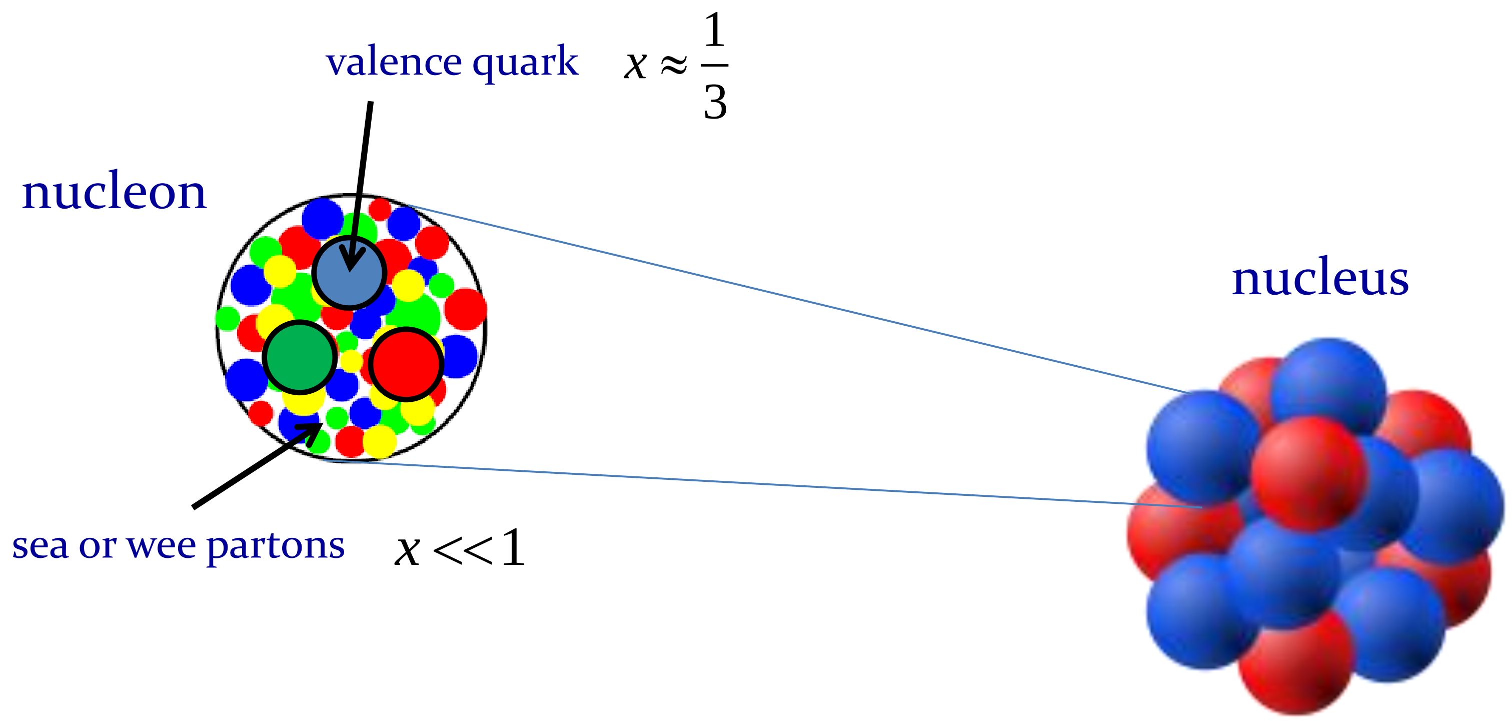
$$Q^2 = -q^2$$





Saturated gluon system can be described in terms of classical chromodynamic fields.

Wee partons & valence quarks



In relativistic heavy-ion collsions

- Saturated wee partons – classical chromodynamic fields
- Valence quarks – classical sources of chromodynamic fields

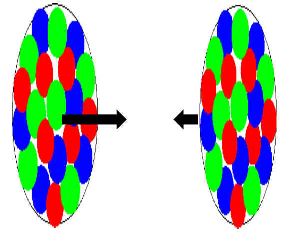
Color Glass Condensate & glasma

Classical Yang-Mills equation

$$\boxed{D_\mu F^{\mu\nu}(x) = j^\nu(x)}$$

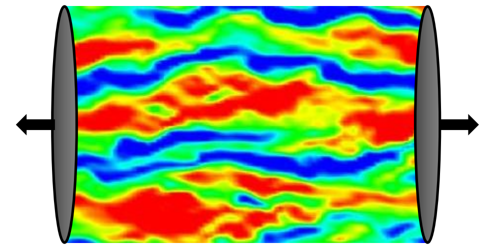
$$j^\mu(x) = j_1^\mu(x) + j_2^\mu(x)$$

$$j_{1,2}^\mu(x) = \pm\delta^{\mu\mp}\delta(x^\pm)\rho_{1,2}(\mathbf{x}_\perp)$$



β_1^i β_2^i

pre-collision potentials

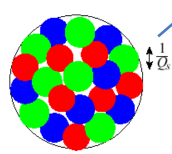


$\alpha, \alpha_\perp^i$

post-collision potentials

Ansatz of gauge potentials

$$\begin{cases} A^+(x) = \Theta(x^+)\Theta(x^-)x^+\alpha(\tau,\mathbf{x}_\perp) \\ A^-(x) = -\Theta(x^+)\Theta(x^-)x^-\alpha(\tau,\mathbf{x}_\perp) \\ A^i(x) = \Theta(x^+)\Theta(x^-)\alpha_\perp^i(\tau,\mathbf{x}_\perp) \\ \qquad + \Theta(-x^+)\Theta(x^-)\beta_1^i(\mathbf{x}_\perp) + \Theta(x^+)\Theta(-x^-)\beta_2^i(\mathbf{x}_\perp) \end{cases}$$



Boundary condition

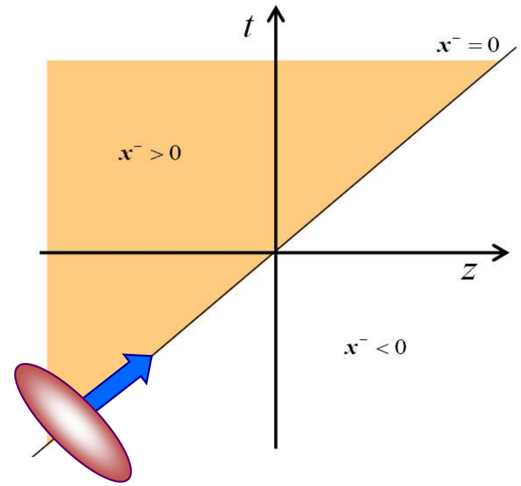
$$\begin{cases} \alpha(0,\mathbf{x}_\perp) = \beta_1^i(\mathbf{x}_\perp) + \beta_2^i(\mathbf{x}_\perp) \\ \alpha_\perp^i(0,\mathbf{x}_\perp) = -\dfrac{ig}{2}[\beta_1^i(\mathbf{x}_\perp), \beta_2^i(\mathbf{x}_\perp)] \end{cases}$$

Gauge condition

$$x^+A^- + x^-A^+ = 0$$

E. Iancu, R. Venugopalan, in *Quark-Gluon Plasma 3*,
ed. by R.C. Hwa, X.-N. Wang (World Scientific, Singapore, 2004

Pre-collision potentials



$$\boxed{j^{\mu}(x^-, \mathbf{x}_\perp) = \delta^{\mu+}\delta(x^-)\rho(\mathbf{x}_\perp)}$$

Gauge condition: $A^i(x^-, \mathbf{x}_\perp) = 0$

$$D_\mu F^{\mu\nu} = j^\nu \quad \Rightarrow \quad \begin{cases} A^-(x^-, \mathbf{x}_\perp) = 0 \\ A^+(x^-, \mathbf{x}_\perp) = \delta(x^-)\Lambda(\mathbf{x}_\perp) \end{cases}$$

Poisson equation

$$\boxed{\nabla_\perp^2 \Lambda(\mathbf{x}_\perp) = -\rho(\mathbf{x}_\perp)}$$

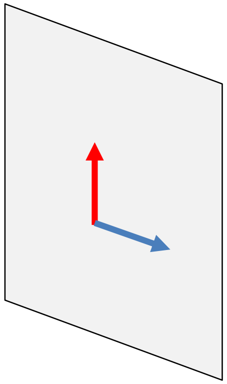
$$\Lambda(\mathbf{x}_\perp) = \int d^2x'_\perp\, G(\mathbf{x}_\perp - \mathbf{x}'_\perp)\rho(\mathbf{x}'_\perp)$$

$$G(\mathbf{x}_\perp) = \int \frac{d^2k_\perp}{(2\pi)^2}\frac{e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp}}{\mathbf{k}_\perp^2 + m^2} = \frac{1}{2\pi}K_0(m|\mathbf{x}_\perp|)$$

IR regulator $m = \Lambda_{\mathrm{QCD}}$

$$E^z = B^z = 0$$

$$\mathbf{E}_\perp, \mathbf{B}_\perp \sim \delta(x^-)$$



$x^- = 0$

Pre-collision potentials cont.

