

The earliest phase of relativistic heavy-ion collisions

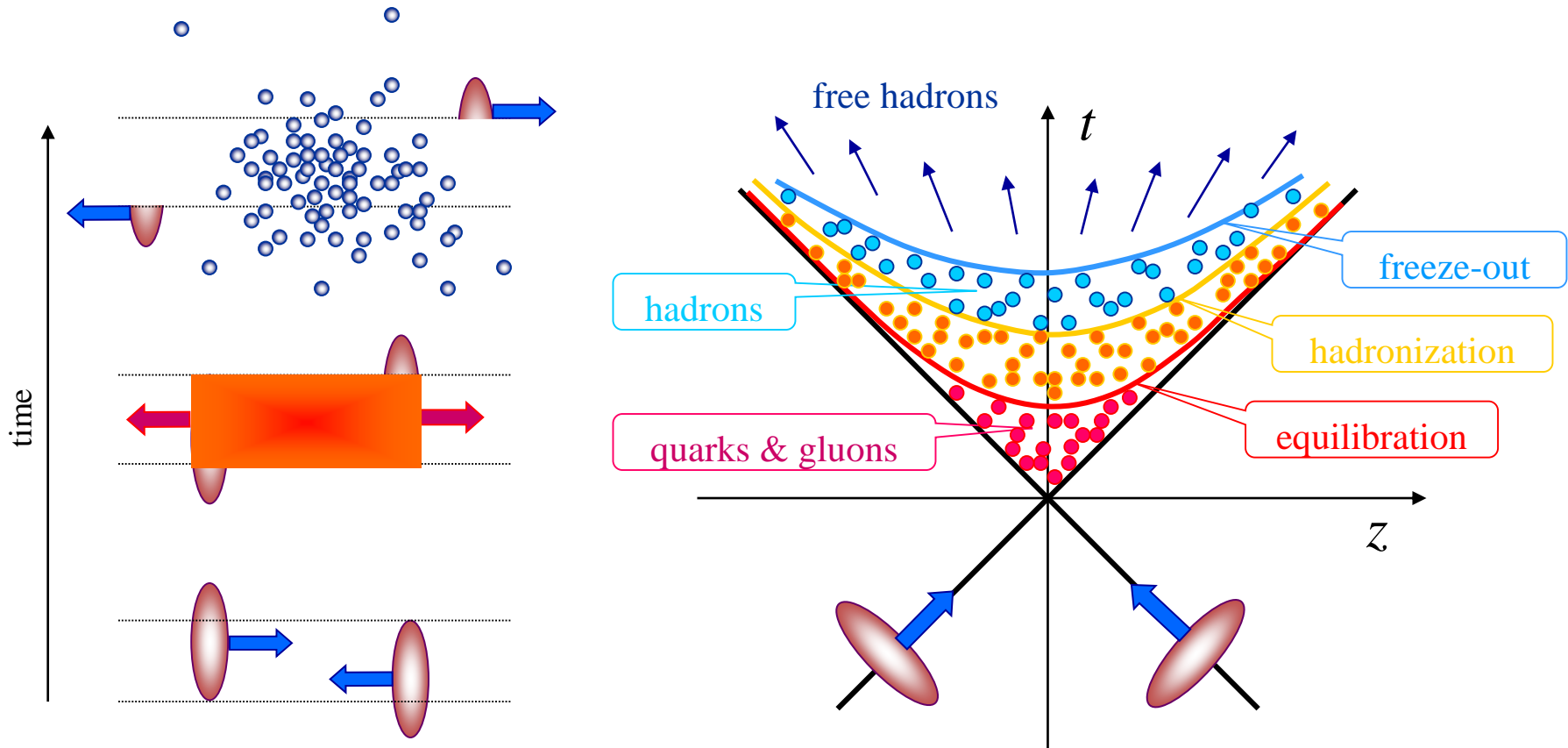
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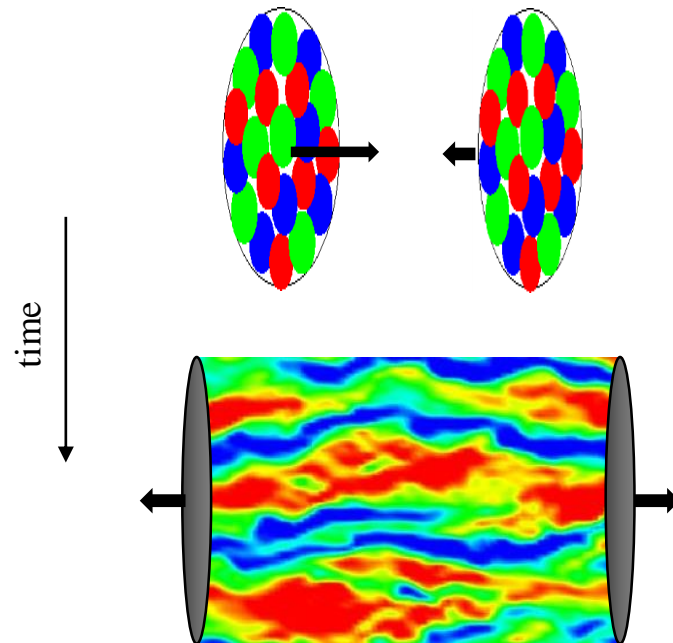
**Alina Czajka, Wade Cowie, Bryce Friesen,
Jean-Yves Ollitrault and Doug Pickering**

Relativistic heavy-ion collisions



Color Glass Condensate

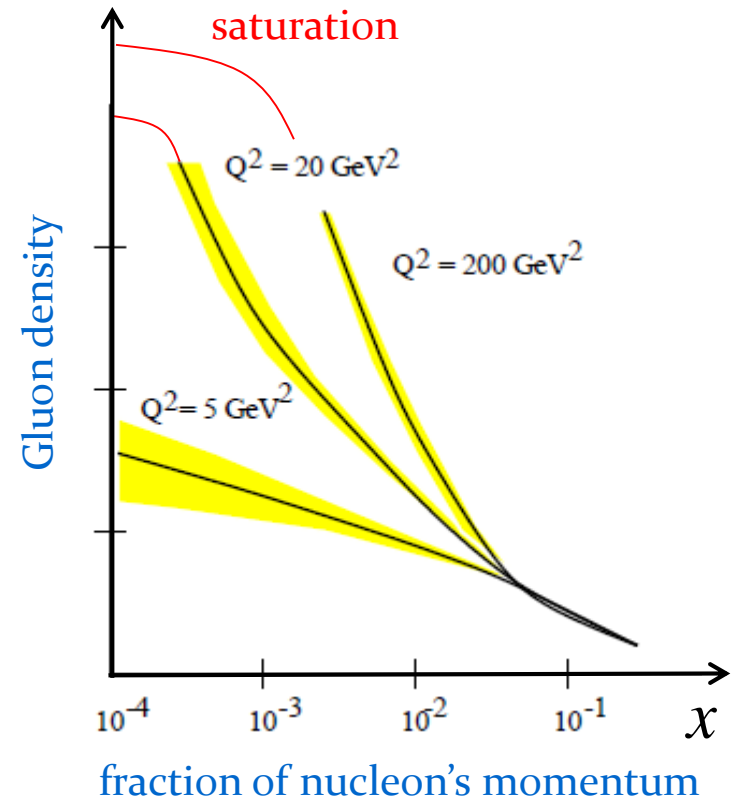
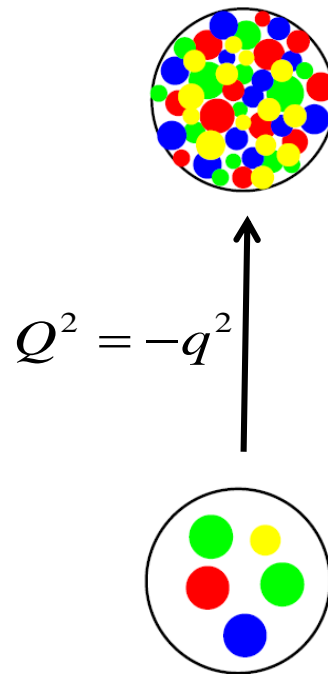
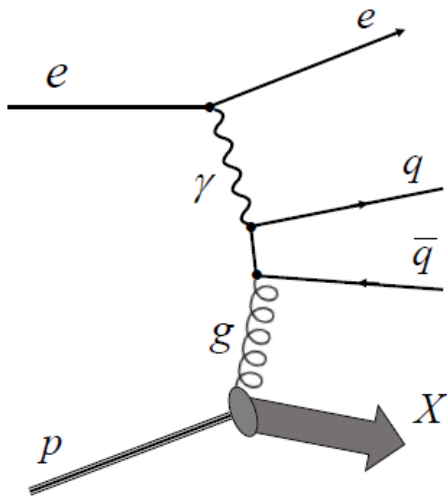
Color charges confined in the colliding nuclei generate **glasma** – the system of strong mostly classical chromodynamic fields which evolves towards equilibrium.



What are characteristics of the earliest ($\tau < 0.1 \text{ fm}/c$) stage of relativistic heavy-ion collisions?

