Femtoscopic correlations of light nuclei

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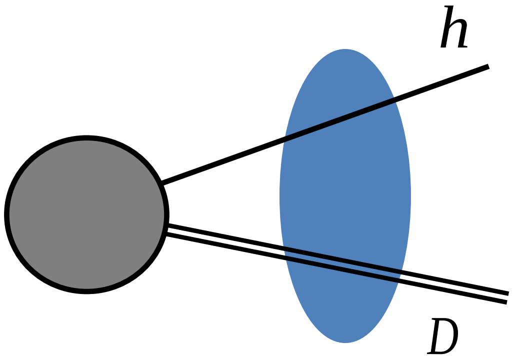
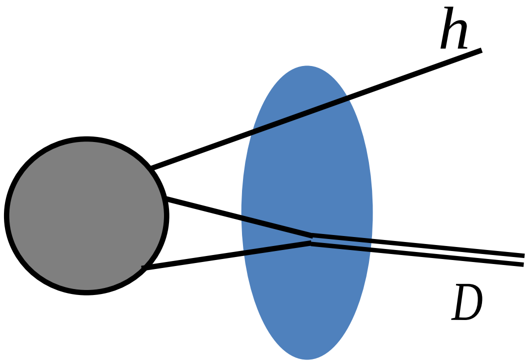
Hadron-deuteron correlations

Hadron-deuteron correlations carry information about a mechanism of deuteron production.



direct production

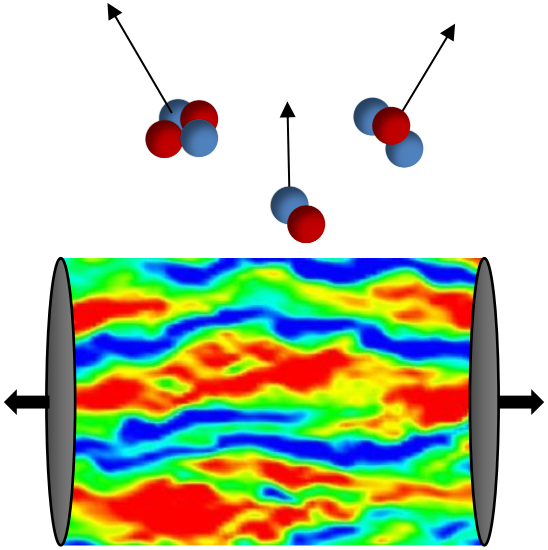
or



final state interaction

St. Mrówczyński & P. Słoń, Acta Phys. Pol. B **51**, 1739 (2020)

Production of light nuclei at RHIC & LHC



baryonless matter

$^{2}\mathrm{H},\ ^{2}\overline{\mathrm{H}},\ ^{3}\mathrm{H},\ ^{3}\overline{\mathrm{H}},\ ^{3}\mathrm{He},\ ^{3}\overline{\mathrm{He}},\ ^{4}\mathrm{He},\ ^{4}\overline{\mathrm{He}},\ {}^{3}_{\Lambda}\mathrm{H},\ {}^{3}_{\bar{\Lambda}}\overline{\mathrm{H}},\ {}^{4}_{\Lambda}\mathrm{H},\ {}^{4}_{\bar{\Lambda}}\overline{\mathrm{H}}$

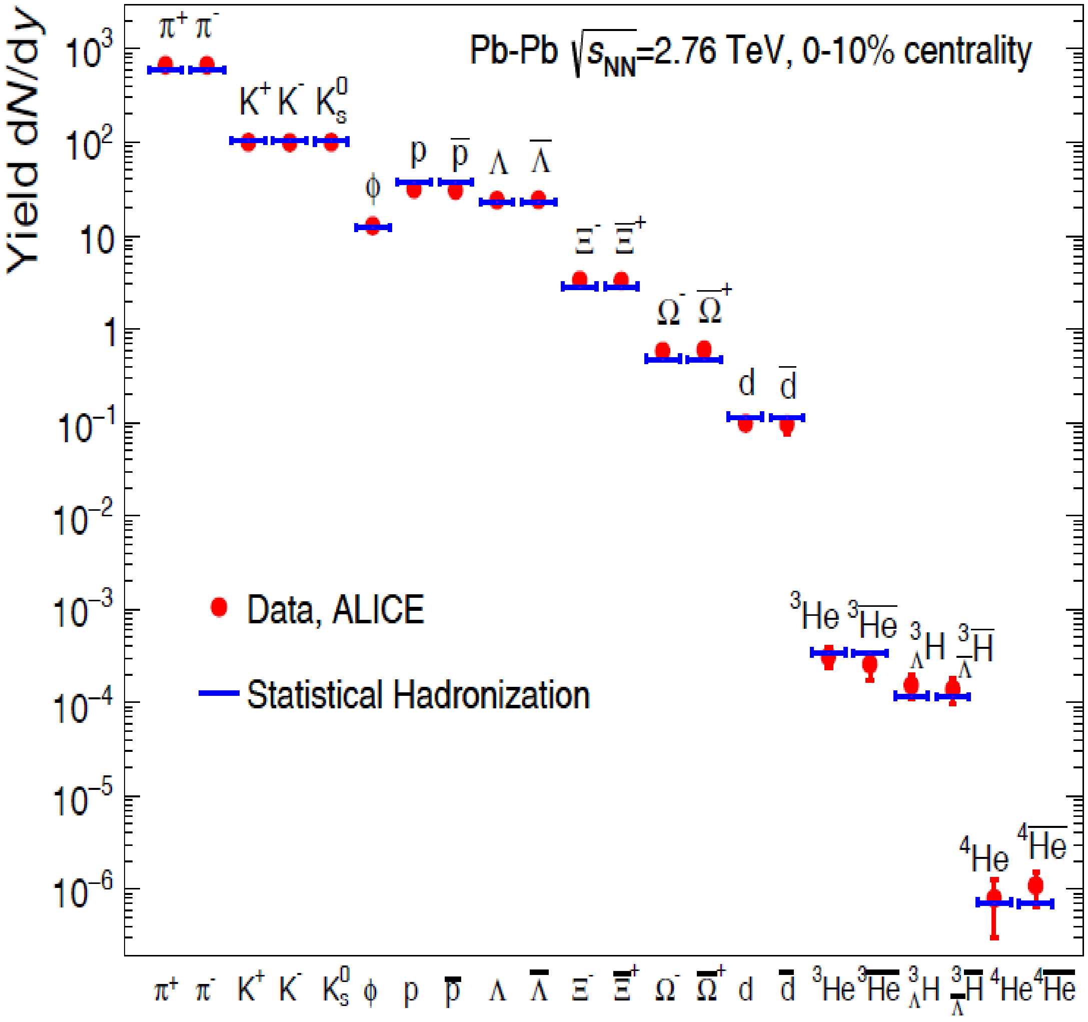
Genuine production!