Gauge transformation: $A^{\mu}(x^-, \mathbf{x}_\perp) \quad \rightarrow \quad \beta^{\mu}(x^-, \mathbf{x}_\perp)$

gauge condition $A^i(x^-, \mathbf{x}_\perp) = 0$

light-come gauge $\beta^+(x^-, \mathbf{x}_\perp) = 0$

$$\beta^{\mu}(x) = U(x) A^{\mu}(x) U^{\dagger}(x) + \frac{i}{g} U(x) \partial^{\mu} U^{\dagger}(x)$$

$$U(x^-, \mathbf{x}_\perp) A^+(x^-, \mathbf{x}_\perp) U^{\dagger}(x^-, \mathbf{x}_\perp) + \frac{i}{g} U(x^-, \mathbf{x}_\perp) \partial^+ U^{\dagger}(x^-, \mathbf{x}_\perp) = 0$$

$$\begin{cases} U(x^-, \mathbf{x}_\perp) = P \exp\left(ig \int_{-\infty}^{x^-} dz^- A^+(z^-, \mathbf{x}_\perp) \right) \\ \beta^i(x^-, \mathbf{x}_\perp) = \dfrac{i}{g} U(x^-, \mathbf{x}_\perp) \partial^i U^{\dagger}(x^-, \mathbf{x}_\perp) \end{cases}$$

pure gauge except $x^- = 0$

J. Jalilian-Marian, A. Kovner, L.D. McLerran & H. Weigert, Phys. Rev. D **55**, 5414 (1997).

Proper time expansion

$$\alpha(\tau,\mathbf{x}_\perp)=\sum_{n=0}^{\infty}\tau^n\alpha_{(n)}(\mathbf{x}_\perp),\qquad \alpha_\perp^i(\tau,\mathbf{x}_\perp)=\sum_{n=0}^{\infty}\tau^n\alpha_{\perp(n)}^i(\mathbf{x}_\perp)$$

Proper time τ is treated as a small parameter $\tau << Q_s^{-1}$

Yang-Mills equations for the expanded potentials are solved recursively

$$\alpha_{(n)}=\alpha_{\perp(n)}^i=0 \quad \text{for} \quad n=1,3,5,\ldots$$

0th order - oboundary conditions

$$\begin{cases}\alpha_{(0)}=-\dfrac{ig}{2}[\beta_1^i,\beta_2^i]\\ \alpha_{\perp(0)}^i=\beta_1^i+\beta_2^i\end{cases}$$

Post-collision potentials are expressed through pre-collision potentials

2nd order

$$\begin{cases}\alpha_{(2)}=-\dfrac{ig}{16}[D^j,[D^j,[\beta_1^i,\beta_2^i]]]\\ \alpha_{\perp(2)}^i=\dfrac{ig}{4}\varepsilon^{zij}\varepsilon^{zkl}[D^j,[\beta_1^k,\beta_2^l]]\end{cases}$$

$$D^i\equiv\partial^i-ig(\beta_1^i+\beta_2^i)$$

Fully analytic approach!

R. J. Fries, J. I. Kapusta, and Y. Li, arXiv:nucl-th/0604054
G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Proper time expansion cont.

Chromoelectric and chromomagnetic fields

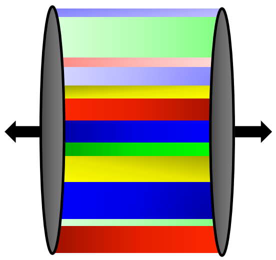
$$E^i = F^{i0}, \quad B^i = \frac{1}{2}\varepsilon^{ijk}F^{kj}$$

0th order

$$\mathbf{E}_{(0)} = (0,0,E), \qquad \mathbf{B}_{(0)} = (0,0,B)$$

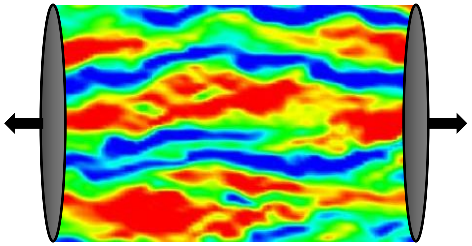
$$E^z_{(0)}(\mathbf{x}_\perp) = -ig[\beta^i_1(\mathbf{x}_\perp), \beta^i_2(\mathbf{x}_\perp)]$$

$$B^z_{(0)}(\mathbf{x}_\perp) = -ig\varepsilon^{zij}[\beta^i_1(\mathbf{x}_\perp), \beta^j_2(\mathbf{x}_\perp)]$$



E & *B* fields along the axis *z*

At higher orders transverse fields show up

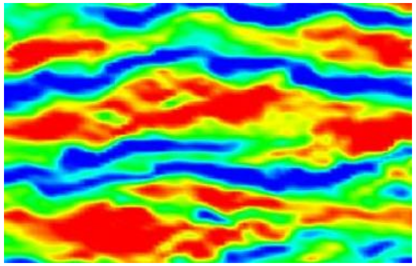


R. J. Fries, J. I. Kapusta, and Y. Li, arXiv:nucl-th/0604054
G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Observables - energy-momentum tensor

$$\boxed{T^{\mu\nu}(x) = 2\mathrm{Tr}[F^{\mu\rho}(x)F_{\rho}{}^{\nu}(x) + \frac{1}{4}g^{\mu\nu}F^{\rho\sigma}(x)F_{\rho\sigma}(x)]}$$

$$F^{\mu\nu}(x) = \partial^{\mu}A^{\nu}(x) - \partial^{\nu}A^{\mu}(x) - ig[A^{\mu}(x), A^{\nu}(x)]$$



Physical quantities

$\langle T^{00}(x)\rangle$ - energy density

$\langle T^{0i}(x)\rangle$ - energy flux, Poynting vector

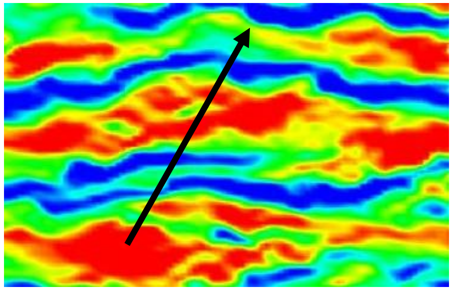
$\langle T^{xx}(x)\rangle, \langle T^{yy}(x)\rangle, \langle T^{zz}(x)\rangle$ - pressures

$\langle T^{ij}(x)\rangle$ - momentum flux

$\langle \ldots \rangle$ averaging over colour configuration of initial nuclei

one-point correlators

Observables - transport coefficients of hard probes



$$\begin{cases} \hat{q} = \dfrac{2}{\mathrm{v}}\left(\delta^{ij} - \dfrac{\mathrm{v}^i \mathrm{v}^j}{\mathrm{v}^2}\right) X^{ji}(\mathbf{v}) \\ \dfrac{dE}{dx} = -\dfrac{\mathrm{v}^i \mathrm{v}^j}{\mathrm{v}T} X^{ij}(\mathbf{v}) \end{cases}$$

$$X^{ij}(\mathbf{v}) = \frac{g}{N_c}\int_0^t dt' \Big\{ \big\langle E^i(t,\mathbf{r}) E^j(t',\mathbf{r}') \big\rangle + \varepsilon^{jkl} \mathrm{v}^k \big\langle E^i(t,\mathbf{r}) B^l(t',\mathbf{r}') \big\rangle + \varepsilon^{ikl} \mathrm{v}^k \big\langle B^l(t,\mathbf{r}) E^j(t',\mathbf{r}') \big\rangle + \varepsilon^{ikl}\varepsilon^{jmn} \mathrm{v}^k \mathrm{v}^m \big\langle B^l(t,\mathbf{r}) B^n(t',\mathbf{r}') \big\rangle \Big\}$$

$\mathbf{r}' \equiv \mathbf{r} - \mathbf{v}(t - t')$

E^i, B^i - chromoelectric and chromomagnetic fields

two-point correlators

St. Mrówczyński, European Physical Journal A **54**, 43 (2018)

Averaging over colour configuration of initial nuclei

- $$Observable = \int \sum \partial_x^i \partial_z^j \left\langle \beta^k(\mathbf{x}_\perp) \beta^l(\mathbf{y}_\perp) \ldots \beta^m(\mathbf{z}_\perp) \right\rangle$$

- $$\beta^i(\mathbf{x}_\perp) \sim \rho(\mathbf{x}_\perp)$$

Wick theorem

- $$\left\langle \rho_a^k(\mathbf{x}_\perp) \rho_b^l(\mathbf{y}_\perp) \ldots \rho_c^m(\mathbf{z}_\perp) \right\rangle = \sum \prod \left\langle \rho_a^i(\mathbf{x}_\perp) \rho_b^j(\mathbf{y}_\perp) \right\rangle$$