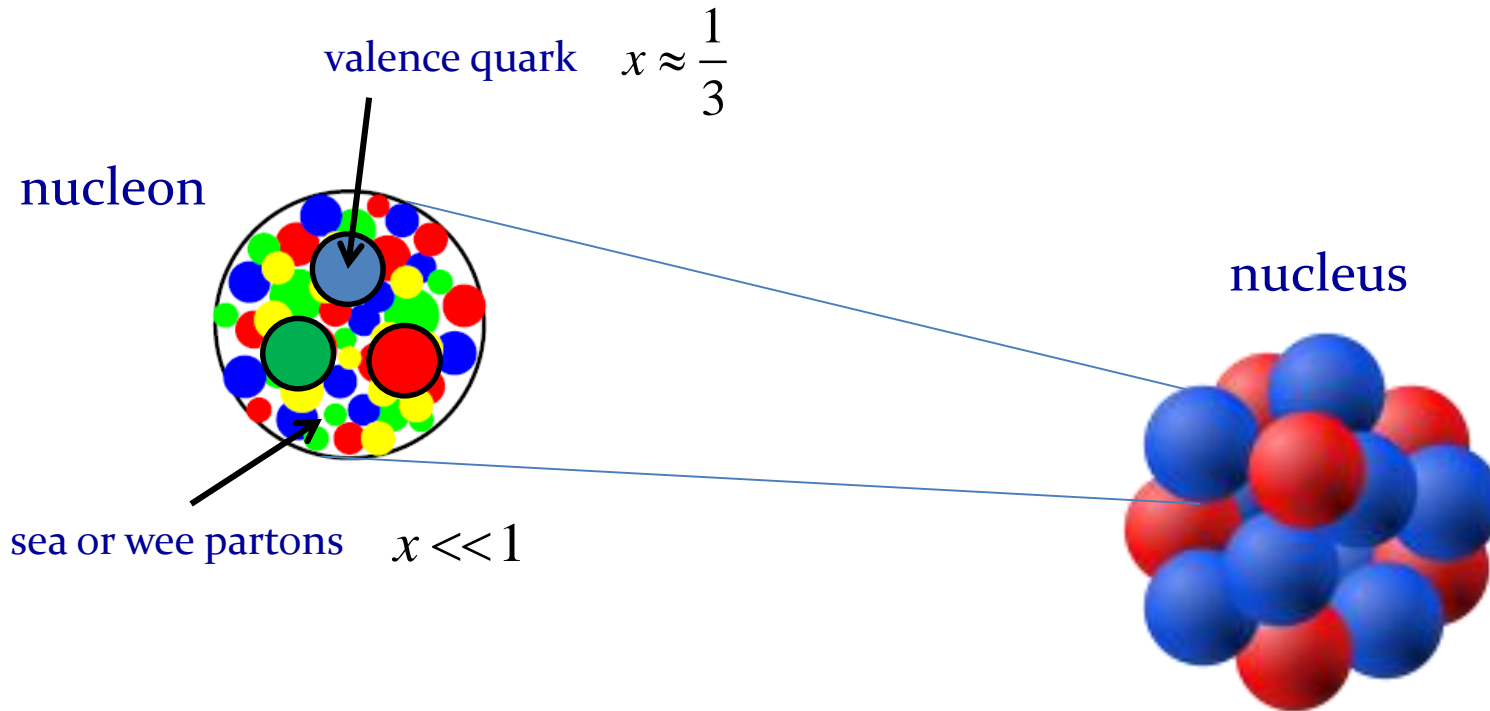
Saturation

Deep Inelastic Scattering



Saturated gluon system can be described in terms of classical chromodynamic fields.

Wee partons & valence quarks



In relativistic heavy-ion collisions

- ▶ Saturated wee partons – classical chromodynamic fields
- ▶ Valence quarks – classical sources of chromodynamic fields

Color Glass Condensate

Classical Yang-Mills equation

$$D_\mu F^{\mu\nu}(x) = j^\nu(x)$$

$$j^\mu(x) = j_1^\mu(x) + j_2^\mu(x)$$

$$j_{1,2}^\mu(x) = \pm \delta^{\mu\mp} \delta(x^\pm) \rho_{1,2}(\mathbf{x}_\perp)$$

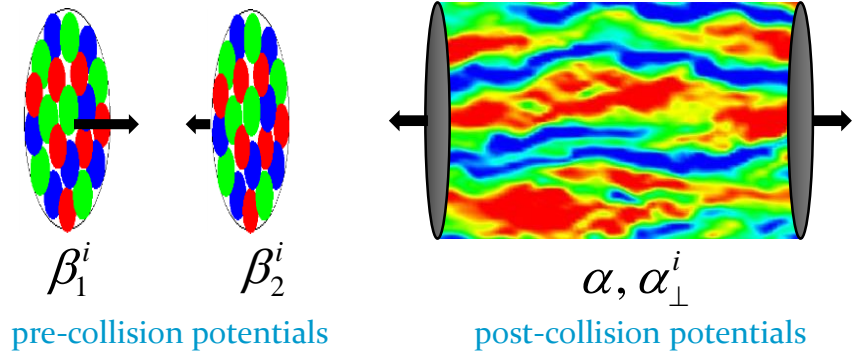
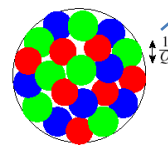
Ansatz of gauge potentials

$$A^+(x) = \Theta(x^+) \Theta(x^-) x^+ \alpha(\tau, \mathbf{x}_\perp)$$

$$A^-(x) = -\Theta(x^+) \Theta(x^-) x^- \alpha(\tau, \mathbf{x}_\perp)$$

$$A^i(x) = \Theta(x^+) \Theta(x^-) \alpha_\perp^i(\tau, \mathbf{x}_\perp)$$

$$+ \Theta(-x^+) \Theta(x^-) \beta_1^i(\mathbf{x}_\perp) + \Theta(x^+) \Theta(-x^-) \beta_2^i(\mathbf{x}_\perp)$$



Boundary condition

$$\begin{cases} \alpha(0, \mathbf{x}_\perp) = \beta_1^i(\mathbf{x}_\perp) + \beta_2^i(\mathbf{x}_\perp) \\ \alpha_\perp^i(0, \mathbf{x}_\perp) = -\frac{ig}{2} [\beta_1^i(\mathbf{x}_\perp), \beta_2^i(\mathbf{x}_\perp)] \end{cases}$$

Gauge condition

$$x^+ A^- + x^- A^+ = 0$$

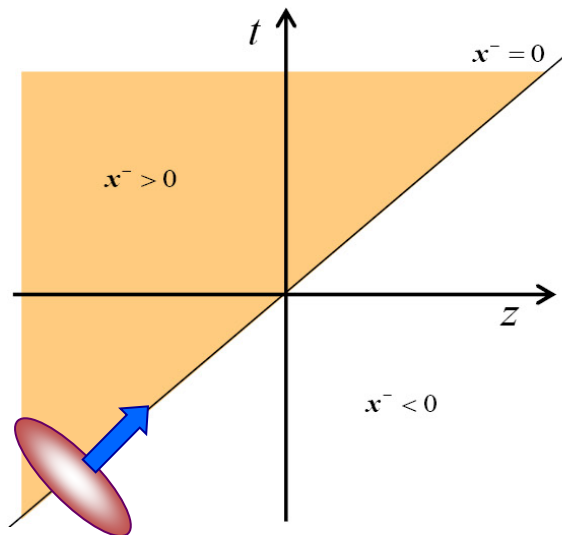
Heavy-ion collisions in light-cone variables

$$x^\pm \equiv \frac{t \pm z}{\sqrt{2}}$$

$$x^\pm = 0 \Rightarrow z = \pm t$$

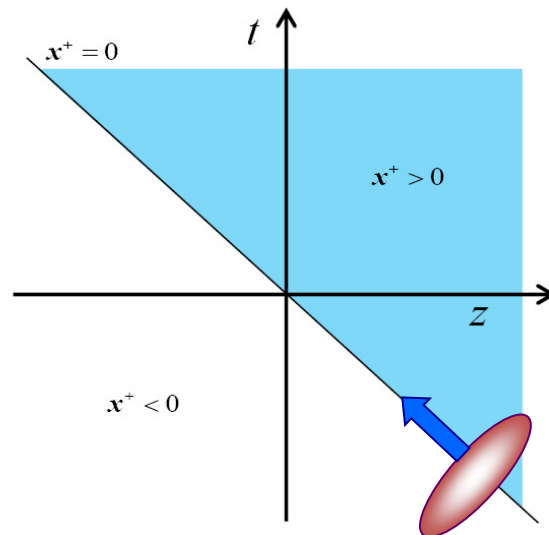
natural units $c = 1$

$$\Theta(x^-) > 0$$



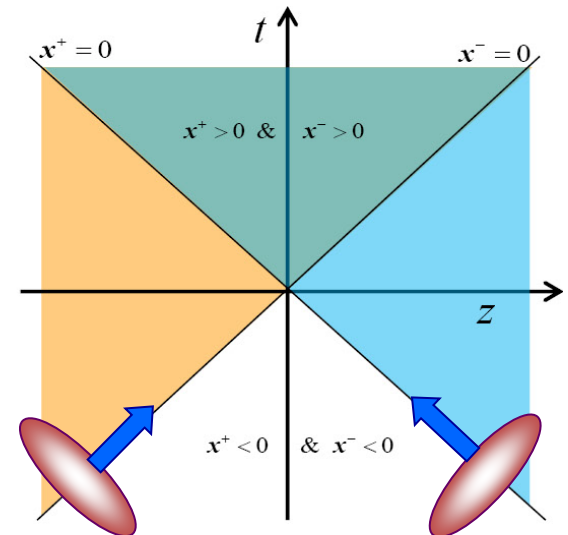
$$x^- = 0$$

$$\Theta(x^+) > 0$$



$$x^+ = 0$$

$$\Theta(x^-)\Theta(x^+) > 0$$



Proper time expansion

$$\alpha(\tau, \mathbf{x}_\perp) = \sum_{n=0}^{\infty} \tau^n \alpha_{(n)}(\mathbf{x}_\perp), \quad \alpha_\perp^i(\tau, \mathbf{x}_\perp) = \sum_{n=0}^{\infty} \tau^n \alpha_{\perp(n)}^i(\mathbf{x}_\perp)$$

Proper time τ is treated as a small parameter $\tau \ll Q_s^{-1}$

Yang-Mills equations for the expanded potentials are solved recursively

$$\alpha_{(n)} = \alpha_{\perp(n)}^i = 0 \quad \text{for } n = 1, 3, 5, \dots$$

0th order - boundary conditions

$$\begin{cases} \alpha_{(0)} = -\frac{ig}{2} [\beta_1^i, \beta_2^i] \\ \alpha_{\perp(0)}^i = \beta_1^i + \beta_2^i \end{cases}$$

Post-collision potentials are expressed through pre-collision potentials

2nd order

$$\begin{cases} \alpha_{(2)} = -\frac{ig}{16} [D^j, [D^j, [\beta_1^i, \beta_2^i]]] \\ \alpha_{\perp(2)}^i = \frac{ig}{4} \varepsilon^{zij} \varepsilon^{zkl} [D^j, [\beta_1^k, \beta_2^l]] \end{cases}$$

$$D^i \equiv \partial^i - ig(\beta_1^i + \beta_2^i)$$

Fully analytic approach!

Proper time expansion cont.