Matter-antimatter symmetry!

Two approaches to production of light nuclei

- Thermal model – direct production from thermalized hadron matter
- Coalescence model – final state interactions of nucleons

Thermal model prediction



baryonless fireball

$$\text{Yield} \sim g\, e^{-\frac{m}{T}}$$

$$T = 156\ \text{MeV}$$

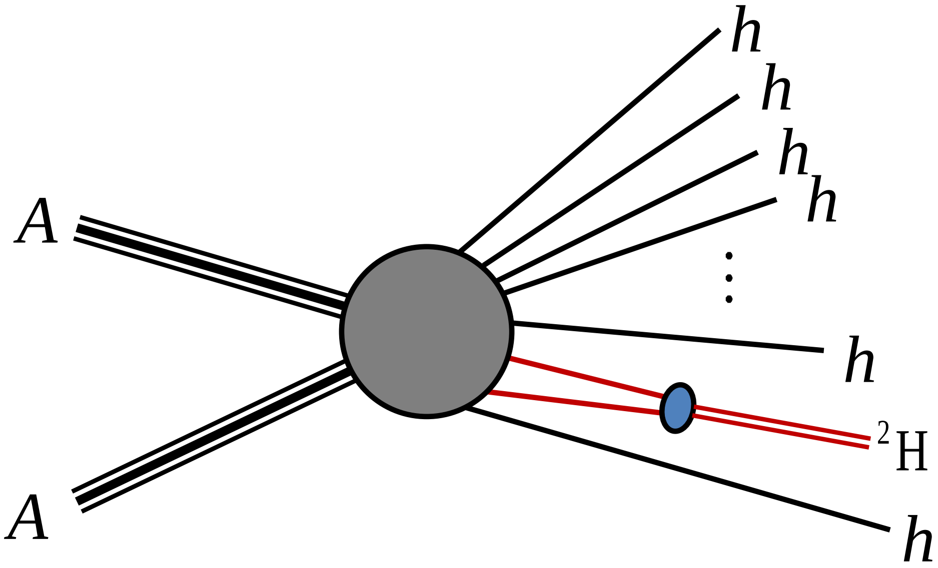
A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Nature **561**, 321 (2018)

Can light nuclei exist in a fireball?

- Interparticle spacing in a hadron gas is about 1.5 fm at T = 156 MeV.
- Root mean square radius of a deuteron is 2.0 fm.
- Binding energy of a deuteron is ε_B = 2.2 MeV.
- A characteristic time of deuteron formation t is longer than 2 fm/c.
- A hadron gas at T = 156 MeV is essentially a classical system.

Snowflakes in hell ?
or
Snowflakes from hell ?

Final state interaction



energy scale
of particle production
- deuteron mass

>>

energy scale
of formation of a nucleus
- binding energy

fast process

slow process

S.T. Butler & C.A. Pearson, Phys. Rev. **129**, 836 (1963)
A. Schwarzschild & C. Zupancic, Phys. Rev. **129**, 854 (1963)

Factorization of production of nucleons and formation of a deuteron

isospin factor

Deuteron yield

$$\frac{dN^D}{d^3\mathbf{P}_D} = \frac{1}{2} A_D \frac{dN^{np}}{d^3\mathbf{p}_n d^3\mathbf{p}_p}$$

$$\frac{1}{2}\mathbf{P}_D = \mathbf{p}_n = \mathbf{p}_p$$

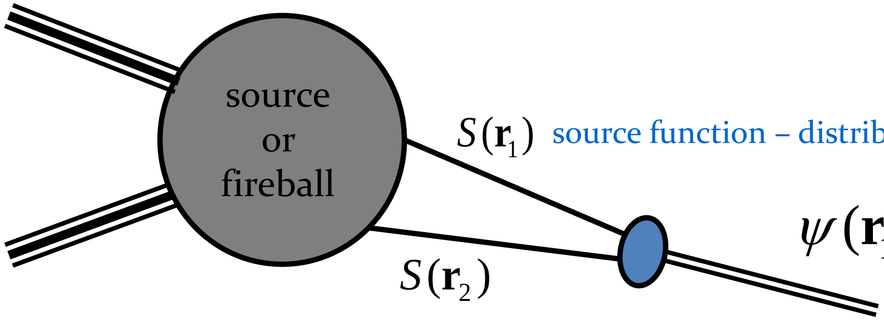
yield of *np* pairs

deuteron
formation rate

$$\frac{1}{2}\frac{dN^{np}}{d^3\mathbf{p}_n d^3\mathbf{p}_p} \approx \frac{dN^{pp}}{d^3\mathbf{p}_p d^3\mathbf{p}_p} \approx \left(\frac{dN^p}{d^3\mathbf{p}_p}\right)^2$$

$$\boxed{\frac{dN^D}{d^3\mathbf{P}_D} = A_D \left(\frac{dN^p}{d^3\mathbf{p}_p}\right)^2}$$