Glasma graph approximation

- $$\left\langle \beta_a^k(\mathbf{x}_\perp) \beta_b^l(\mathbf{y}_\perp) \ldots \beta_c^m(\mathbf{z}_\perp) \right\rangle = \sum \prod \left\langle \beta_a^i(\mathbf{x}_\perp) \beta_b^j(\mathbf{y}_\perp) \right\rangle = \sum \prod B_{ab}^{ij}(\mathbf{x}_\perp, \mathbf{y}_\perp)$$

Basic two-point correlator in transversaly uniform system

$$B_{ab}^{ij}(\mathbf{x}_\perp - \mathbf{y}_\perp) \equiv \left\langle \beta_a^i(\mathbf{x}_\perp)\beta_b^j(\mathbf{y}_\perp)\right\rangle = \int d^2x'_\perp\, d^2y'_\perp \cdots\cdots \left\langle \rho_a^i(\mathbf{x}'_\perp)\rho_b^j(\mathbf{y}'_\perp)\right\rangle$$

$$\left\langle \rho_a^i(\mathbf{x}_\perp)\rho_b^j(\mathbf{y}_\perp)\right\rangle = g^2\mu\,\delta^{ab}\delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp)$$

color charge surface density

$$\mu = g^{-4}Q_s^2$$

$$\boxed{B_{ab}^{ij}(\mathbf{x}_\perp - \mathbf{y}_\perp) \equiv \delta^{ab}\left(\delta^{ij}C_1(r) - \hat{r}^i\hat{r}^j C_2(r)\right)}$$

$$\mathbf{r} \equiv \mathbf{x}_\perp - \mathbf{y}_\perp, \quad r \equiv |\mathbf{r}|, \quad \hat{r}^i \equiv \frac{r^i}{r}$$

$$\begin{cases} C_1(r) \equiv \dfrac{m^2K_0(mr)}{g^2N_c\left(mrK_1(mr)-1\right)}\left\{\exp\left[\dfrac{g^4N_c\mu\left(mrK_1(mr)-1\right)}{4\pi m^2}\right]-1\right\} \underset{r<<m^{-1}}{\approx} \#\log(mr) \\[2ex] C_2(r) \equiv \dfrac{m^3rK_1(mr)}{g^2N_c\left(mrK_1(mr)-1\right)}\left\{\exp\left[\dfrac{g^4N_c\mu\left(mrK_1(mr)-1\right)}{4\pi m^2}\right]-1\right\} \end{cases}$$

UV regularization required

$$r > Q_s^{-1}$$

J. Jalilian-Marian, A. Kovner, L. D. McLerran and H. Weigert, Physical Review D **55**, 5414 (1997)

Basic correlator of real nucleus

- System nonuniform in the transverse plane

Projected Woods-Saxon distribution

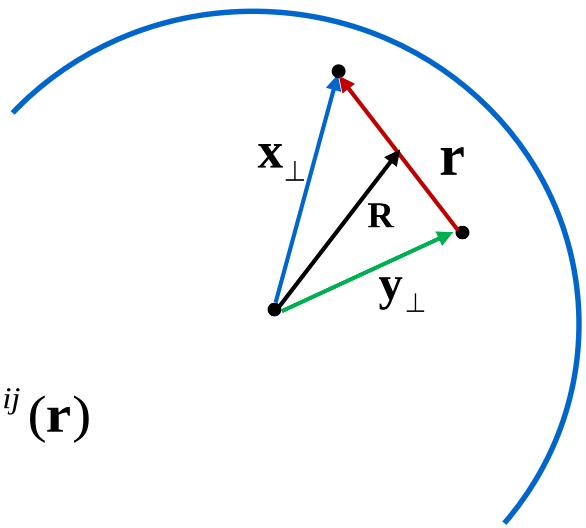
$$\mu(\mathbf{x}_\perp) = \frac{\bar{\mu}}{\ln(1+e^{R_A/a})} \int_{-\infty}^{\infty} \frac{dz}{1+\exp\left[\left(\sqrt{\mathbf{x}_\perp{}^2 + z^2} - R_A\right)/a\right]}$$

$$\mathbf{R} \equiv \frac{1}{2}(\mathbf{x}_\perp + \mathbf{y}_\perp), \qquad \mathbf{r} \equiv \mathbf{x}_\perp - \mathbf{y}_\perp$$

$$B^{ij}_{ab}(\mathbf{x}_\perp, \mathbf{y}_\perp) = \delta^{ab} f^{ij}(\mathbf{R}, \mathbf{r}) \approx \text{``}\textit{gradient expansion in } \mathbf{R}\text{''}$$

strong dependence on $\mathbf{r}$

weak dependence of $\mathbf{R}$



System uniform in the transverse plane

$$B^{ij}_{ab}(\mathbf{x}_\perp, \mathbf{y}_\perp) = \delta^{ab} f^{ij}(\mathbf{x}_\perp - \mathbf{y}_\perp) = \delta^{ab} f^{ij}(\mathbf{r})$$