Chromoelectric and chromomagnetic fields

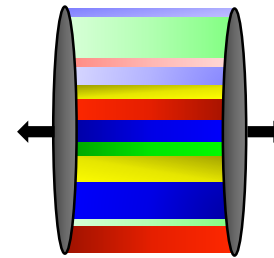
$$E^i = F^{i0}, \quad B^i = \frac{1}{2} \varepsilon^{ijk} F^{kj}$$

0th order

$$\mathbf{E}_{(0)} = (0, 0, E), \quad \mathbf{B}_{(0)} = (0, 0, B)$$

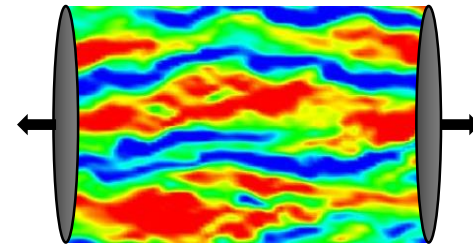
$$E_{(0)}^z(\mathbf{x}_\perp) = -ig[\beta_1^i(\mathbf{x}_\perp), \beta_2^i(\mathbf{x}_\perp)]$$

$$B_{(0)}^z(\mathbf{x}_\perp) = -ig\varepsilon^{zij}[\beta_1^i(\mathbf{x}_\perp), \beta_2^j(\mathbf{x}_\perp)]$$



E & *B* fields along the axis *z*

At higher orders transverse fields show up



Energy-momentum tensor

▶
$$T^{\mu\nu} = 2\text{Tr}[F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma}]$$

▶
$$F^{\mu\nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu} - ig[A^{\mu}, A^{\nu}]$$

The energy-momentum tensor is symmetric, gauge invariant and obeys

▶
$$\partial_{\mu} T^{\mu\nu} = 0$$

T^{00} - energy density

T^{0i} - energy flux, Poynting vector

T^{xx}, T^{yy}, T^{zz} - pressures

T^{ij} - momentum flux

Averaging over collisions

$$T^{\mu\nu} \sim \sum \partial^i \partial^j \beta^k \beta^l \dots \beta^m \Rightarrow \langle T^{\mu\nu} \rangle \sim \sum \partial^i \partial^j \langle \beta^k \beta^l \dots \beta^m \rangle$$

The pre-collision potentials in covariant gauge $\partial_\mu \beta^\mu = 0$ obey

$$-\nabla^2 \beta^+(\mathbf{x}_\perp) = \rho(\mathbf{x}_\perp) \Rightarrow \beta^+(\mathbf{x}_\perp) = \frac{1}{2\pi} \int d^2 x'_\perp K_0(m|\mathbf{x}_\perp - \mathbf{x}'_\perp|) \rho(\mathbf{x}'_\perp)$$

IR regulator $m = \Lambda_{\text{QCD}}$

The potentials are transformed from the covariant to light-cone gauge $\beta_1^+ = \beta_2^- = 0$

Wick theorem

$$\langle \rho_a^k(\mathbf{x}_\perp) \rho_b^l(\mathbf{y}_\perp) \dots \rho_c^m(\mathbf{z}_\perp) \rangle = \sum \Pi \langle \rho_a^i(\mathbf{x}_\perp) \rho_b^j(\mathbf{y}_\perp) \rangle$$

Glasma graph approximation

$$\langle \beta_a^k(\mathbf{x}_\perp) \beta_b^l(\mathbf{y}_\perp) \dots \beta_c^m(\mathbf{z}_\perp) \rangle = \sum \Pi \langle \beta_a^i(\mathbf{x}_\perp) \beta_b^j(\mathbf{y}_\perp) \rangle = \sum \Pi B_{ab}^{ij}(\mathbf{x}_\perp, \mathbf{y}_\perp)$$

Basic correlator in transversally uniform system

$$B_{ab}^{ij}(\mathbf{x}_\perp - \mathbf{y}_\perp) \equiv \langle \beta_a^i(\mathbf{x}_\perp) \beta_b^j(\mathbf{y}_\perp) \rangle = \int d^2 x'_\perp d^2 y'_\perp \cdots \langle \rho_a^i(\mathbf{x}'_\perp) \rho_b^j(\mathbf{y}'_\perp) \rangle$$

$$\langle \rho_a^i(\mathbf{x}_\perp) \rho_b^j(\mathbf{y}_\perp) \rangle = g^2 \mu \delta^{ab} \delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp)$$

color charge surface density

$$\mu = g^{-4} Q_s^2$$

$$B_{ab}^{ij}(\mathbf{x}_\perp - \mathbf{y}_\perp) \equiv \delta^{ab} \left(\delta^{ij} C_1(r) - \hat{r}^i \hat{r}^j C_2(r) \right)$$

$$\mathbf{r} \equiv \mathbf{x}_\perp - \mathbf{y}_\perp, \quad r \equiv |\mathbf{r}|, \quad \hat{r}^i \equiv \frac{r^i}{r}$$

$$\left\{ \begin{array}{l} C_1(r) \equiv \frac{m^2 K_0(mr)}{g^2 N_c (mr K_1(mr) - 1)} \left\{ \exp \left[\frac{g^4 N_c \mu (mr K_1(mr) - 1)}{4\pi m^2} \right] - 1 \right\} \\ C_2(r) \equiv \frac{m^3 r K_1(mr)}{g^2 N_c (mr K_1(mr) - 1)} \left\{ \exp \left[\frac{g^4 N_c \mu (mr K_1(mr) - 1)}{4\pi m^2} \right] - 1 \right\} \end{array} \right. \approx_{r \ll m^{-1}} \# \log(mr)$$

UV regularization required

$$r > Q_s^{-1}$$

Basic correlator in transversally non-uniform system

$$B_{ab}^{ij}(\mathbf{x}_\perp, \mathbf{y}_\perp) \equiv \langle \beta_a^i(\mathbf{x}_\perp) \beta_b^j(\mathbf{y}_\perp) \rangle = \int d^2 x'_\perp d^2 y'_\perp \cdots \langle \rho_a^i(\mathbf{x}'_\perp) \rho_b^j(\mathbf{y}'_\perp) \rangle$$

$$\langle \rho_a^i(\mathbf{x}_\perp) \rho_b^j(\mathbf{y}_\perp) \rangle = g^2 \mu(\mathbf{x}_\perp) \delta^{ab} \delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp)$$

color charge surface density

$$\mu = g^{-4} Q_s^2$$

Projected Woods-Saxon distribution

$$\mu(\mathbf{x}_\perp) = \frac{\bar{\mu}}{\ln(1 + e^{R_A/a})} \int_{-\infty}^{\infty} \frac{dz}{1 + \exp\left[\left(\sqrt{\mathbf{x}_\perp^2 + z^2} - R_A\right)/a\right]}$$

System uniform in the transverse plane

$$B_{ab}^{ij}(\mathbf{x}_\perp, \mathbf{y}_\perp) = \delta^{ab} f^{ij}(\mathbf{x}_\perp - \mathbf{y}_\perp) = \delta^{ab} f^{ij}(\mathbf{r})$$

$$\begin{cases} \mathbf{R} = \frac{1}{2}(\mathbf{x}_\perp + \mathbf{y}_\perp) \\ \mathbf{r} = \mathbf{x}_\perp - \mathbf{y}_\perp \end{cases}$$

System weakly nonuniform in the transverse plane

$$B_{ab}^{ij}(\mathbf{x}_\perp, \mathbf{y}_\perp) = \delta^{ab} f^{ij}(\mathbf{R}, \mathbf{r}) \approx \text{``gradient expansion in } \mathbf{R}\text{''}$$