Deuteron formation rate



$S(\mathbf{r}_1)$ source function – distribution of emission points

$\psi(\mathbf{r}_1,\mathbf{r}_2)$ deuteron wave fuction

$S(\mathbf{r}_2)$

spin factor

$$A_D = \frac{3}{4}(2\pi)^3 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2\, S(\mathbf{r}_1)S(\mathbf{r}_2)\left|\psi(\mathbf{r}_1,\mathbf{r}_2)\right|^2$$

$$\mathbf{R} \equiv \frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2), \qquad \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$$

$$\psi(\mathbf{r}_1,\mathbf{r}_2) = e^{i\mathbf{P}\cdot\mathbf{R}}\varphi_D(\mathbf{r})$$

$$\boxed{A_D = \frac{3}{4}(2\pi)^3 \int d^3\mathbf{r}\, S_r(\mathbf{r})\left|\varphi_D(\mathbf{r})\right|^2}$$

$$S_r(\mathbf{r}) \equiv \int d^3\mathbf{R}\, S\left(\mathbf{R}-\frac{1}{2}\mathbf{r}\right)S\left(\mathbf{R}+\frac{1}{2}\mathbf{r}\right)$$

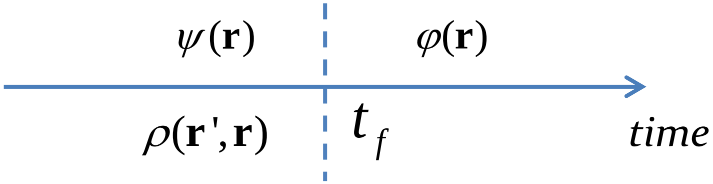
distribution of relative distance of *n* and *p*

H. Sato and K. Yazaki, Phys. Lett. B **98**, 153 (1981)

Quantum-mechanical meaning of the formation rate formula

Sudden approximation

$$E\Delta t << 1$$



Transition matrix element

$$M = \left| \int d^3\mathbf{r}\, \psi^*(\mathbf{r})\, \varphi(\mathbf{r}) \right|^2 = \int d^3\mathbf{r}\, d^3\mathbf{r}'\, \varphi^*(\mathbf{r}')\, \underbrace{\psi(\mathbf{r}')\psi^*(\mathbf{r})}_{\rho(\mathbf{r}',\mathbf{r})}\, \varphi(\mathbf{r})$$

density matrix

$$M = \int d^3\mathbf{r}\, d^3\mathbf{r}'\, \varphi^*(\mathbf{r}')\, \rho(\mathbf{r}',\mathbf{r})\, \varphi(\mathbf{r})$$

If density matrix is diagonal

$$\rho(\mathbf{r}',\mathbf{r}) = S(\mathbf{r})\, \delta^{(3)}(\mathbf{r}'-\mathbf{r}) \qquad \Rightarrow \qquad \boxed{M = \int d^3\mathbf{r}\, S(\mathbf{r}) \left| \varphi(\mathbf{r}) \right|^2}$$

Diagonal density matrix

$$\langle\psi|\hat{A}|\psi\rangle = \sum_{i,j} c_i^* c_j \langle\alpha_i|\hat{A}|\alpha_j\rangle = \sum_{i,j} \rho_{ji} A_{ij}$$

$$|\psi\rangle = \sum_i c_i |\alpha_i\rangle$$

$$\rho_{ji} \equiv c_i^* c_j \qquad A_{ij} \equiv \langle\alpha_i|\hat{A}|\alpha_j\rangle$$

density matrix

$\overline{\cdots\cdots}$ – averaging over time or events

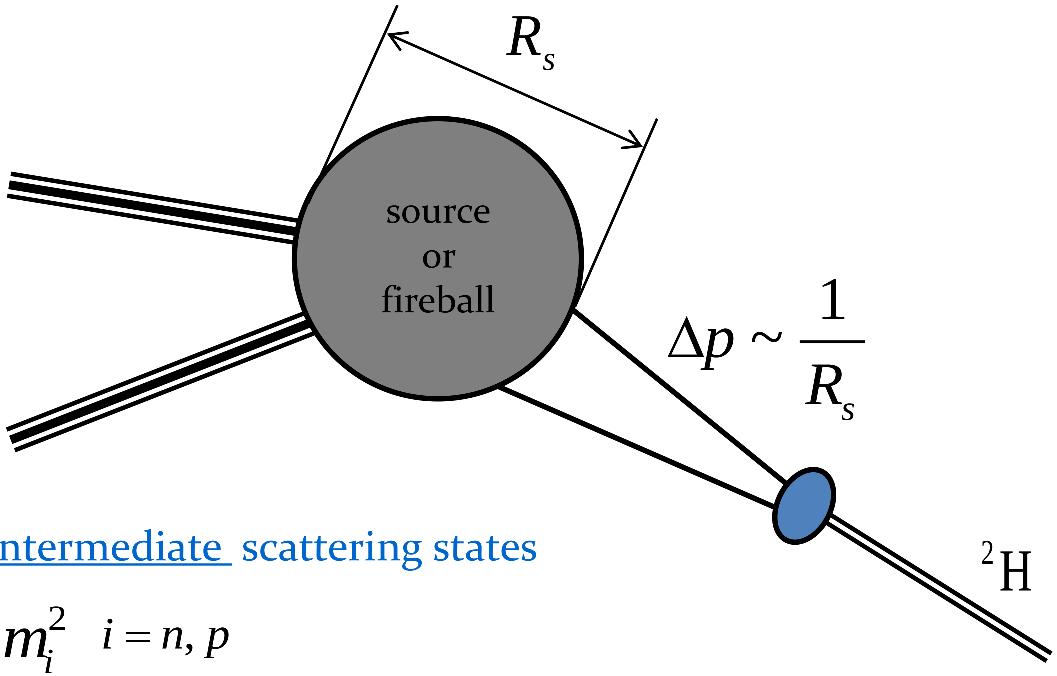
$$\overline{\langle\psi|\hat{A}|\psi\rangle} = \sum_{i,j} \overline{c_i^* c_j} \langle\alpha_i|\hat{A}|\alpha_j\rangle = \sum_i |c_i|^2 A_{ii}$$

$$\overline{\rho_{ji}} = \overline{c_i^* c_j} = \delta^{ij} |c_i|^2$$

random phase approximation

diagonal density matrix

Energy-momentum conservation



Nucleons are intermediate scattering states

$$E_i^2 - \mathbf{p}_i^2 \neq m_i^2 \quad i = n, p$$

Energy-momentum conservation

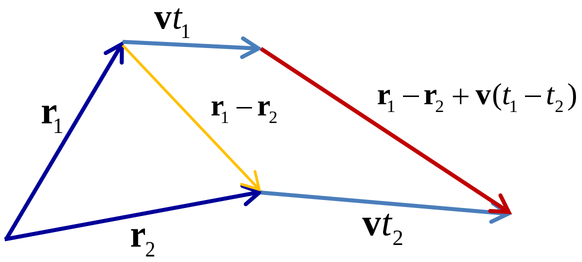
$$\begin{cases} \mathbf{p}_p + \mathbf{p}_n = \mathbf{p}_D \\ E_p + E_n = E_D \end{cases}$$