G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Numerical results

Pb-Pb collisions at LHC

$N_c = 3$

$g = 1$

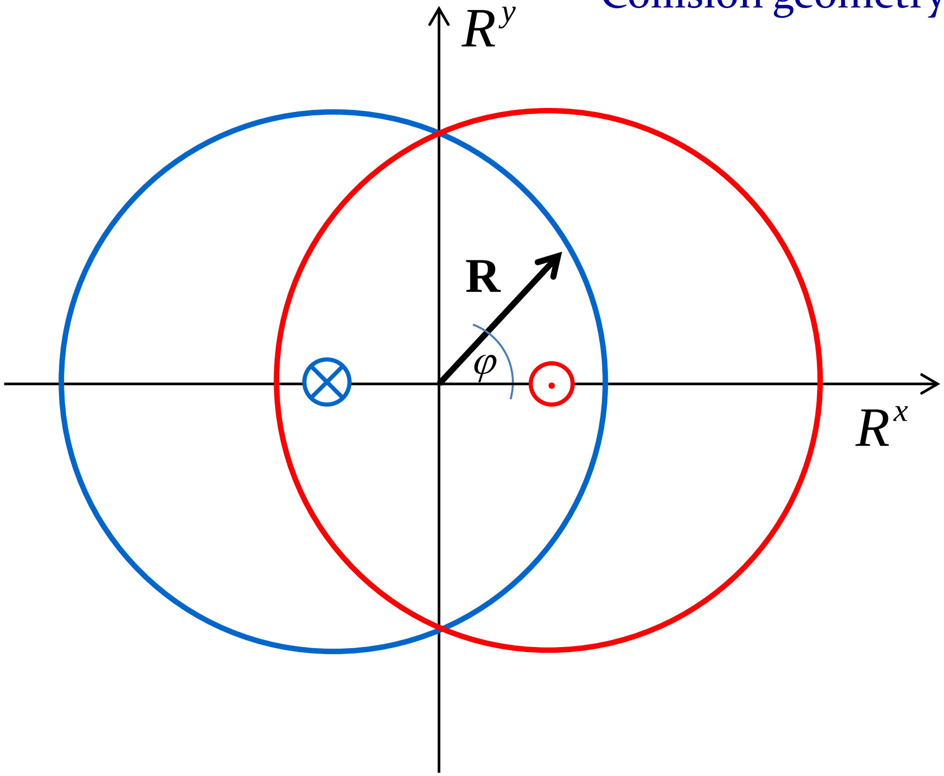
$Q_s = 2$ GeV

$m = 0.2$ GeV

$R_A = 7.4$ fm

$a = 0.5$ fm

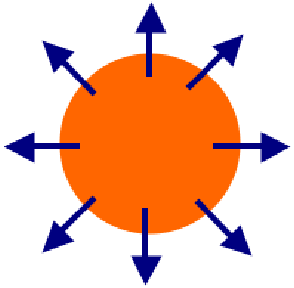
Collision geometry

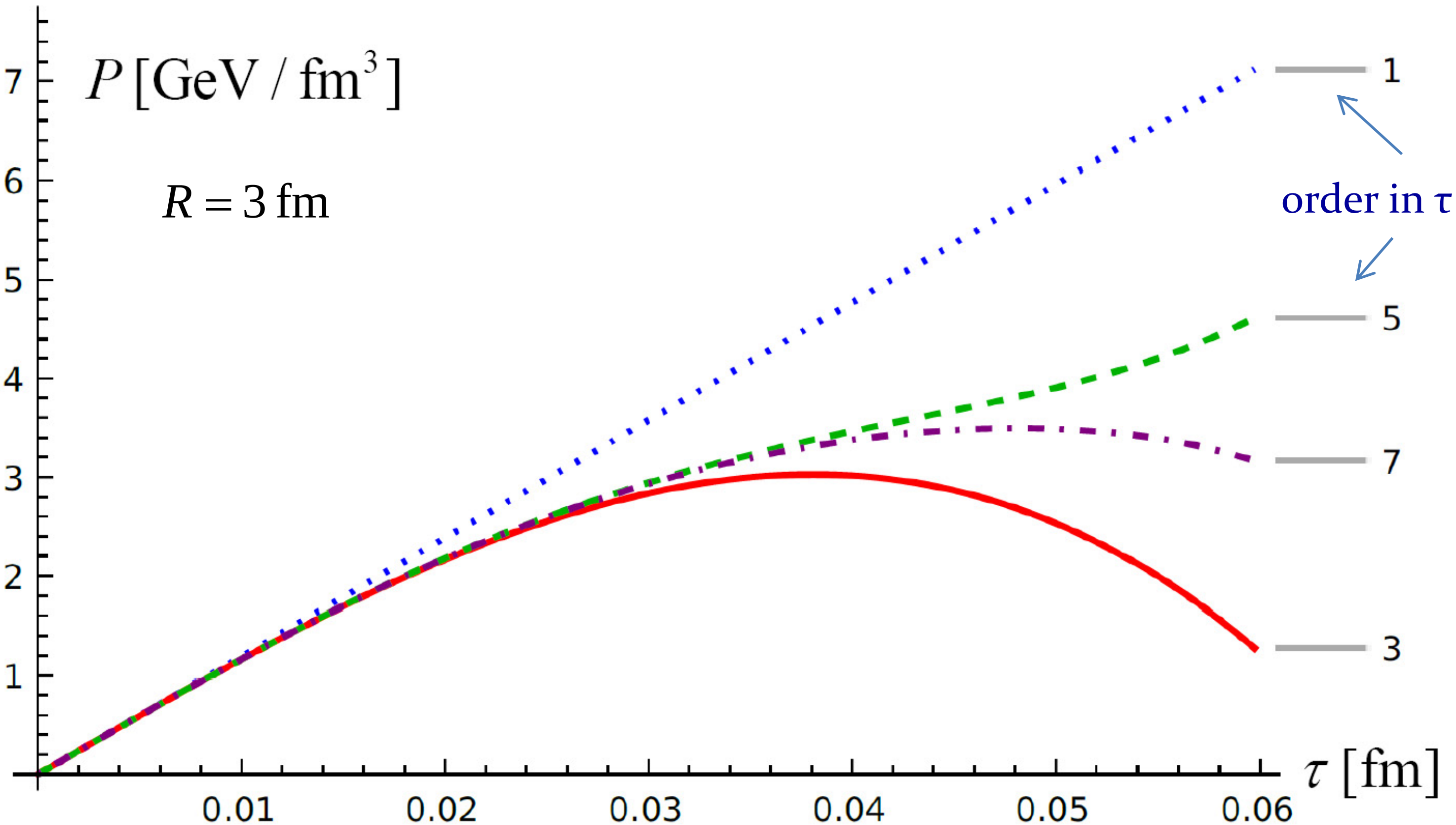


Radial flow

Pb-Pb collisions at b = 6 fm

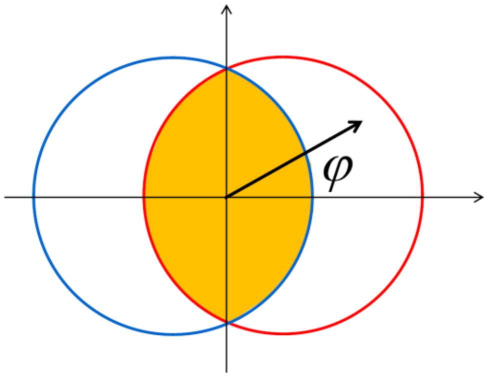
$$P \equiv R^i T^{0i}$$





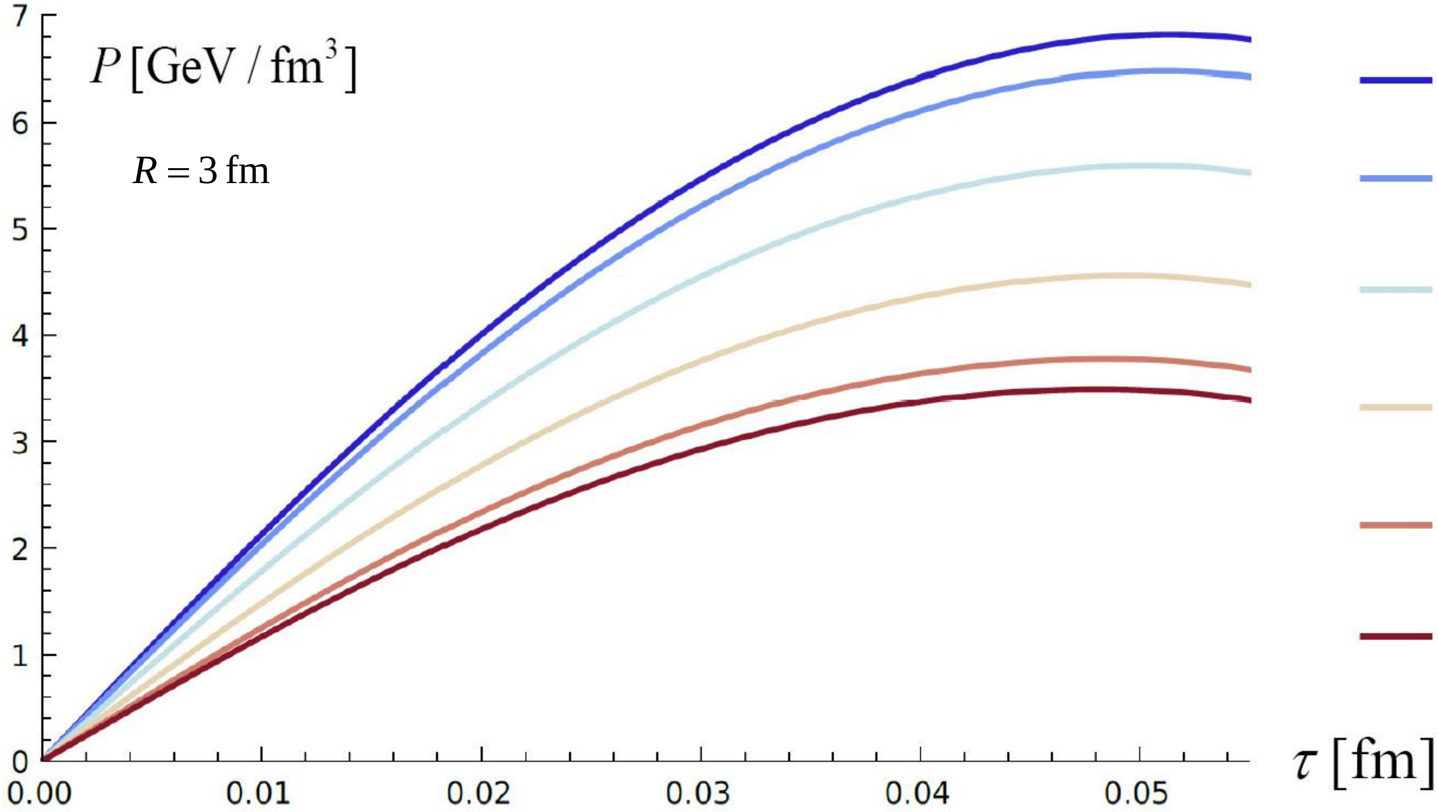
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Radial flow cont.