Numerical results

Pb-Pb collisions at LHC

$$N_c = 3$$

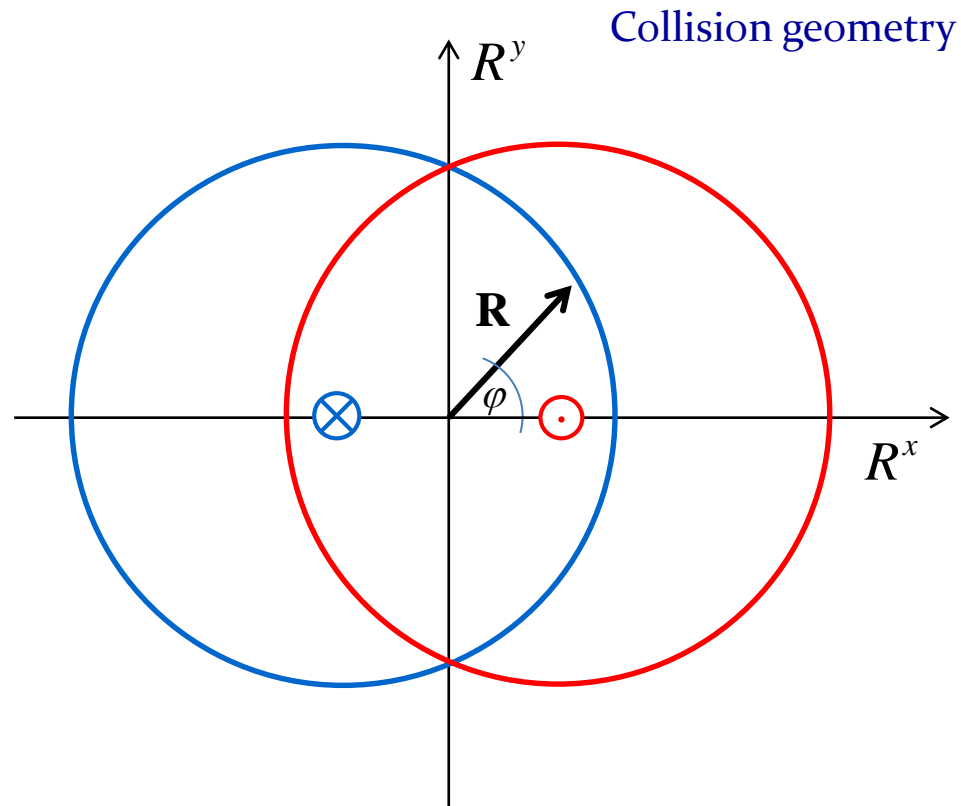
$$g = 1$$

$$Q_s = 2 \text{ GeV}$$

$$m = 0.2 \text{ GeV}$$

$$R_A = 7.4 \text{ fm}$$

$$a = 0.5 \text{ fm}$$

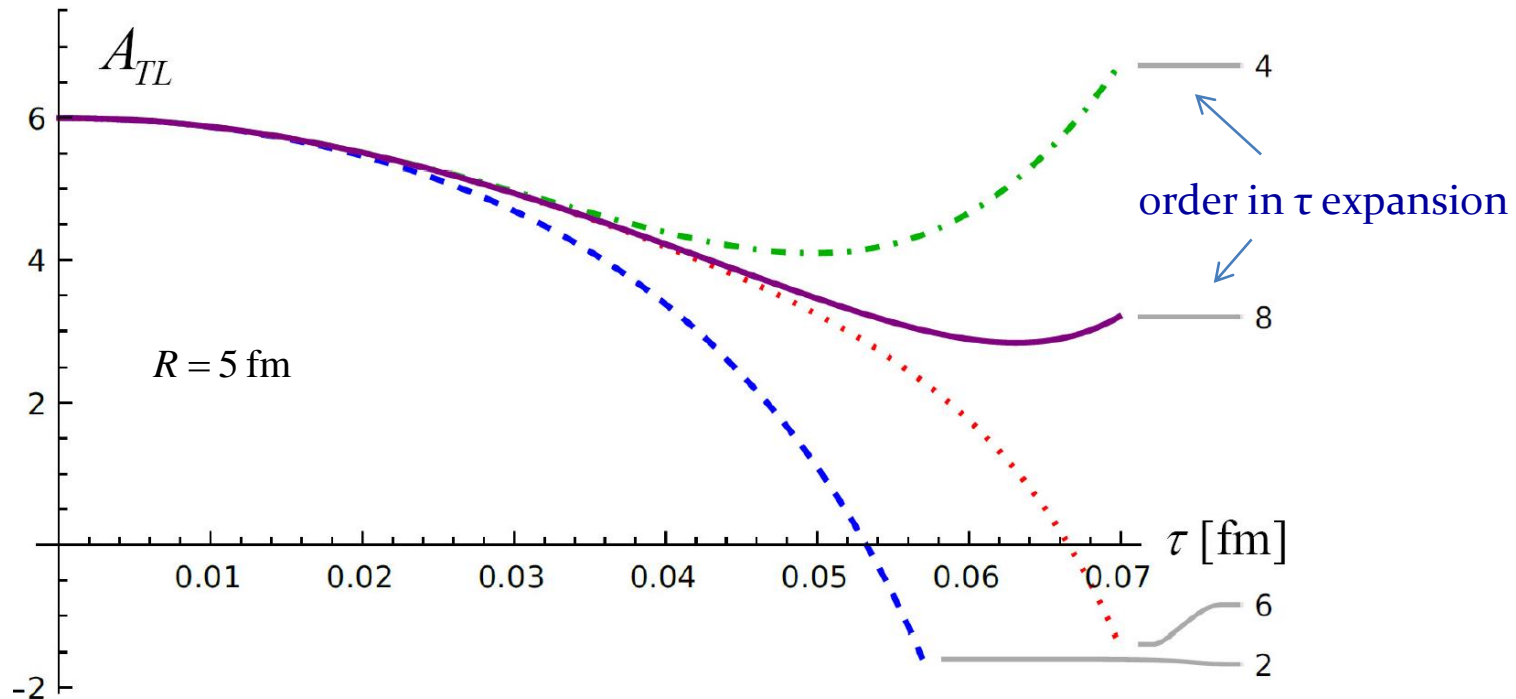


Anisotropy

Central Pb-Pb collisions

$$A_{TL} \equiv \frac{3(p_T - p_L)}{2p_T + p_L} \quad p_T \equiv \langle T^{xx} \rangle, \quad p_L \equiv \langle T^{zz} \rangle$$

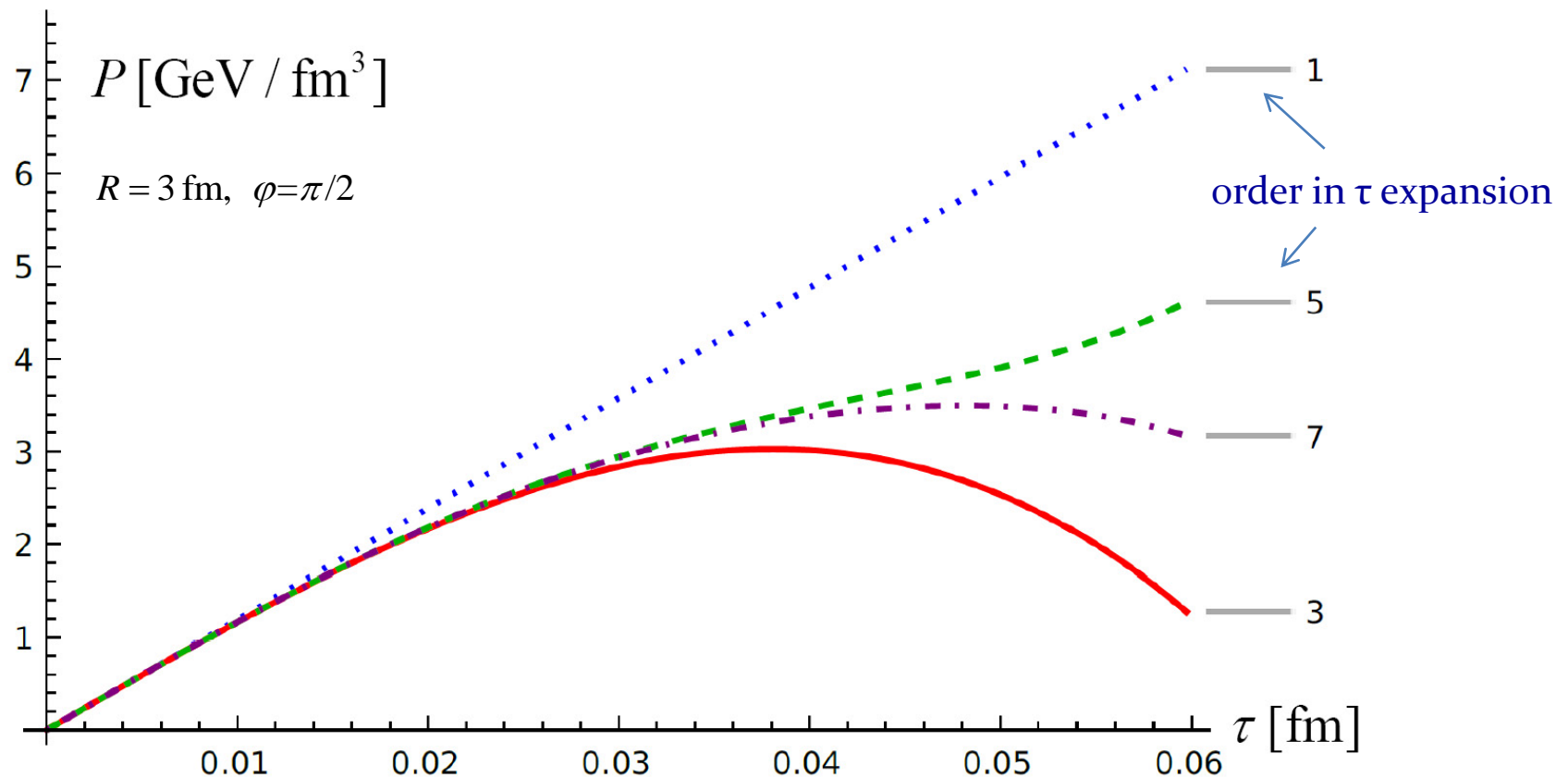
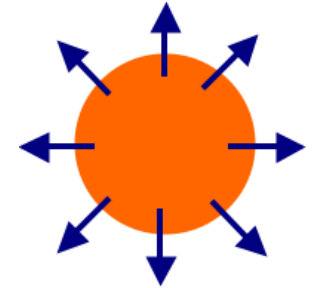
$$\tau = 0 \Rightarrow p_T = -p_L = \varepsilon \Rightarrow A_{TL} = 6$$



Radial flow

Pb-Pb collisions at $b = 6$ fm

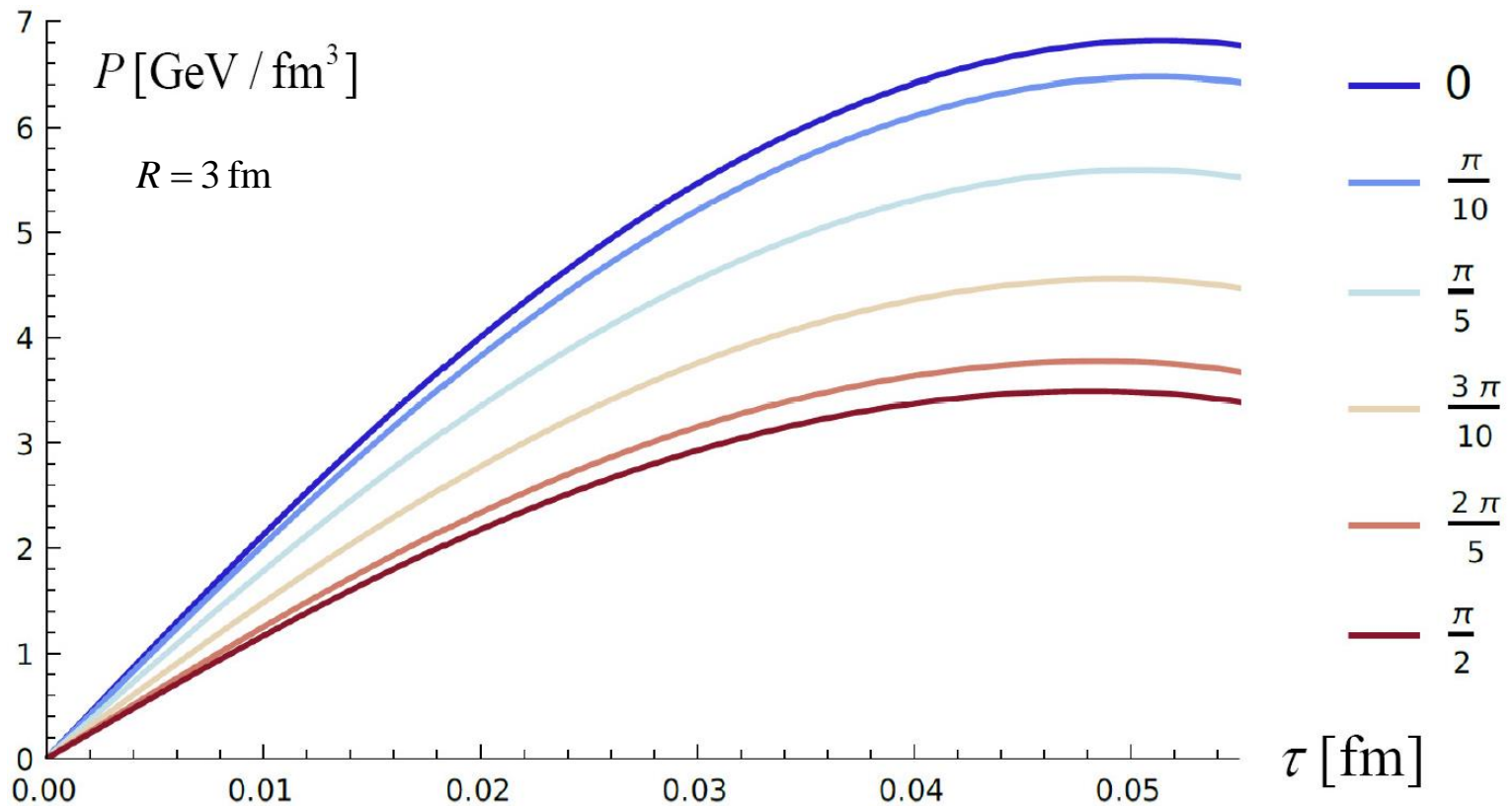
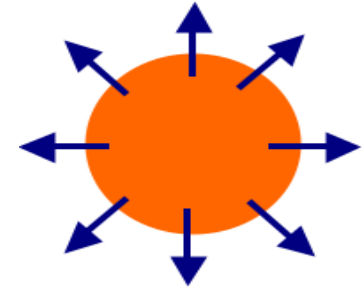
$$P \equiv R^i T^{0i}$$



Radial flow cont.