St. Mrówczyński, J. Phys. G **11**, 1087 (1987)

Emission time

Instantaneous emission

$$A_D = \frac{3}{4}(2\pi)^3 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2\, S(\mathbf{r}_1)S(\mathbf{r}_2)\left|\psi(\mathbf{r}_1,\mathbf{r}_2)\right|^2$$



Emission extended in time

$$A_D = \frac{3}{4}(2\pi)^3 \int dt_1 d^3\mathbf{r}_1 dt_2 d^3\mathbf{r}_2\, S(t_1,\mathbf{r}_1)S(t_2,\mathbf{r}_2)\left|\psi(\mathbf{r}_1 + \mathbf{v}t_1, \mathbf{r}_2 + \mathbf{v}t_2)\right|^2$$

$$\int dt\, d^3\mathbf{r}\, S(t,\mathbf{r}) = 1 \qquad \mathbf{v} = \frac{\mathbf{P}_D}{E_D} \qquad \begin{cases} \mathbf{R} \equiv \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2), & \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2 \\ T \equiv \frac{1}{2}(t_1 + t_2), & t \equiv t_1 - t_2 \end{cases} \qquad \psi(\mathbf{r}_1,\mathbf{r}_2) = e^{i\mathbf{P}\cdot\mathbf{R}}\varphi(\mathbf{r})$$

$$S_r(t,\mathbf{r}) \equiv \int dT\, d^3\mathbf{R}\, S\left(T - \frac{1}{2}t, \mathbf{R} - \frac{1}{2}\mathbf{r}\right) S\left(T + \frac{1}{2}t, \mathbf{R} + \frac{1}{2}\mathbf{r}\right)$$

$$S_r(\mathbf{r}) \equiv \int dt\;\; S_r\left(t, \mathbf{r} - \mathbf{v}t\right)$$

$$S(t,\mathbf{r}) = \left(\frac{1}{2\pi\tau^2}\right)^{1/2} \left(\frac{1}{2\pi R_s^2}\right)^{3/2} \exp\left(-\frac{t^2}{2\tau^2}\right) \exp\left(-\frac{\mathbf{r}^2}{2R_s^2}\right)$$

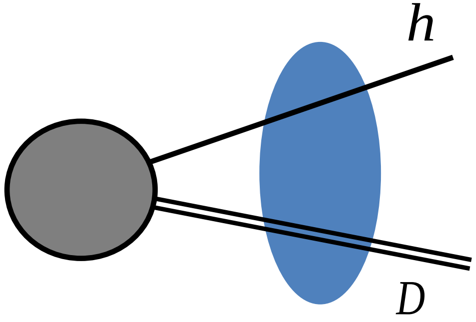
$$\boxed{A_D = \frac{3}{4}(2\pi)^3 \int d^3\mathbf{r}\, S_r(\mathbf{r})\left|\varphi(\mathbf{r})\right|^2}$$

$$\boxed{R_s \to \sqrt{R_s^2 + v^2\tau^2}}$$

S.E. Koonin, Phys. Lett. B **70**, 43 (1977)

Hadron-deuteron correlation function

1) Deuteron is treated as an elementary particle



Experimental definition

$$\boxed{\frac{dN_{hD}}{d\mathbf{p}_h d\mathbf{p}_D} = C(\mathbf{p}_h, \mathbf{p}_D) \frac{dN_h}{d\mathbf{p}_h} \frac{dN_D}{d\mathbf{p}_D}}$$

Theoretical formula

$$\boxed{C(\mathbf{p}_h, \mathbf{p}_D) = \int d^3 r_h \, d^3 r_D \, S(\mathbf{r}_h) \, S(\mathbf{r}_D) \left| \psi(\mathbf{r}_h, \mathbf{r}_D) \right|^2}$$

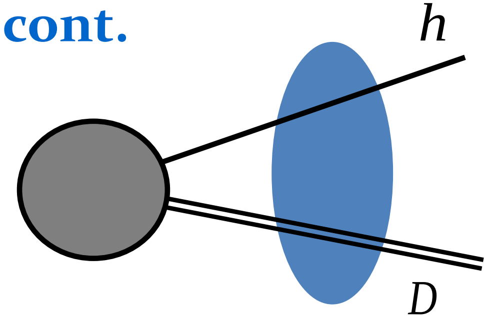
distribution
of emission points

h-*D* wave function

S.E. Koonin, Phys. Lett. B **70**, 43 (1977)
R. Lednicky and V.L. Lyuboshitz, Yad. Fiz. **35**, 1316 (1982)

Hadron-deuteron correlation function

1) Deuteron is treated as an elementary particle cont.