Pb-Pb collisions at b = 6 fm

$$P \equiv R^i T^{0i}$$

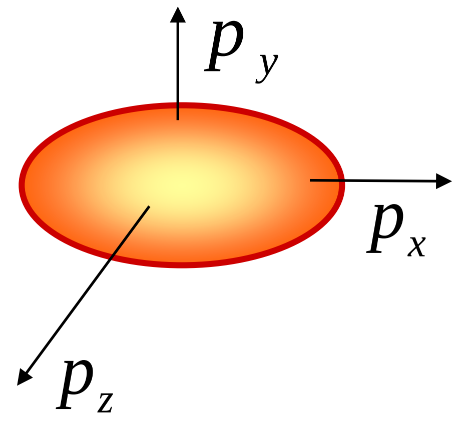


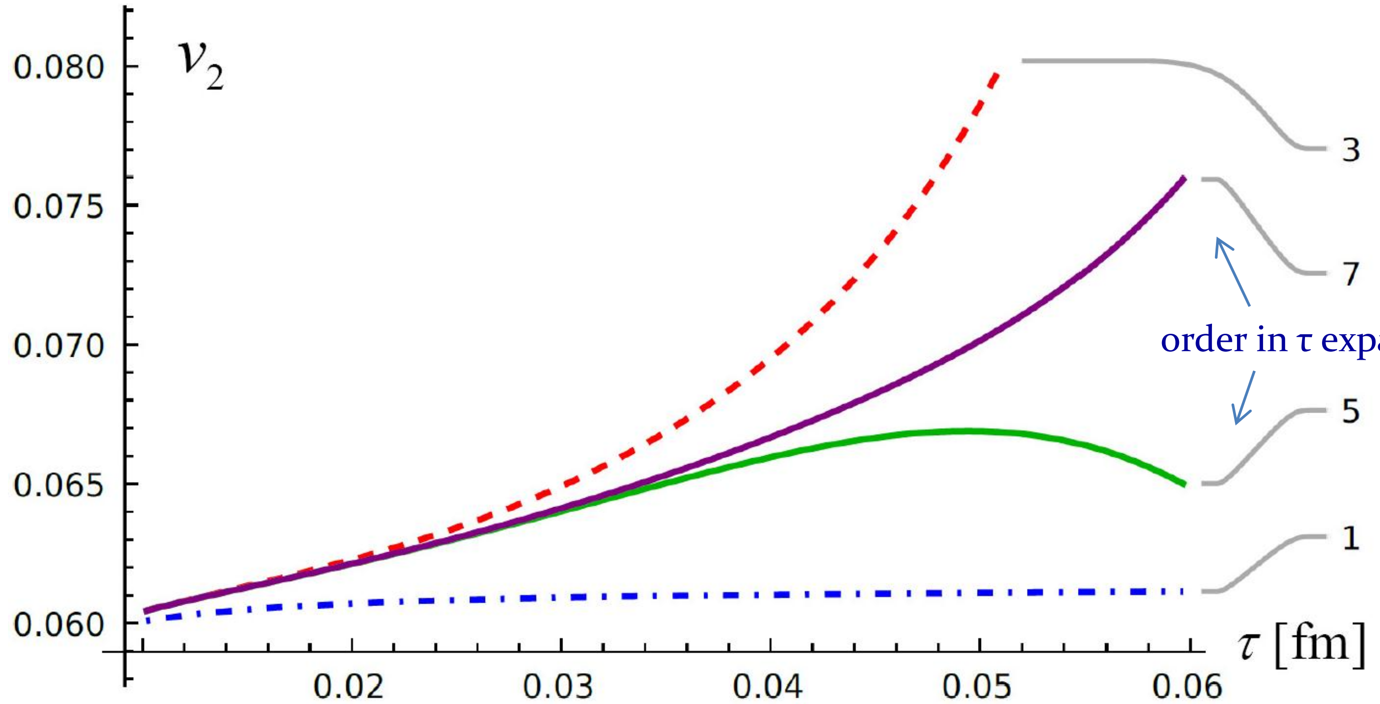
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Elliptic flow

Pb-Pb collisions at b = 2 fm

$$v_2 = \frac{\int d^2R \frac{T_{0x}^2 - T_{0y}^2}{\sqrt{T_{0x}^2 + T_{0y}^2}}}{\int d^2R \sqrt{T_{0x}^2 + T_{0y}^2}}$$



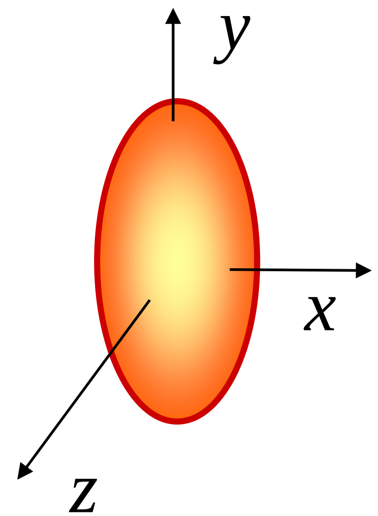


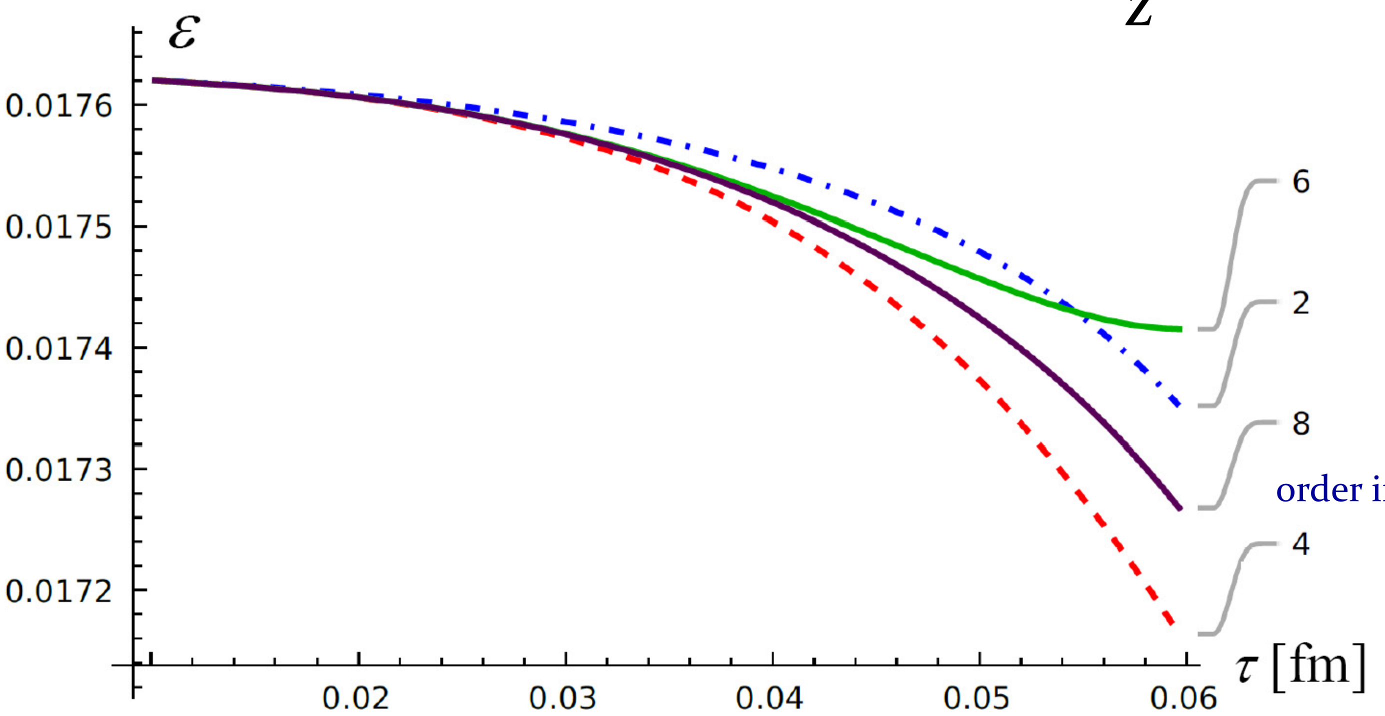
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Eccentricity

Pb-Pb collisions at b = 2 fm

$$\varepsilon = \frac{\int d^2R \frac{R_x^2 - R_y^2}{\sqrt{R_x^2 + R_y^2}} T^{00}}{\int d^2R \sqrt{R_x^2 + R_y^2}\, T^{00}}$$