Pb-Pb collisions at $b = 6$ fm

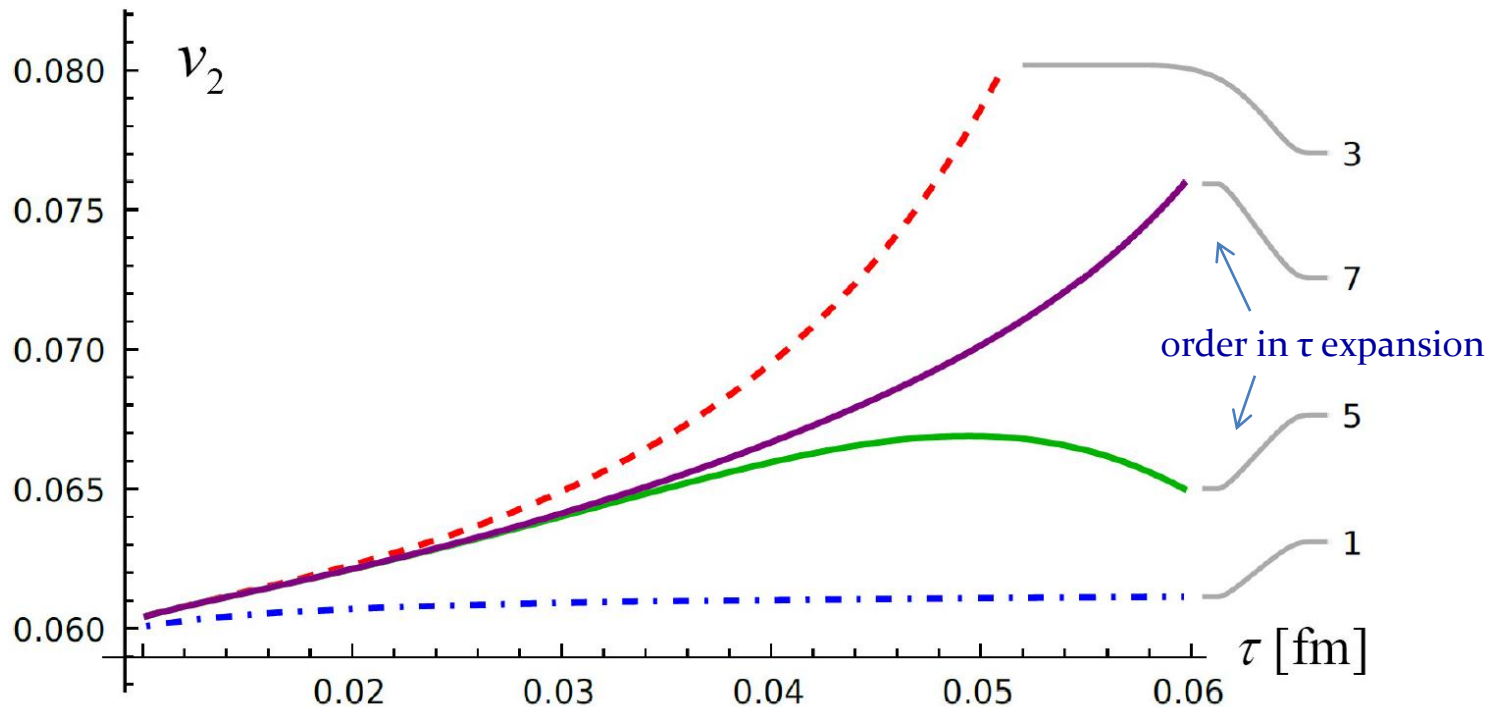
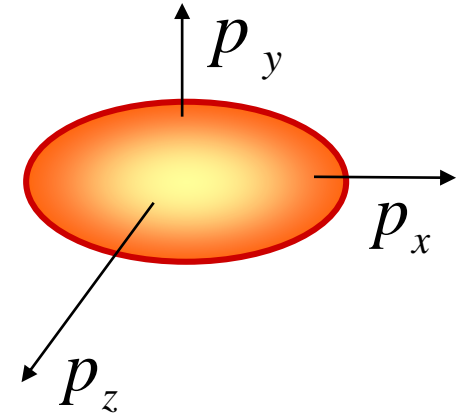
$$P \equiv R^i T^{0i}$$



Elliptic flow

Pb-Pb collisions at $b = 2$ fm

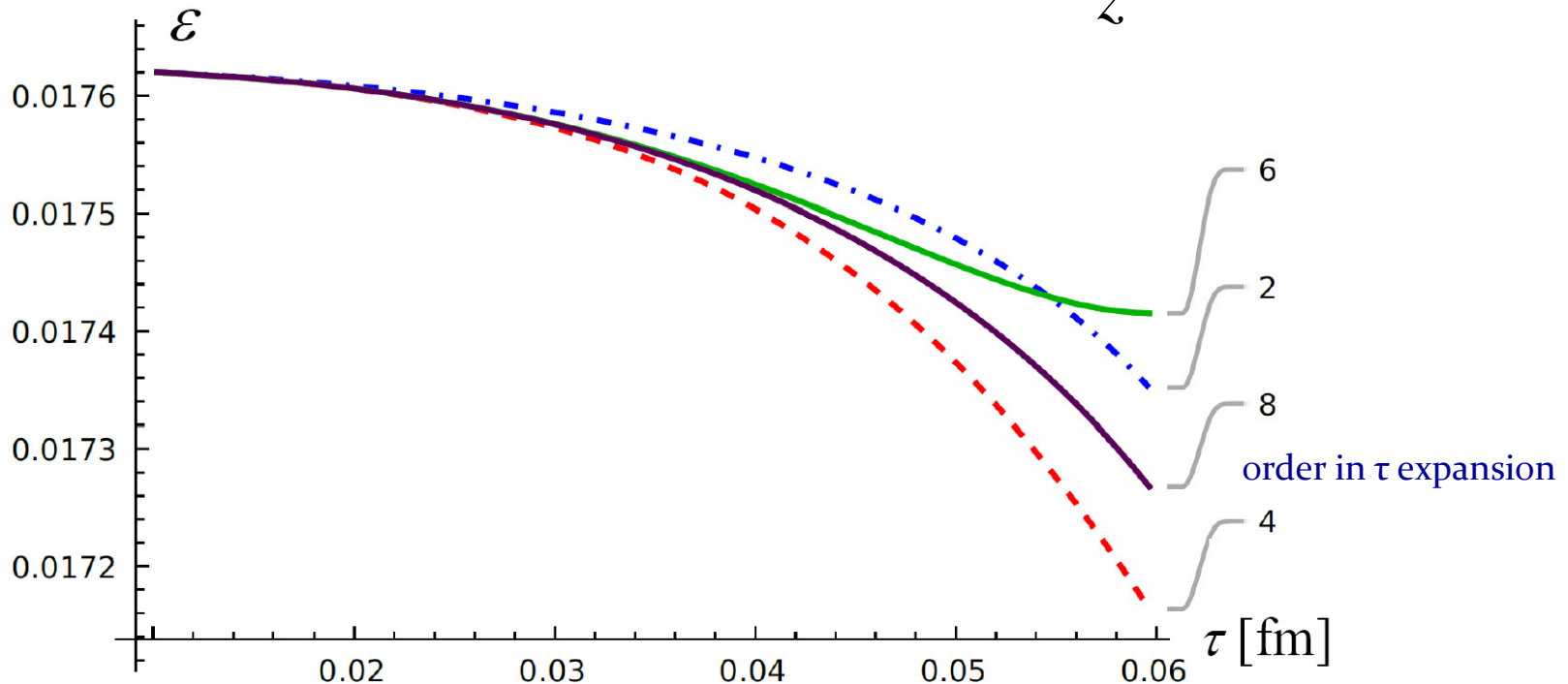
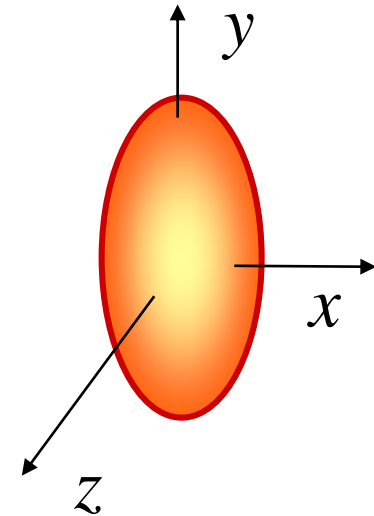
$$v_2 = \frac{\int d^2R \frac{T_{0x}^2 - T_{0y}^2}{\sqrt{T_{0x}^2 + T_{0y}^2}}}{\int d^2R \sqrt{T_{0x}^2 + T_{0y}^2}}$$



Eccentricity

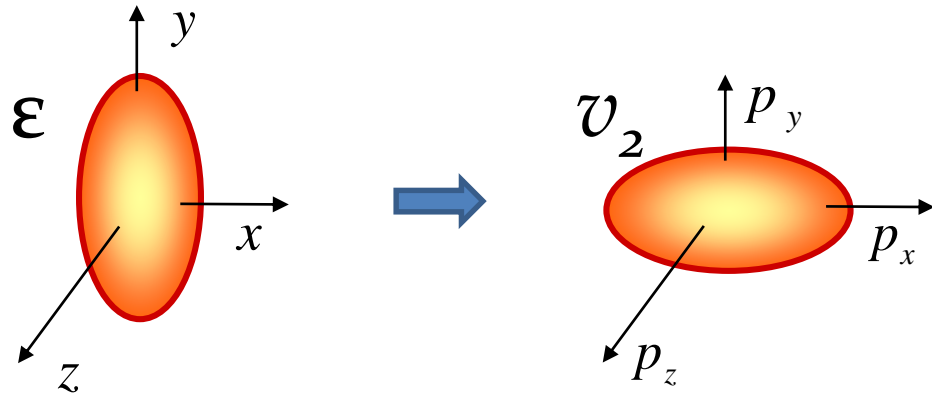
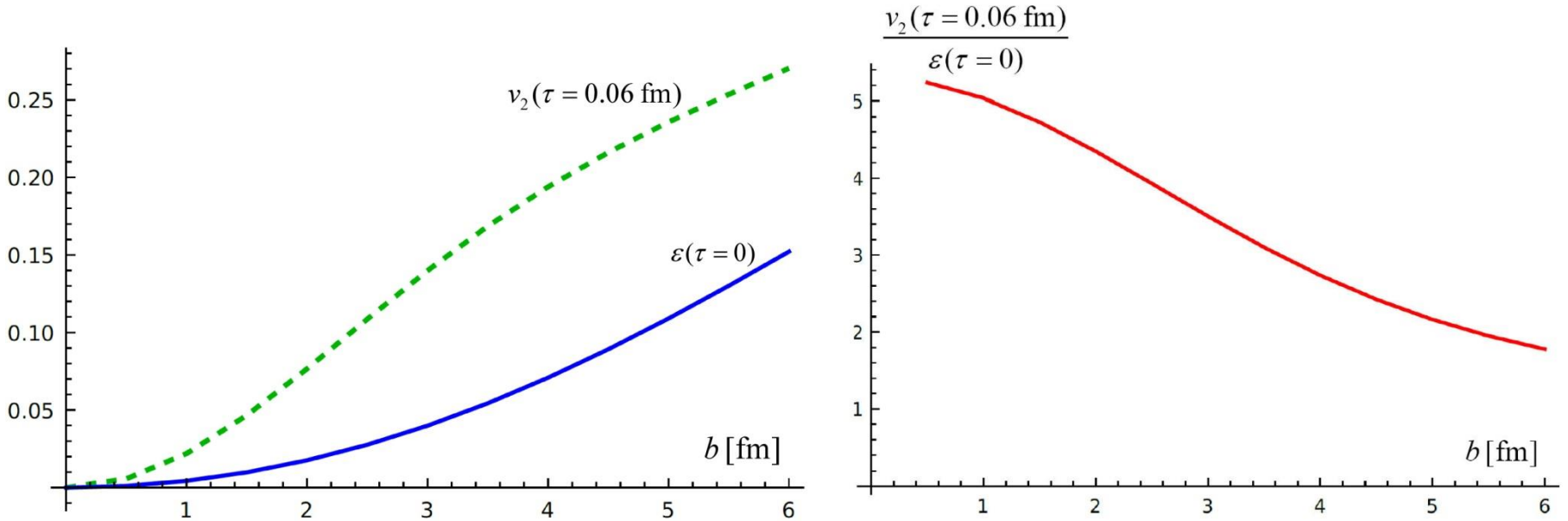
Pb-Pb collisions at $b = 2$ fm