Separation of CM and relative motion

$$\begin{cases} \mathbf{R} \equiv \dfrac{m_D \mathbf{r}_D + m_h \mathbf{r}_h}{m_D + m_h} \\ \mathbf{r} \equiv \mathbf{r}_D - \mathbf{r}_h \end{cases}$$

$$\psi(\mathbf{r}_h, \mathbf{r}_D) = e^{i\mathbf{PR}} \phi_{\mathbf{q}}(\mathbf{r})$$

$$\boxed{C(\mathbf{q}) = \int d^3r \; S_r(\mathbf{r}) \left| \phi_{\mathbf{q}}(\mathbf{r}) \right|^2}$$

„Relative" source function

$$S_r(\mathbf{r}) \equiv \int d^3R \; S\left(\mathbf{R} - \frac{m_D}{m_D + m_h}\mathbf{r}\right) S\left(\mathbf{R} + \frac{m_h}{m_D + m_h}\mathbf{r}\right) = \left(\frac{1}{4\pi R_s^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R_s^2}\right)$$

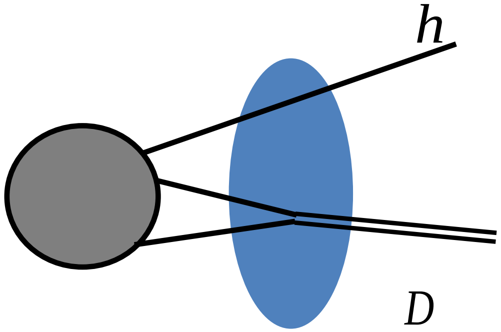
$$S(\mathbf{r}) = \left(\frac{1}{2\pi R_s^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R_s^2}\right)$$

Hadron-deuteron correlation function

2) Deuteron is treated as a bound state of neutron and proton

Experimental definition

$$\frac{dN_{hD}}{d\mathbf{p}_h d\mathbf{p}_D} = C(\mathbf{p}_h,\mathbf{p}_D)\, A_D \frac{dN_h}{d\mathbf{p}_h} \frac{dN_n}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p}$$



Theoretical formula

$$C(\mathbf{p}_h,\mathbf{p}_D)\, A_D = \int d^3r_h\, d^3r_n\, d^3r_p\, S(\mathbf{r}_h) S(\mathbf{r}_n)\, S(\mathbf{r}_p) \left|\psi_{hD}(\mathbf{r}_h,\mathbf{r}_n,\mathbf{r}_p)\right|^2$$

Deuteron formation rate $\frac{dN_D}{d\mathbf{p}_D} = A_D \frac{dN_n}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p}$ $\frac{1}{2}\mathbf{P}_D = \mathbf{p}_n = \mathbf{p}_p$

$$A_D = \frac{3}{8}(2\pi)^3 \int d^3\mathbf{r}_n d^3\mathbf{r}_p\, S(\mathbf{r}_n) S(\mathbf{r}_p) \left|\psi_D(\mathbf{r}_n,\mathbf{r}_p)\right|^2 = \frac{3}{8}(2\pi)^3 \int d^3r_{np}\, S_r(\mathbf{r}_{np}) \left|\phi_D(\mathbf{r}_{np})\right|^2$$

spin-isopsin factor

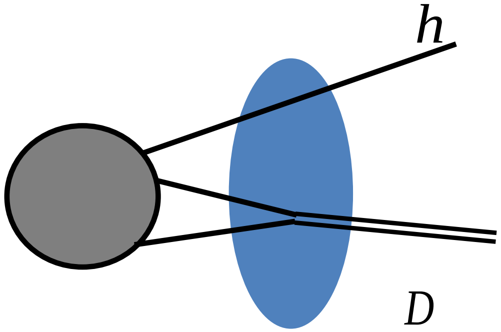
$$\psi_D(\mathbf{r}_n,\mathbf{r}_p) = e^{i\mathbf{PR}} \phi_D(\mathbf{r}_{np})$$

St. Mrówczyński & P. Słoń, Acta Phys. Pol. B **51**, 1739 (2020)

Hadron-deuteron correlation function

2) Deuteron is treated as a bound state of neutron and proton cont

Separation of CM and relative motion



$$\begin{cases} \mathbf{R} \equiv \dfrac{m_p \mathbf{r}_p + m_n \mathbf{r}_n + m_h \mathbf{r}_h}{m_p + m_n + m_h} \\ \mathbf{r}_{np} \equiv \mathbf{r}_p - \mathbf{r}_n \\ \mathbf{r} \equiv \mathbf{r}_h - \dfrac{m_p \mathbf{r}_p + m_n \mathbf{r}_n}{m_p + m_n} \end{cases}$$

assumed factorization

$$\boxed{\psi(\mathbf{r}_h, \mathbf{r}_n, \mathbf{r}_p) = e^{i\mathbf{PR}} \phi_{\mathbf{q}}(\mathbf{r}) \varphi_D(\mathbf{r}_{np})}$$

$$C(\mathbf{q}) = \frac{1}{A_D} \int d^3R\, d^3r_{np}\, d^3r\, S(\mathbf{r}_h) S(\mathbf{r}_n) S(\mathbf{r}_p) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2 \left|\varphi_D(\mathbf{r}_{np})\right|^2$$

For Gaussian source

$$\boxed{C(\mathbf{q}) = \int d^3r\, S_{3r}(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2}$$

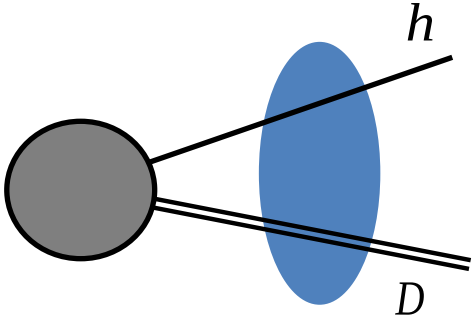
$$S_{3r}(\mathbf{r}) = \left(\frac{1}{3\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{3R^2}\right)$$

For a non-Gaussian source, A_D remains in the correlation function!