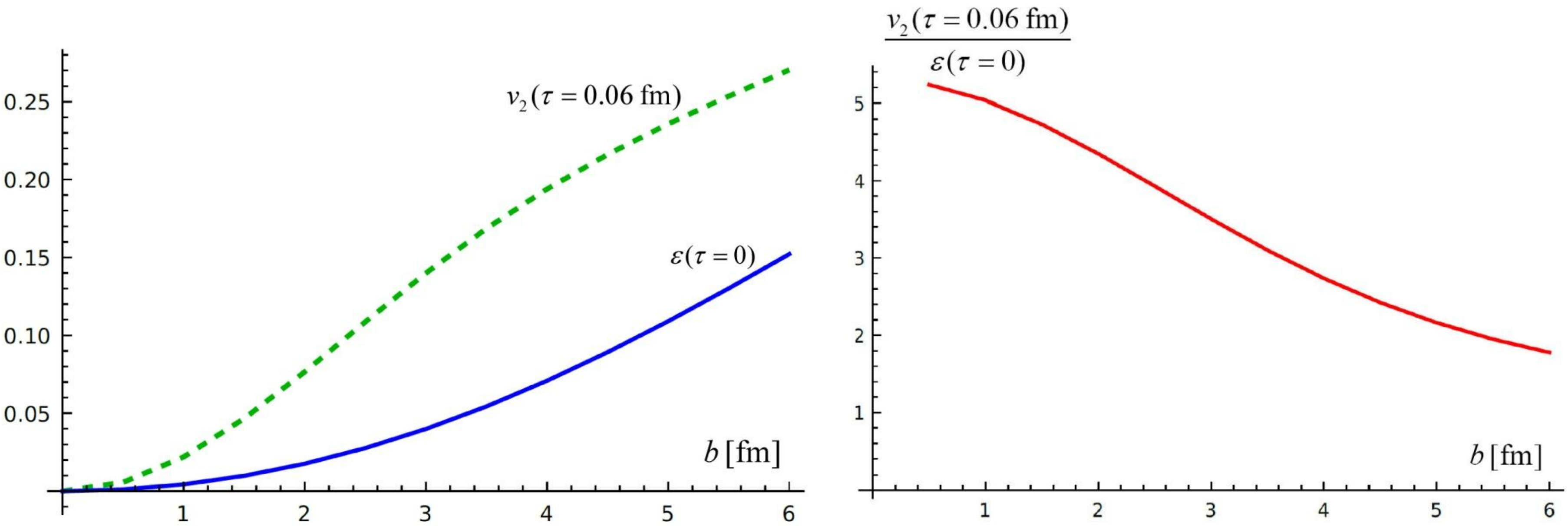


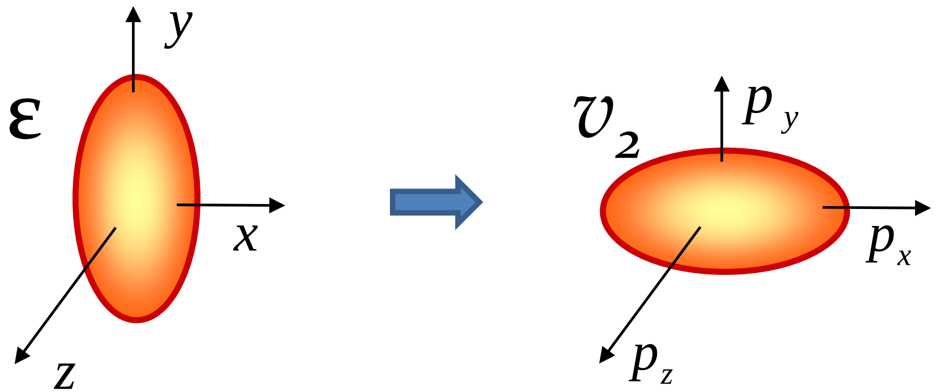


M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Hydrodynamic-like behavior

Pb-Pb collisions



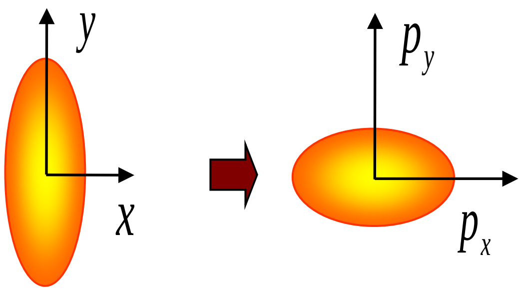


M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Equation of universal flow

$$\boxed{T^{0x} \approx -\frac{1}{2} t \frac{\partial T^{00}}{\partial x}}$$

Energy flow is generated by gradient of energy density.



Assumptions:

- $\partial_\mu T^{\mu\nu} = 0$
- Energy momentum tensor is mostly diagonal.
- The system is boost-invariant.

Glasma in ultrarelativistic collisions

$$T^{\mu\nu}(t=0) = \mathrm{diag}(\varepsilon, \varepsilon, \varepsilon, -\varepsilon)$$

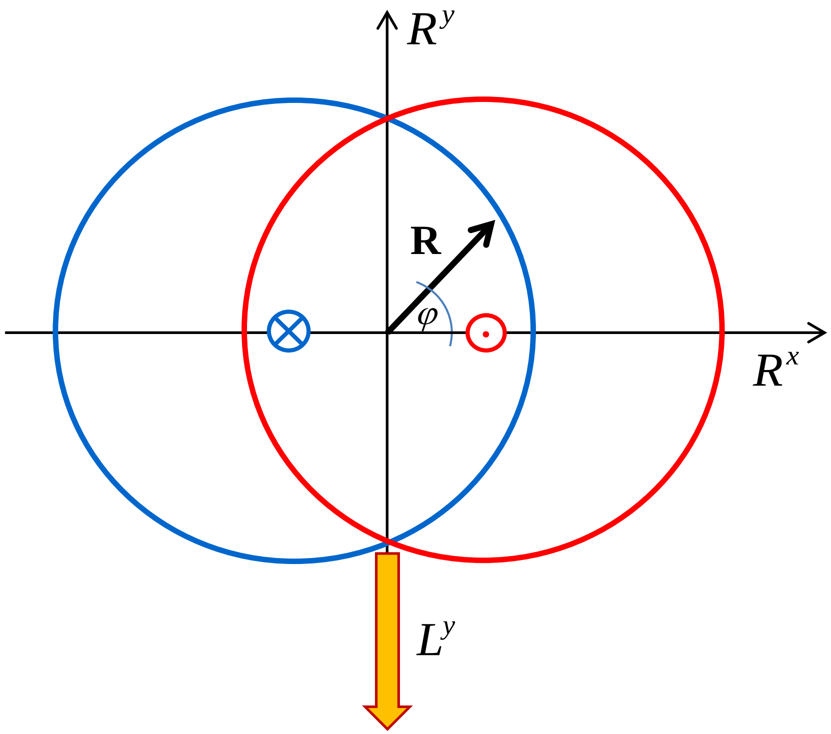
short-time evolution

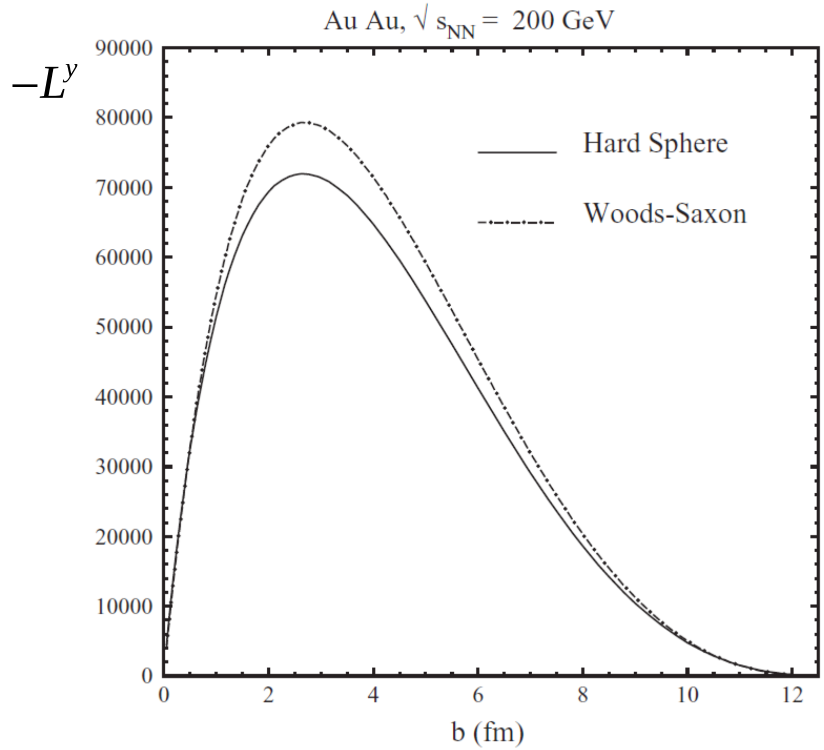
For glasma the equation of universal flow is satsified exactly order by order in proper time expansion.

J. Vredevoogd & S. Pratt, Phys. Rev. C **79**, 044915 (2009);

M. Carrington, St. Mrówczyński &J.-Y. Ollitrault, Phys. Rev. C **110**, 054903 (2024)

Angular momentum





$$L^y \sim 10^7 \hbar \text{ @ LHC } ?$$

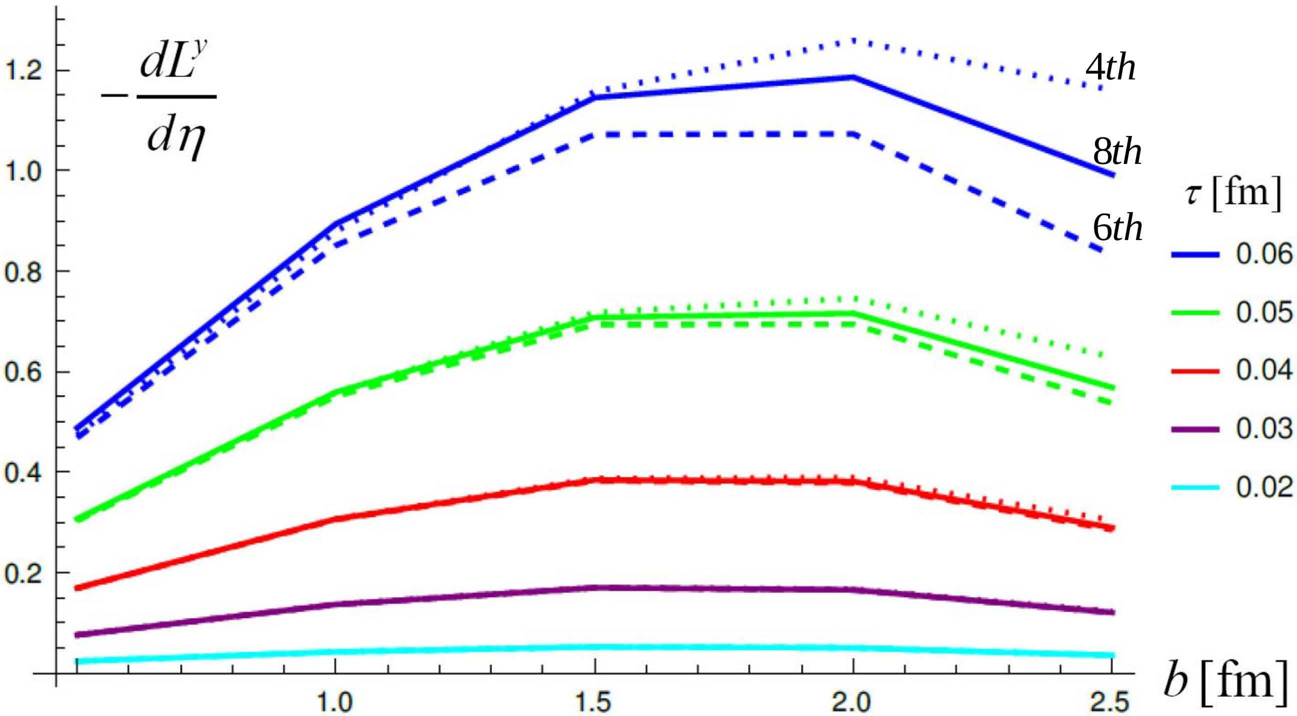
F. Becattini, F.Piccini, J. Rizzo, Physical Review C **77**, 024906 (2008)