$$\varepsilon = \frac{\int d^2R \frac{R_x^2 - R_y^2}{\sqrt{R_x^2 + R_y^2}} T^{00}}{\int d^2R \sqrt{R_x^2 + R_y^2} T^{00}}$$



Hydrodynamic-like behavior

Pb-Pb collisions



Universal flow

Continuity equation: $\partial_\mu T^{\mu\nu} = 0$

- short-time evolution
- $T^{\mu\nu}(\tau = 0) = \text{diag}(\varepsilon, \varepsilon, \varepsilon, -\varepsilon)$
- boost invariance

$$T^{tx} \approx -\frac{1}{2} \tau \frac{\partial T^{tt}}{\partial x}$$

Universal flow of glasma

$$T^{\mu\nu} = \sum_{n=0}^{\infty} \tau^n T_n^{\mu\nu}$$

$$T_{n+1}^{tx} = -\frac{1}{2} \tau \frac{\partial T_n^{tt}}{\partial x}$$

$$n = 1, 2, \dots, 7$$

M. Carrington, St. Mrówczyński
& J.-Y. Ollitrault, arXiv:2406.14463

Hydrodynamic-like behavior

Mapping of glasma $T_{\text{glasma}}^{\mu\nu}(\tau, \mathbf{x}_T)$ on hydrodynamic $T_{\text{hydro}}^{\mu\nu}(\tau, \mathbf{x}_T)$

Eigenvalue problem:

$$T_{\text{glasma}}^{\mu\nu} w_\nu = \lambda w^\mu$$

Ideal hydrodynamics

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu}$$

$$T^\mu{}_\mu = 0 \Rightarrow p = \frac{1}{3}\varepsilon$$

$$T^{\mu\nu} u_\nu = \varepsilon u^\mu$$

Anisotropic hydrodynamics

$$T^{\mu\nu} = (\varepsilon + p_T)u^\mu u^\nu - p_T g^{\mu\nu} - (p_T - p_L)z^\mu z^\nu$$

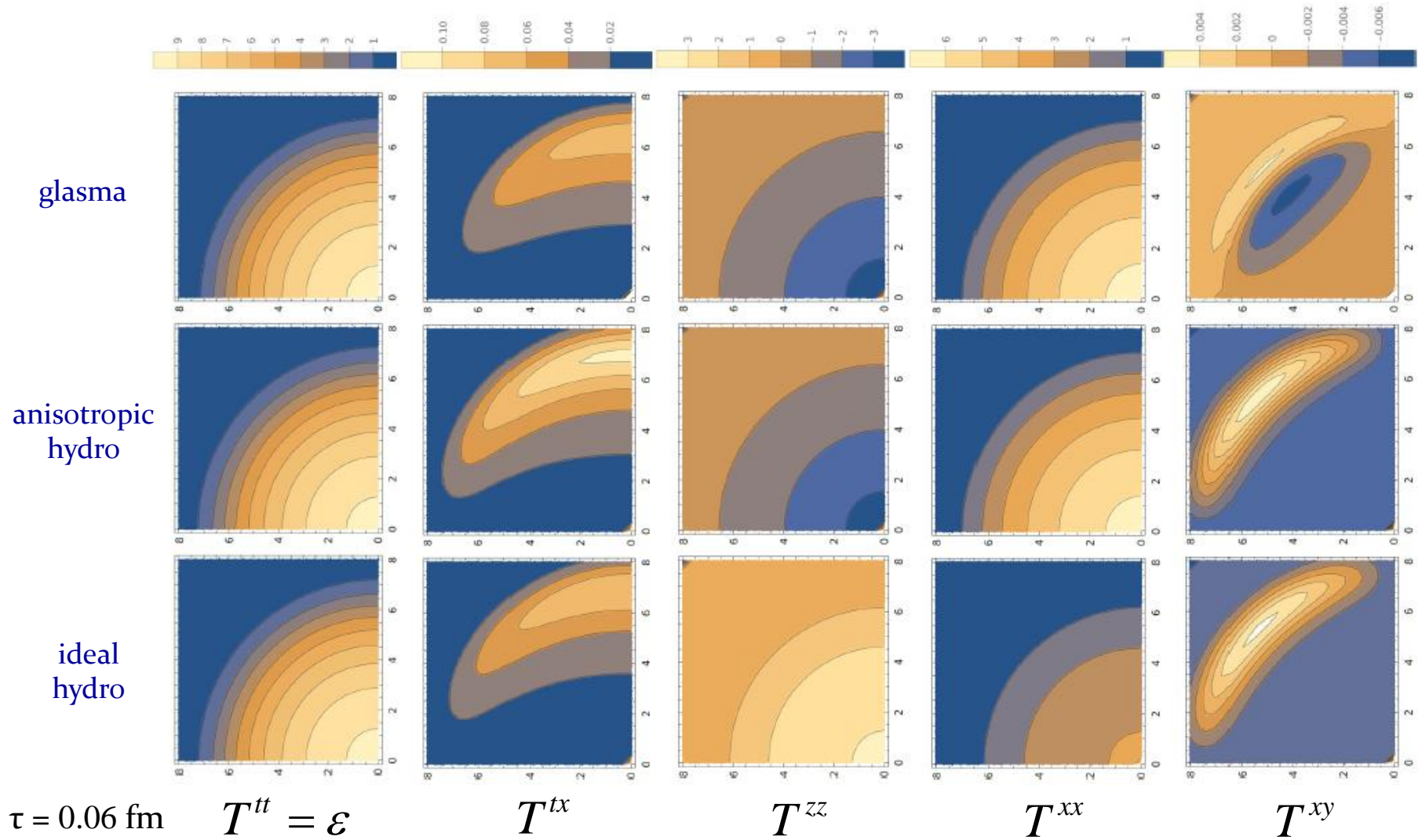
$$T^\mu{}_\mu = 0 \Rightarrow p_L = \varepsilon - 2p_T$$

$$T^{\mu\nu} u_\nu = \varepsilon u^\mu, \quad T^{\mu\nu} z_\nu = -p_L z^\mu$$

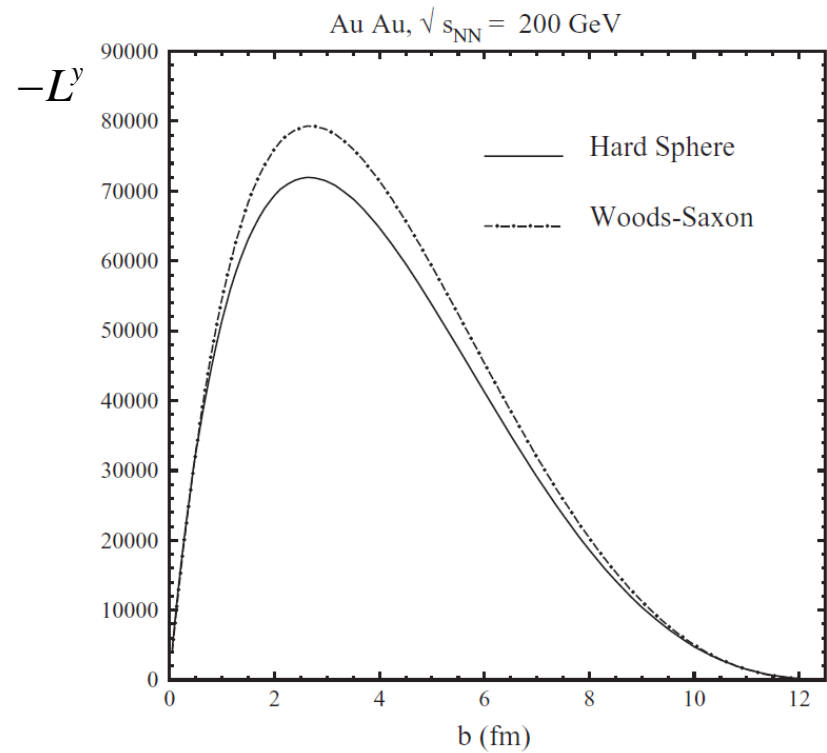
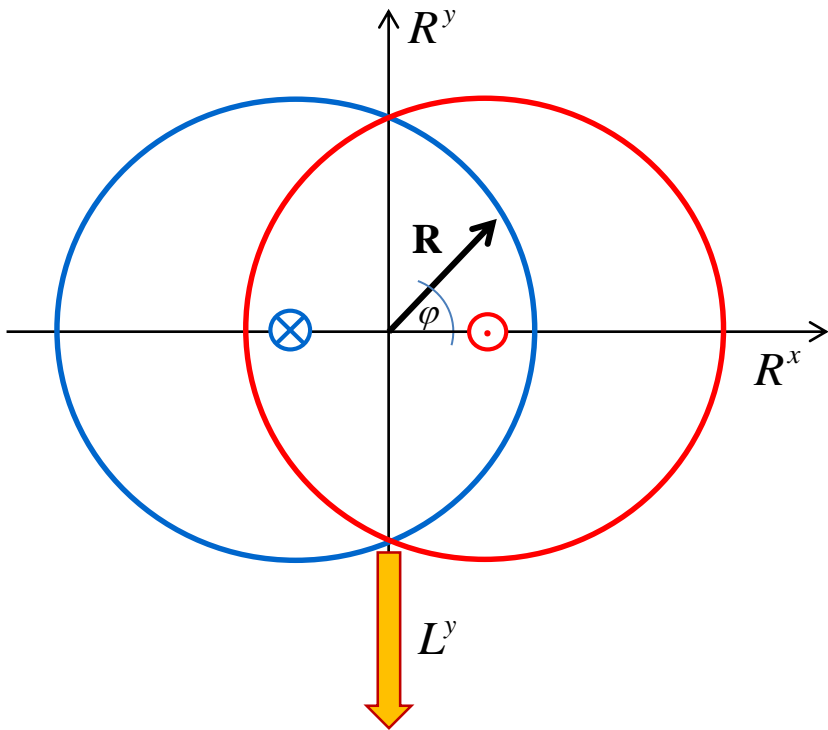
W. Florkowski & R. Ryblewski, Phys. Rev. C **83**, 034907 (2011)