Direct vs. final state interaction

Direct production

$$C(\mathbf{q}) = \int d^3r S_r(\mathbf{r}) \left| \phi_{\mathbf{q}}(\mathbf{r}) \right|^2$$



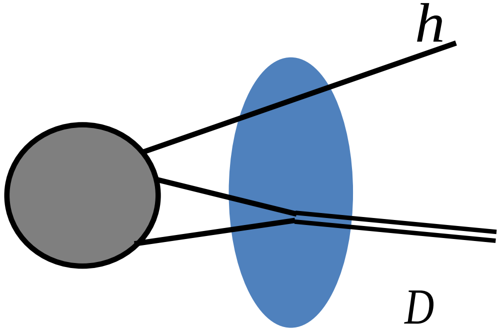
$$S_r(\mathbf{r}) = \left(\frac{1}{4\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R^2} \right)$$

$$\sqrt{\frac{4}{3}} \approx 1.15$$

$$S_{3r}(\mathbf{r}) = \left(\frac{1}{3\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{3R^2} \right)$$

Final state interaction & factorization

$$C(\mathbf{q}) = \int d^3r S_{3r}(\mathbf{r}) \left| \phi_{\mathbf{q}}(\mathbf{r}) \right|^2$$



Lednicky-Lyuboshitz formula

$$C(\mathbf{q}) = \int d^3 r S_r(\mathbf{r}) \left| \phi_{\mathbf{q}}(\mathbf{r}) \right|^2$$

The wave function in scattering asymptotic state

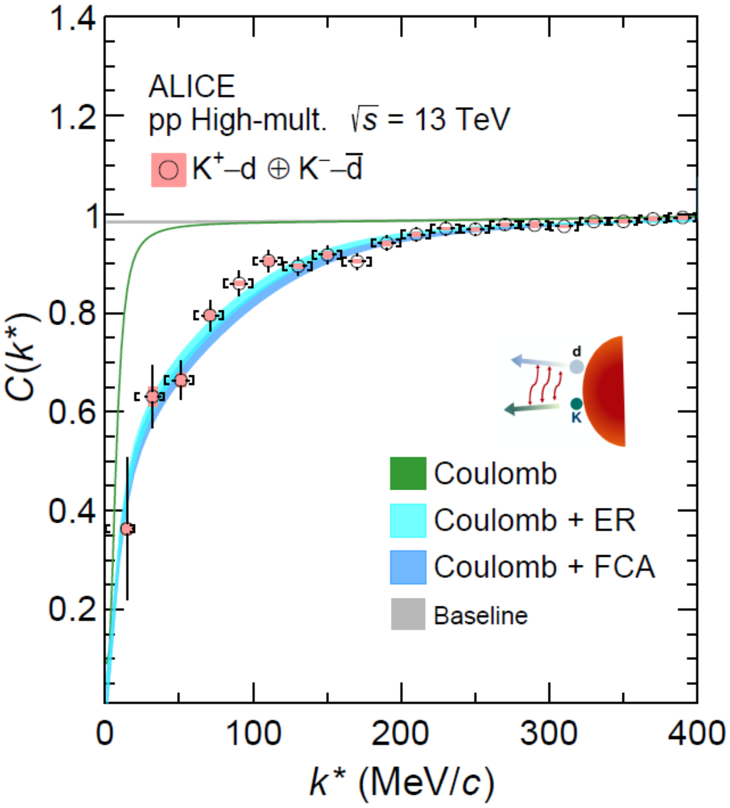
$$\phi_{\mathbf{q}}(\mathbf{r}) = e^{i\mathbf{qr}} + f(\mathbf{q}) \frac{e^{iqr}}{r}$$

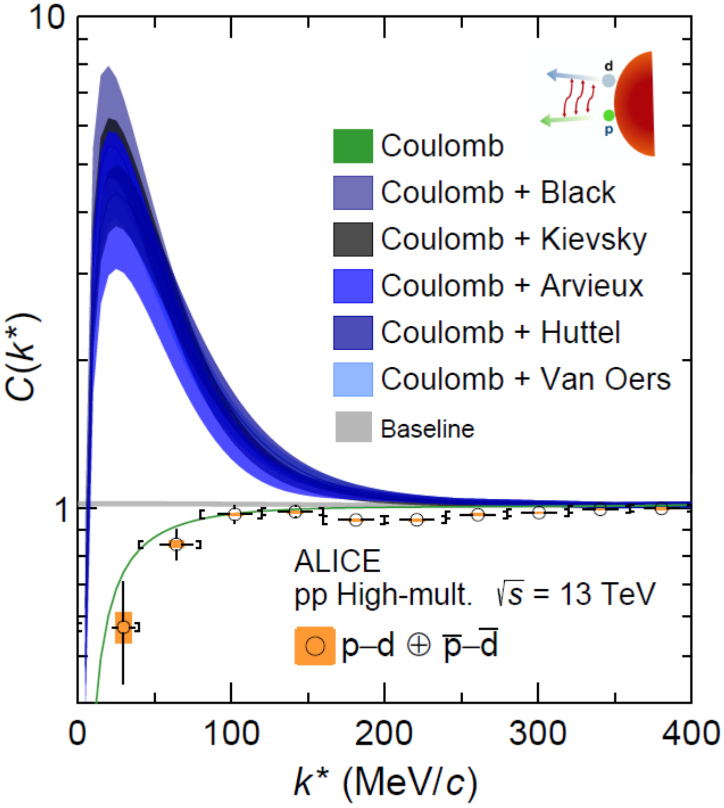
The *s*-wave amplidtue $f(\mathbf{q}) = -\frac{a}{1 - \frac{1}{2} d a q^2 + iqa}$

a – scattering length, *d* – effective range

R. Lednicky and V.L. Lyuboshitz, Yad. Fiz. **35**, 1316 (1982)

K-D vs. p-D correlations





$$a_{KD} \approx 0.5 \text{ fm}$$

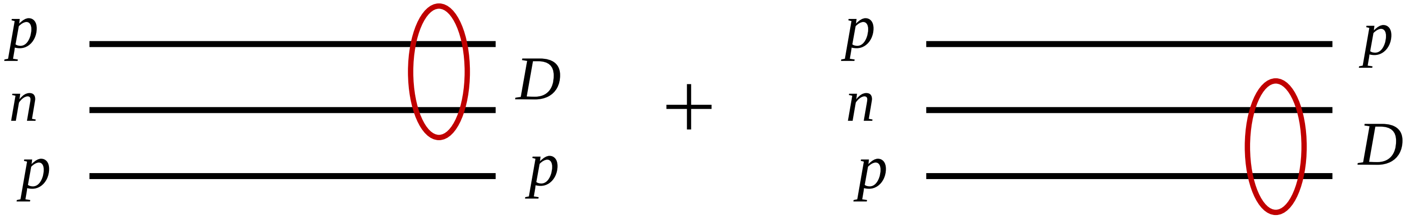
$$a_{pD}^{1/2} \approx 2 \text{ fm}$$

$$a_{pD}^{3/2} \approx 12 \text{ fm}$$

ALICE, Phys. Rev. X **14**, 031051 (2024)