Angular momentum cont.

Pb-Pb collisions

$$\frac{dL^\eta}{d\eta} = -\tau^2 \int d^2R\, R^x\, T^{\tau\eta}$$

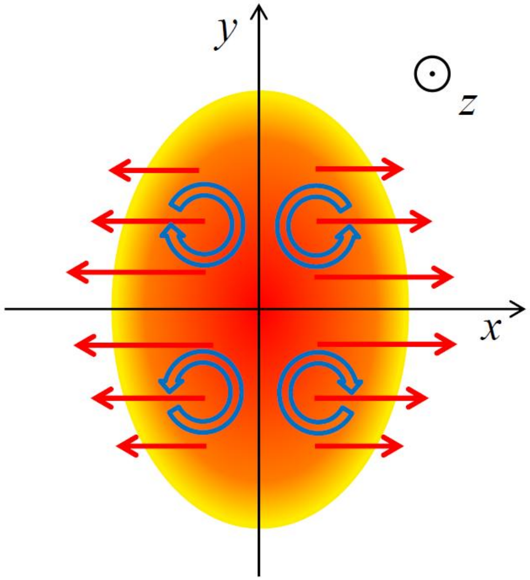
Milne coordinates

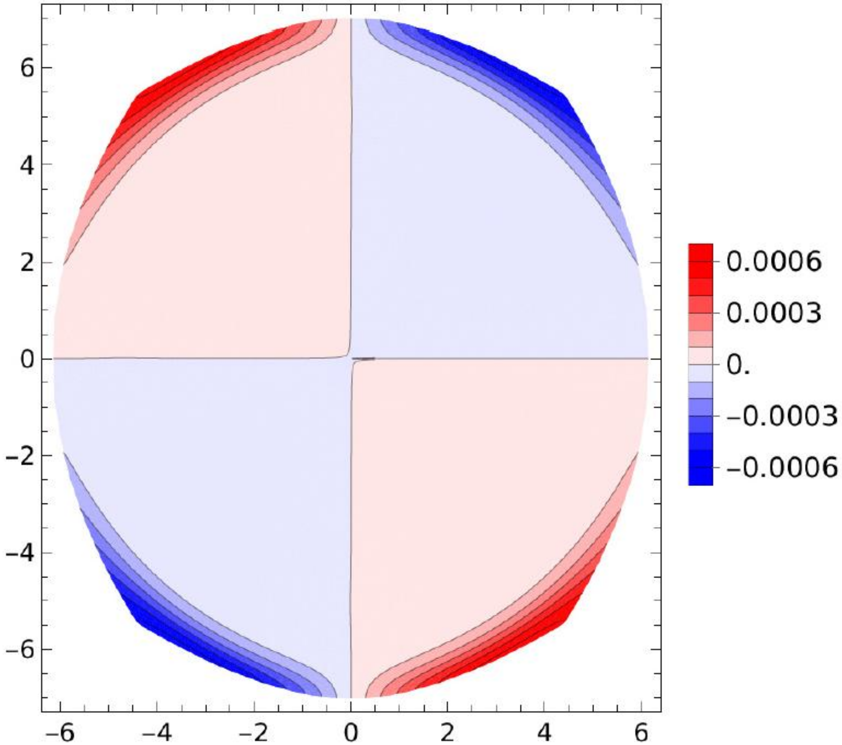


Glasma does not rotate!

M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Vorticity





$$V^i(\vec{r}) \equiv \frac{T^{0i}(\vec{r})}{T^{00}(\vec{r})}$$

ω^z at $b = 2$ fm & $\tau = 0.06$ fm in τ^8 order

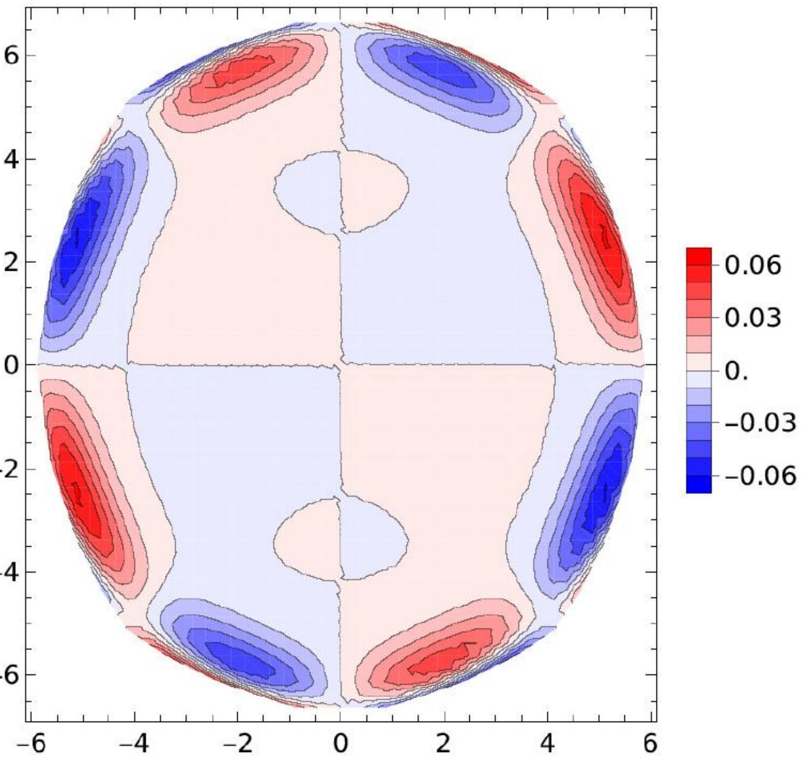
$$\omega(\vec{r}) = \nabla \times \vec{V}(\vec{r})$$

$$\omega_{\rm th}(\vec{r}) = \nabla \times \frac{\vec{u}(\vec{r})}{T(\vec{r})}$$

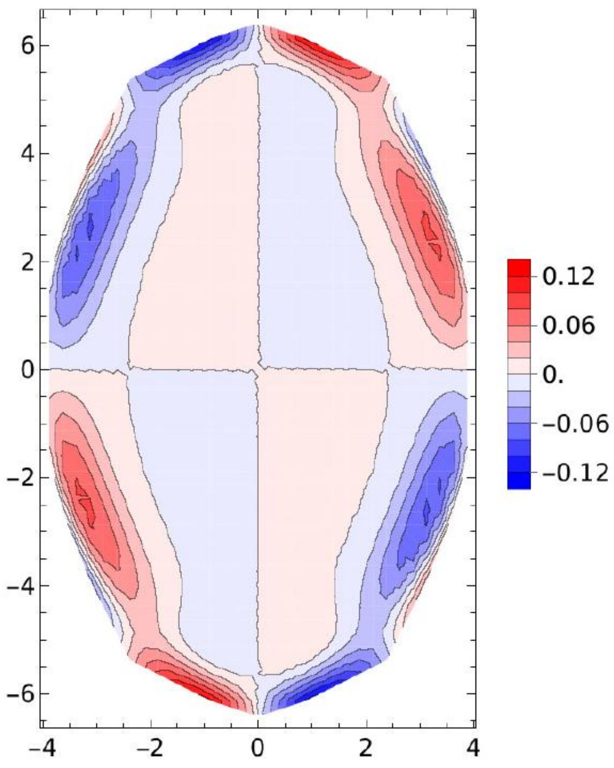
M. Carrington & St. Mrówczyński, arXiv:2505.07324