M. Martinez & M. Strickland, Nucl. Phys. A **848**, 183 (2010)

Hydrodynamic-like behavior



Angular momentum

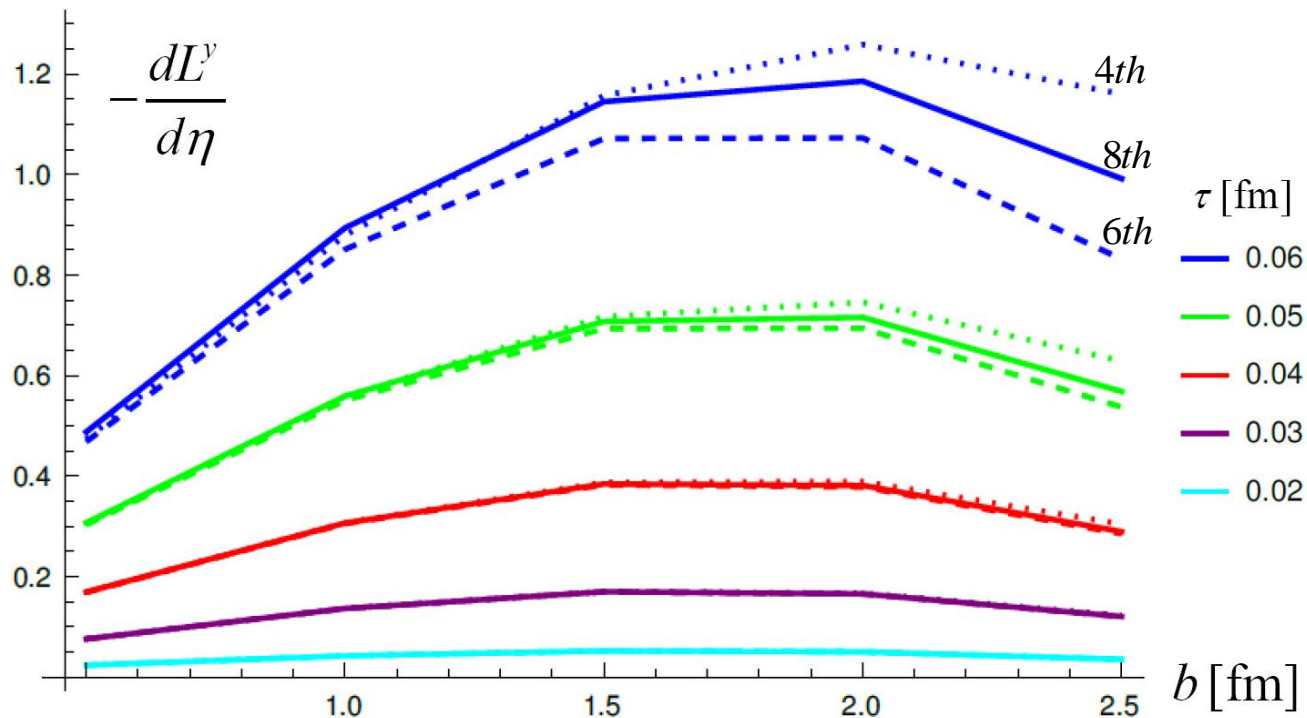


$$L^y \sim 10^7 \hbar @ \text{LHC} ?$$

Angular momentum cont.

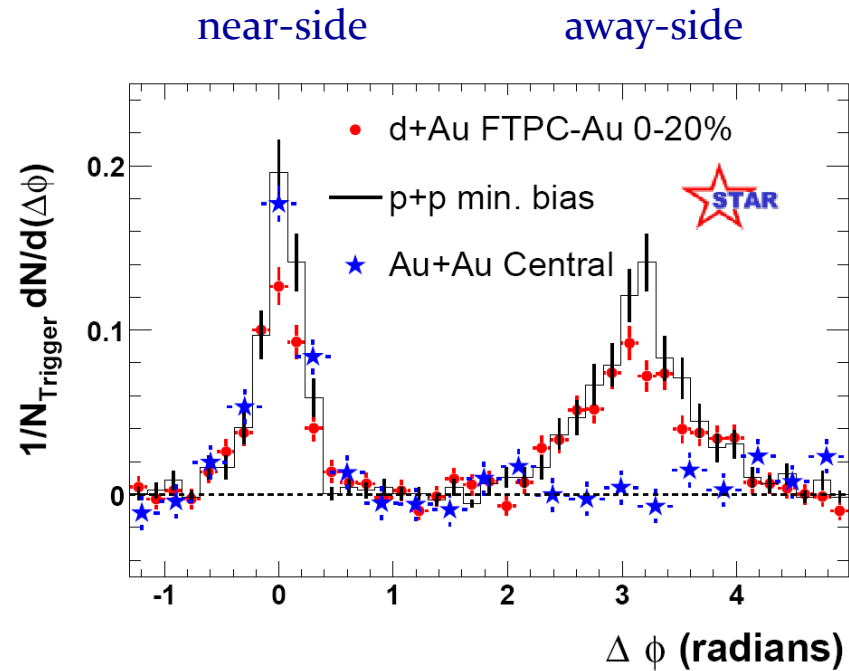
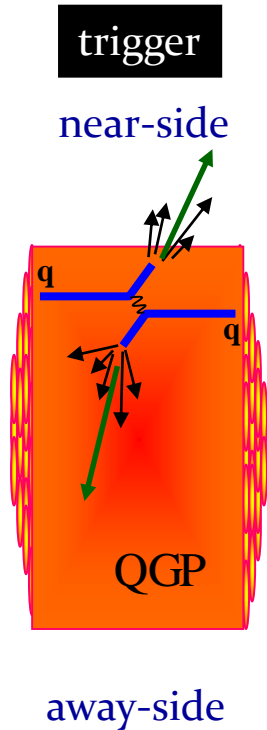
Pb-Pb collisions at $b = 2$ fm

$$\frac{dL^\eta}{d\eta} = -\tau^2 \int d^2R R^x T^{\tau\eta} \quad \text{Milne coordinates}$$



Glasma does not rotate!

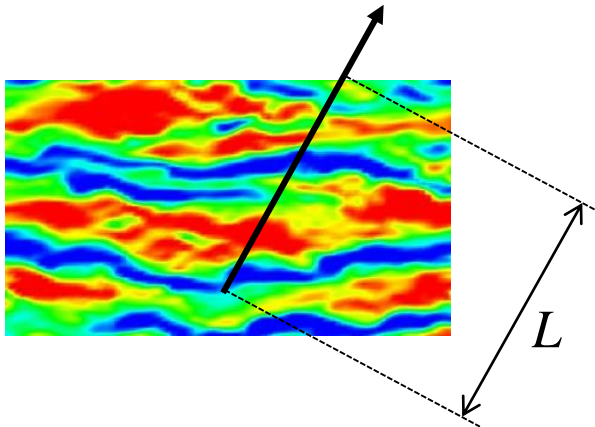
Jet quenching



Away-side jet is suppressed
in central collisions!

Jet quenching in glasma

How hard probes propagate through the glasma?



$$\frac{dE}{dx}, \hat{q} \text{ ?}$$

$$\frac{dE}{dx} - \text{collisional energy loss}$$

$$\hat{q} - \text{transverse momentum broadening}$$

$$\frac{dE^{\text{rad}}}{dx} = -\frac{1}{8} \alpha_s N_c \hat{q} L - \text{radiative energy loss}$$

Fokker-Planck equation

- Transport of hard probes can be described using the Fokker-Planck equation.

$$\overbrace{\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right)}^{\text{drift}} n(t, \mathbf{r}, \mathbf{p}) = \overbrace{\left(\nabla_p^i X^{ij}(\mathbf{v}) \nabla_p^j + \nabla_p^i Y^i(\mathbf{v}) \right)}^{\text{collisions}} n(t, \mathbf{r}, \mathbf{p})$$

$n(t, \mathbf{r}, \mathbf{p})$ - distribution function of hard probes

$$\mathbf{v} \equiv \frac{\mathbf{p}}{E_p}, \quad \nabla_p^i \equiv \frac{\partial}{\partial p_i}$$

$$X^{ij}(\mathbf{v}), Y^i(\mathbf{v}) \Rightarrow \begin{cases} \frac{dE}{dx} = -\frac{v^i}{v} Y^i(\mathbf{v}) & \text{collisional energy loss} \\ \hat{q} = \frac{2}{v} \left(\delta^{ij} - \frac{v^i v^j}{v^2} \right) X^{ji}(\mathbf{v}) & \text{momentum broadening} \end{cases}$$

$$n(t, \mathbf{r}, \mathbf{p}) = n_{\text{eq}}(\mathbf{p}) \sim e^{-\frac{E_p}{T}}$$

solves FK equation

\Leftrightarrow

$$Y^j(\mathbf{v}) = \frac{v^i}{T} X^{ij}(\mathbf{v})$$

Fokker-Planck equation of a hard probe in glasma

▶ Lorentz force $\mathbf{F} \equiv g(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

▶
$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) n(t, \mathbf{r}, \mathbf{p}) = \left(\nabla_p^i X^{ij}(\mathbf{v}) \nabla_p^j + \nabla_p^i Y^i(\mathbf{v}) \right) n(t, \mathbf{r}, \mathbf{p})$$

▶
$$X^{ij}(\mathbf{v}) = \frac{1}{N_c} \int_0^t dt' \langle F^i(t, \mathbf{r}) F^j(t', \mathbf{r} - \mathbf{v}(t-t')) \rangle, \quad Y^j(\mathbf{v}) = \frac{v^i}{T} X^{ij}(\mathbf{v})$$

▶ The collision term is given by field correlators $\langle E^i E^j \rangle, \langle B^i E^j \rangle, \langle B^i B^j \rangle$

▶ Gauge covariance requires: $\langle E_a^i(t, \mathbf{r}) E_a^j(t', \mathbf{r}') \rangle \rightarrow \langle E_a^i(t, \mathbf{r}) \Omega_{ab}(t, \mathbf{r} | t', \mathbf{r}') E_b^j(t', \mathbf{r}') \rangle$