p-*D* correlation function

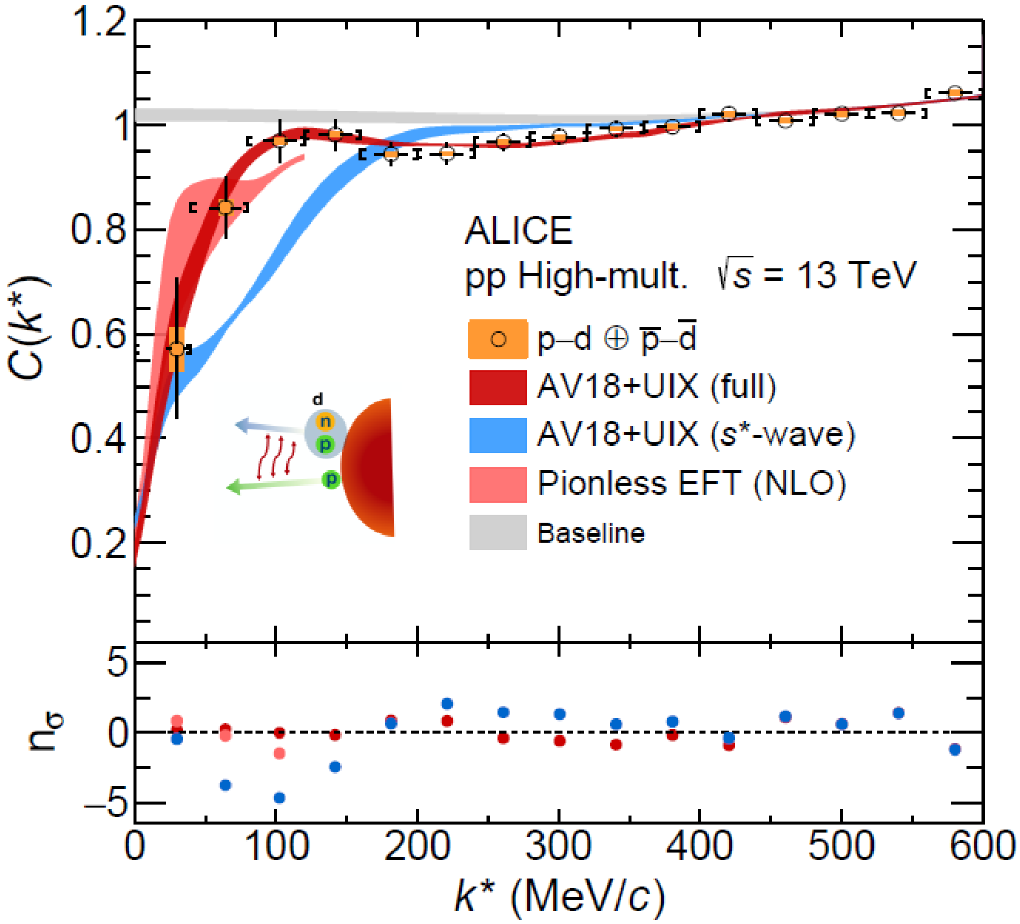
$\psi^{\mathbf{q}}_{pD}(\mathbf{r}_n, \mathbf{r}_{p_1}, \mathbf{r}_{p_2})$



Full three-body calculations

$$C(\mathbf{q}) = \frac{1}{A_D}\int d^3r_n\, d^3r_{p_1}\, d^3r_{p_2}\, S(\mathbf{r}_n)\, S(\mathbf{r}_{p_1})\, S(\mathbf{r}_{p_2}) \left|\psi^{\mathbf{q}}_{pD}(\mathbf{r}_n, \mathbf{r}_{p_1}, \mathbf{r}_{p_2})\right|^2$$

p-D correlation function



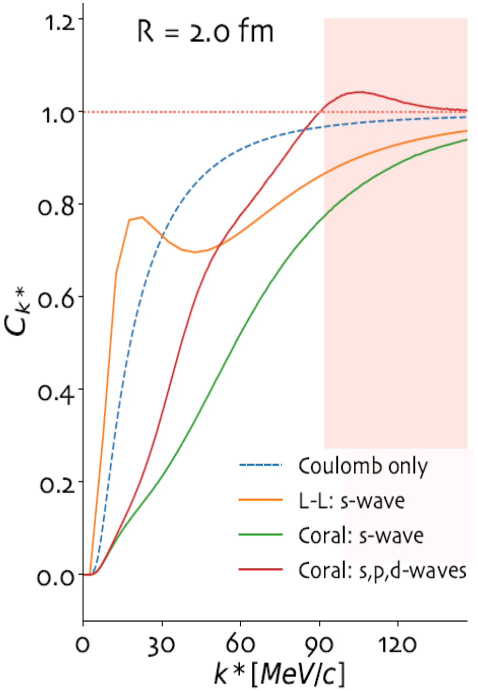
$R_s = 1.43 \pm 0.16$ fm

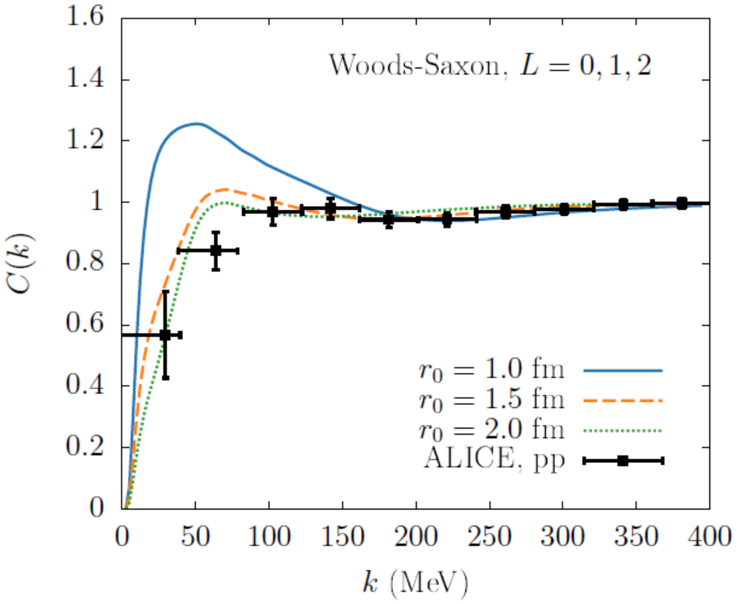
ALICE, Phys. Rev. X **14**, 031051 (2024)

M. Viviani et al, Phys. Rev. C **108**, 064002 (2023)

p-D correlation function

Two-body approaches beyond the Lednicky-Lyuboshitz formula





- The asymptotic wave function is insufficient.
- The *p* & *d* waves are important.