Local angular momentum

$$\frac{dL^z(\vec{r}_0)}{d\eta} = -\tau \int_{\Delta^2} d^2 r \Big((r^y - r_0^y) T^{0x} - (r^x - r_0^x) T^{0y} \Big)$$



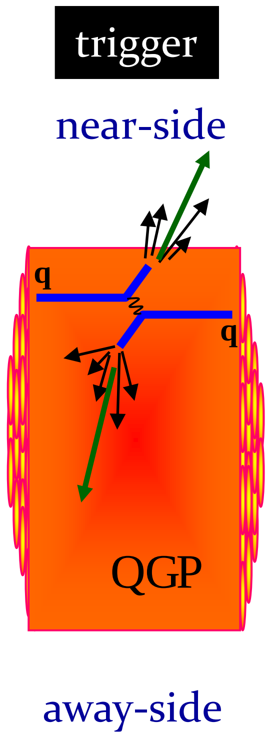
$b = 2$ fm



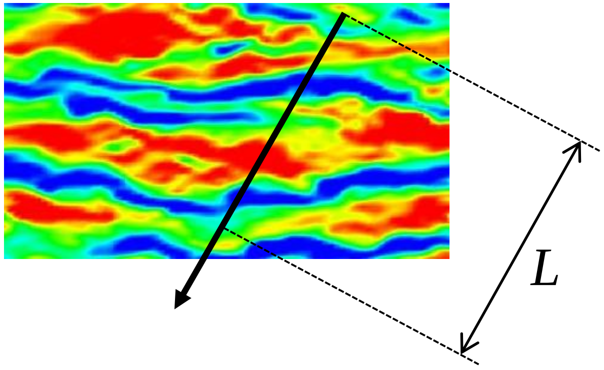
$b = 6$ fm

M. Carrington & St. Mrówczyński, arXiv:2505.07324

Jet quenching in glasma



How hard probes propagate through the glasma?

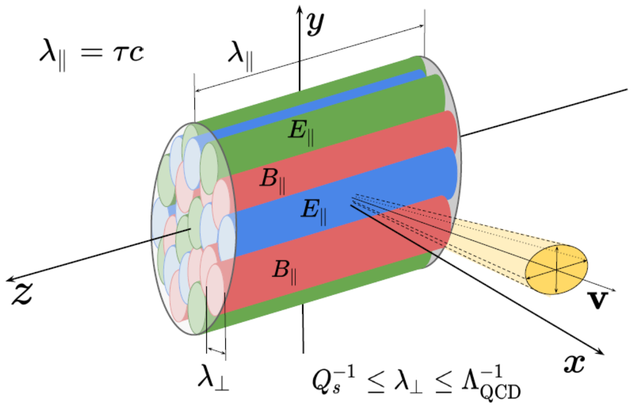


$\frac{dE}{dx}$ - collisional energy loss

$\hat{q}$ - transverse momentum broadening

$$\frac{dE^{\text{rad}}}{dx} = -\frac{1}{8}\alpha_s N_c \hat{q} L$$ - radiative energy loss

Hard probes in glasma - $\hat{q}$



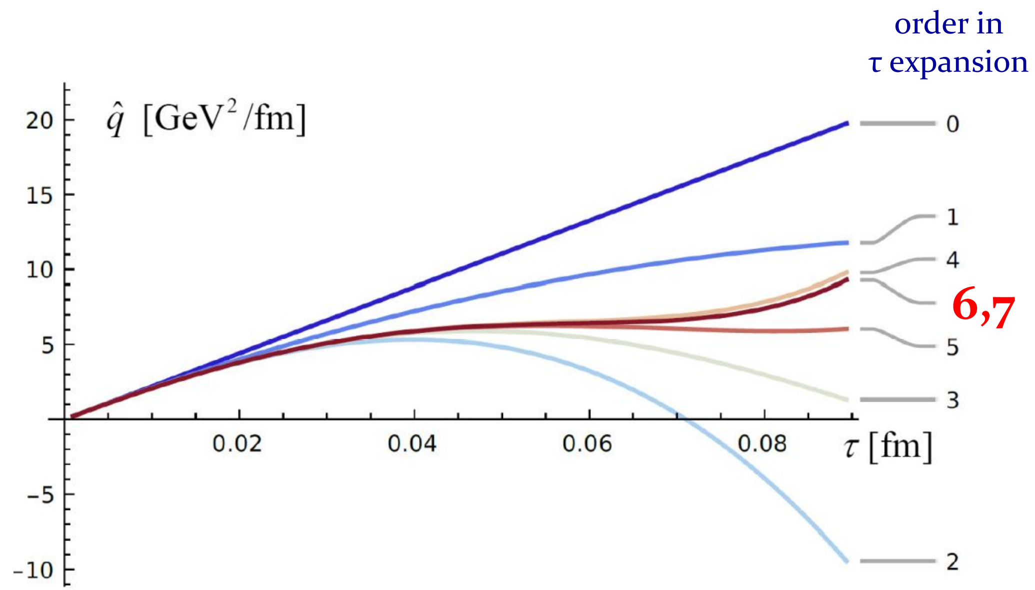
$$\hat{q} = \frac{2}{\mathrm{v}}\left(\delta^{ij} - \frac{\mathrm{v}^i \mathrm{v}^j}{\mathrm{v}^2}\right) X^{ji}(\mathbf{v})$$

$N_c = 3, \quad g = 1$

$Q_s = 2 \text{ GeV}$

$m = 0.2 \text{ GeV}$

$\mathrm{v} = \mathrm{v}_\perp = 1$



M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Glasma impact on jet quenching

Glasma

$$\hat{q}_{\max} = 6 \;\; \mathrm{GeV}^2 / \mathrm{fm}$$

$$t_{\max} = 0.06 \;\; \mathrm{fm}$$

Equilibrium QGP

$$\hat{q} = 3T^3$$

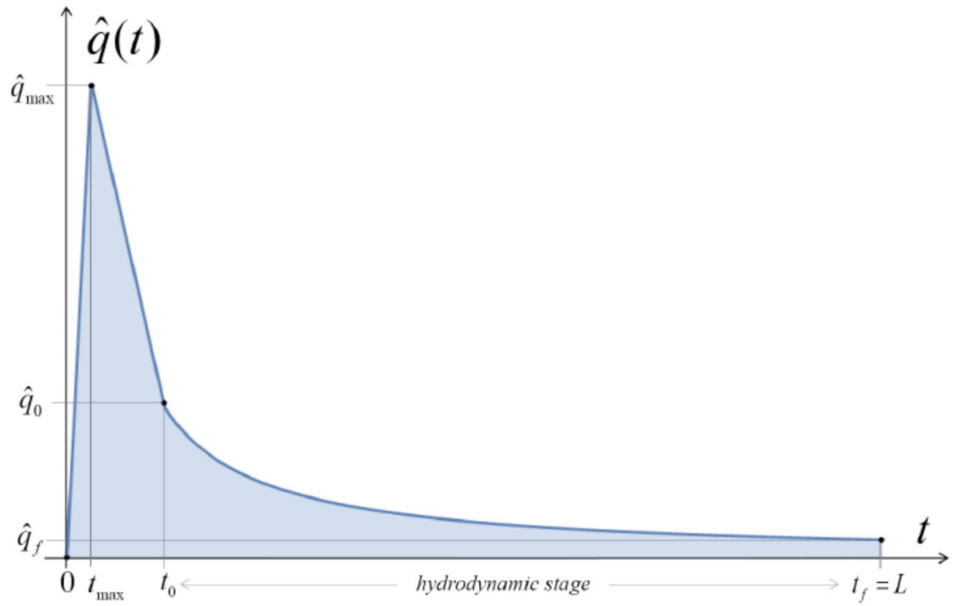
$$t_0 = 0.6 \; \mathrm{fm}$$

$$T_0 = 450 \; \mathrm{MeV}$$

$$\hat{q}_0 = 1.4 \;\; \mathrm{GeV}^2 / \mathrm{fm}$$

$$T = T_0 \left(\frac{t_0}{t} \right)^{1/3}$$

$$L = 10 \; \mathrm{fm}$$



$$\Delta p_T^2 \Big|^{\text{non-eq}} = \int_0^{t_0} dt \, \hat{q}(t)$$

$$\Delta p_T^2 \Big|^{\text{eq}} = \int_{t_0}^{L} dt \, \hat{q}(t)$$

$$\boxed{\frac{\Delta p_T^2 \Big|^{\text{non-eq}}}{\Delta p_T^2 \Big|^{\text{eq}}} = 0.93}$$

S. Cao et al. [JETSCAPE], Physical Review C **104**, 024905 (2021),
C. Shen, U. Heinz, P. Huovinen and H. Song, Physical Review C **84**, 044903 (2011).

Conclusions

- The glasma evolves in a hydrodynamic-like way.
- The glasma's global orbital momentum is small,
the system does not rotate.
- There is a local angular momentum of glasma along the beam
consistent with a sign consistent of the Λ polarization.
- In spite of its short lifetime the glasma provides
a significant contribution to the jet quenching.