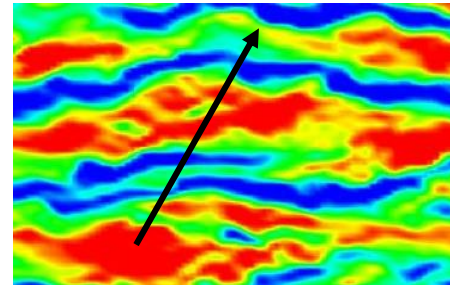
$$\Omega(t, \mathbf{r} | t', \mathbf{r}') \equiv P \exp \left[ig \int_{(t', \mathbf{r}')}^{(t, \mathbf{r})} ds_\mu A^\mu(s) \right]$$

Transport of hard probes in glasma

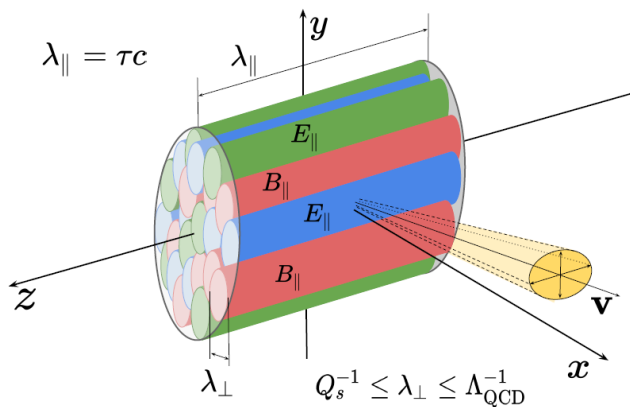
$$X^{ij}(\mathbf{v}) = \frac{g}{N_c} \int_0^t dt' \left\{ \left\langle E^i(t, \mathbf{r}) E^j(t', \mathbf{r}') \right\rangle + \varepsilon^{jkl} v^k \left\langle E^i(t, \mathbf{r}) B^l(t', \mathbf{r}') \right\rangle \right. \\ \left. + \varepsilon^{ikl} v^k \left\langle B^l(t, \mathbf{r}) E^j(t', \mathbf{r}') \right\rangle + \varepsilon^{ikl} \varepsilon^{jmn} v^k v^m \left\langle B^l(t, \mathbf{r}) B^n(t', \mathbf{r}') \right\rangle \right\}$$

$$\mathbf{r}' \equiv \mathbf{r} - \mathbf{v}(t - t')$$

$$\left\{ \begin{array}{l} \hat{q} = \frac{2}{v} \left(\delta^{ij} - \frac{v^i v^j}{v^2} \right) X^{ji}(\mathbf{v}) \\ \frac{dE}{dx} = - \frac{v^i v^j}{vT} X^{ij}(\mathbf{v}) \end{array} \right.$$

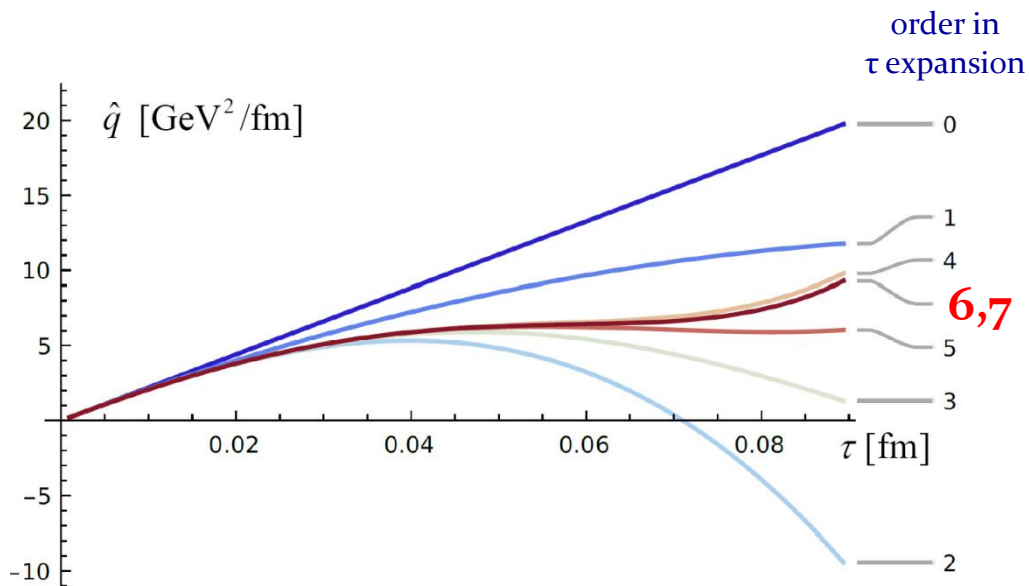


Hard probes in glasma - \hat{q}



$$\hat{q} = \frac{2}{v} \left(\delta^{ij} - \frac{v^i v^j}{v^2} \right) X^{ji}(\mathbf{v})$$

$$\begin{aligned}
 N_c &= 3, \quad g = 1 \\
 Q_s &= 2 \text{ GeV} \\
 m &= 0.2 \text{ GeV} \\
 v &= v_{\perp} = 1
 \end{aligned}$$



Glasma impact on jet quenching

Glasma

$$\hat{q}_{\max} = 6 \text{ GeV}^2 / \text{fm}$$

$$t_{\max} = 0.06 \text{ fm}$$

Equilibrium QGP

$$\hat{q} = 3T^3$$

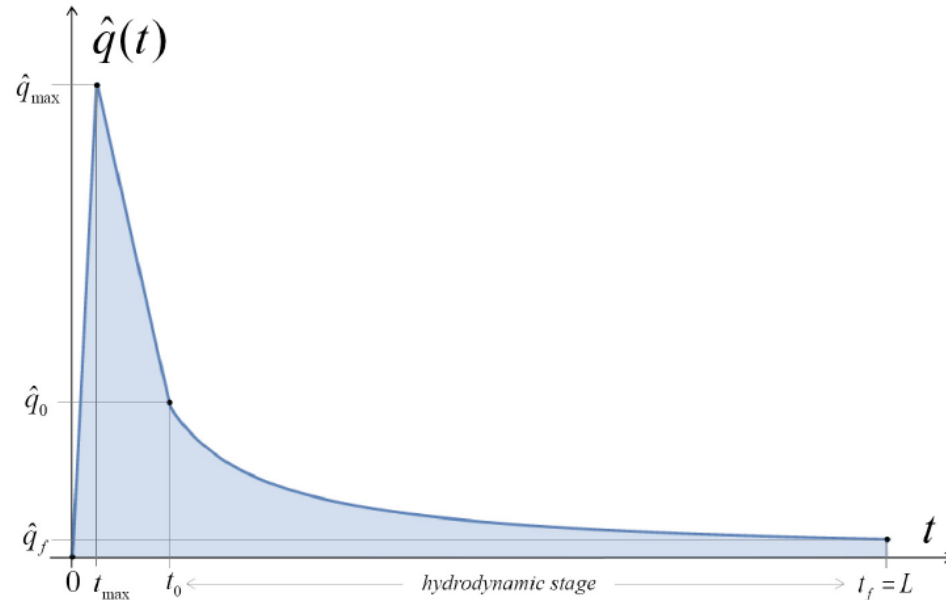
$$t_0 = 0.6 \text{ fm}$$

$$T_0 = 450 \text{ MeV}$$

$$\hat{q}_0 = 1.4 \text{ GeV}^2 / \text{fm}$$

$$T = T_0 \left(\frac{t_0}{t} \right)^{1/3}$$

$$L = 10 \text{ fm}$$



$$\Delta p_T^2 \Big|_{\text{non-eq}} = \int_0^{t_0} dt \hat{q}(t)$$

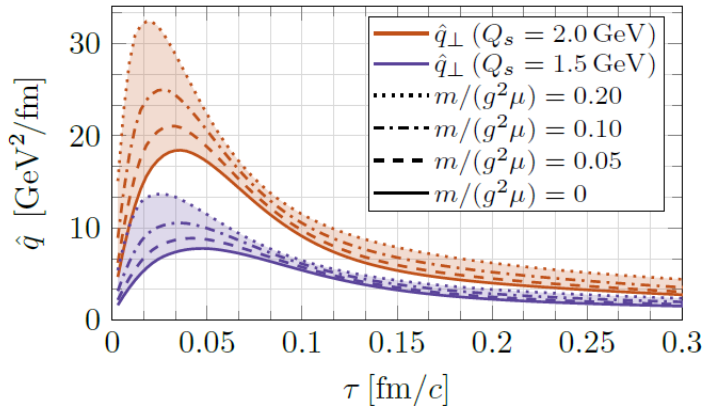
$$\Delta p_T^2 \Big|_{\text{eq}} = \int_{t_0}^L dt \hat{q}(t)$$

$$\frac{\Delta p_T^2 \Big|_{\text{non-eq}}}{\Delta p_T^2 \Big|_{\text{eq}}} = 0.93$$

S. Cao et al. [JETSCAPE], Physical Review C **104**, 024905 (2021),

C. Shen, U. Heinz, P. Huovinen and H. Song, Physical Review C **84**, 044903 (2011).

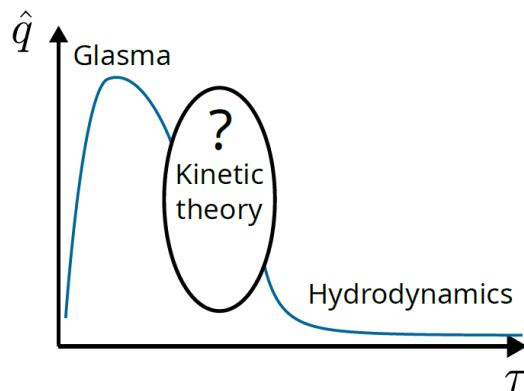
Glasma impact on jet quenching cont.



Full simulations of glasma

A. Ipp, D.I. Müller and D. Schuh, Phys. Lett. B **810**, 135810 (2020)

D. Avramescu, V. Băran, V. Greco, A. Ipp, D.I. Müller & M. Ruggieri, Phys. Rev. D **107**, 114021 (2023)



Kinetic theory interpolates between glasma and equilibrium QGP

K. Boguslavski, A. Kurkela, T. Lappi, F. Lindenbauer & J. Peuron, Phys. Lett. B **850**, 138525 (2024)

Conclusions

- ▶ The glasma evolves in a hydrodynamic-like way.
- ▶ The glasma's orbital momentum is small, the system does not rotate.
- ▶ Momentum broadening and energy loss in the glasma are significantly bigger than in equilibrated QGP.
- ▶ In spite of its short lifetime the glasma provides a significant contribution to the jet quenching.