There is an ambiguity of deuteron source radius.

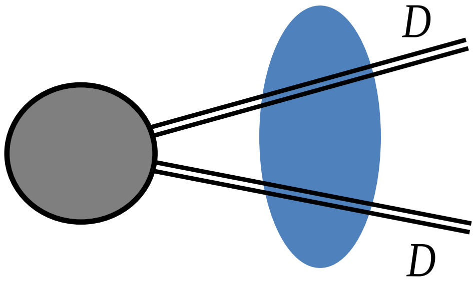
W. Rzęsa, M. Stefaniak & S.Pratt, arXiv:2410.13983

J. M.Torres-Rincon, A.Ramos & J.Rufi, arXiv:2410.23853

Deuteron-deuteron correlation function

Direct production

$$\boxed{C(\mathbf{q}) = \int d^3r\, S_r(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2}$$

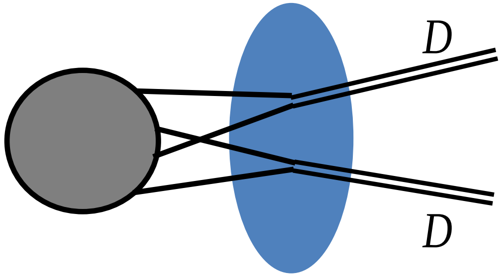


↑

$$S_r(\mathbf{r}) = \left(\frac{1}{4\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R^2}\right)$$

$$\sqrt{2} \approx 1.41$$

$$S_{4r}(\mathbf{r}) = \left(\frac{1}{2\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R^2}\right)$$



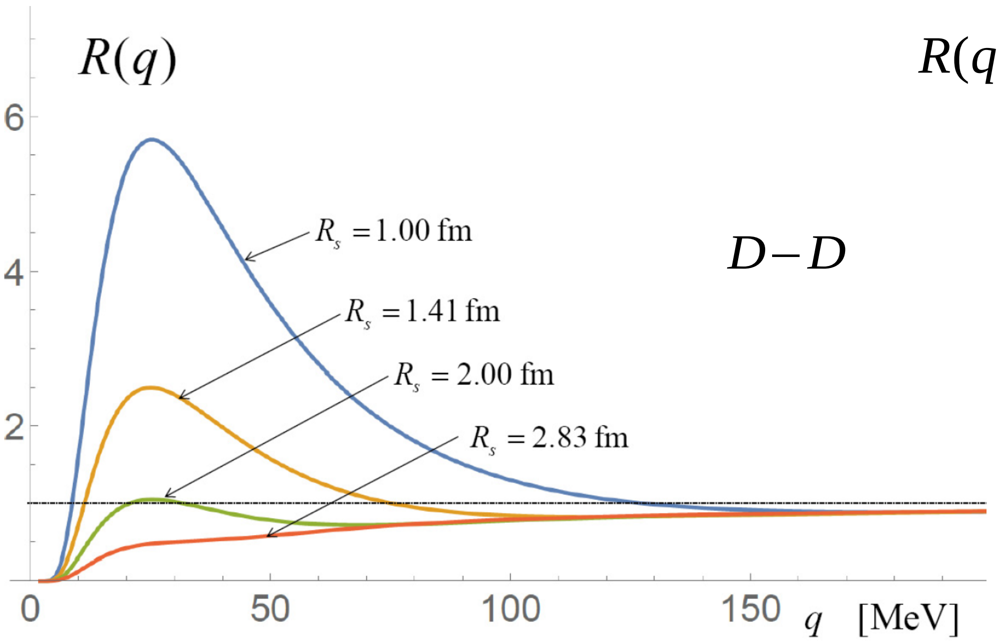
Final state interaction
& factorization

↓

$$\boxed{C(\mathbf{q}) = \int d^3r\, S_{4r}(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2}$$

St. Mrówczyński & P. Słoń, Phys. Rev. C **104**, 024909 (2021)

Deuteron-deuteron correlation function



spin 0 spin 1 spin 2

$$R(q) = \frac{1}{9} R_0(q) + \frac{3}{9} R_1(q) + \frac{5}{9} R_2(q)$$

$$a_0 = (10.2 + 0.2i) \text{ fm}$$

$$a_2 = 7.5 \text{ fm}$$

$$2.83 = \sqrt{2} \cdot 2.00 = \left(\sqrt{2}\right)^2 \cdot 1.41 = \left(\sqrt{2}\right)^3 \cdot 1.00$$

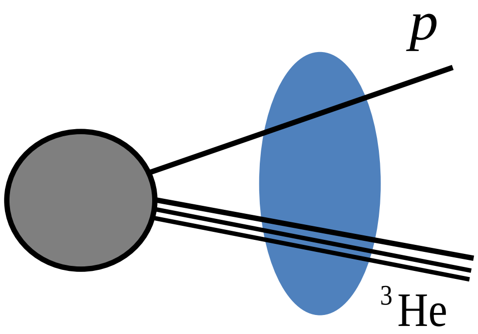
R_s from *D-D* correlation function vs. R_s from *p-p* & *p-D* correlation function

St. Mrówczyński & P. Słoń, Phys. Rev. C **104**, 024909 (2021)

Proton-^{3}He correlation function

Direct production

$$R(\mathbf{q}) = \int d^3r\, D_r(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2$$



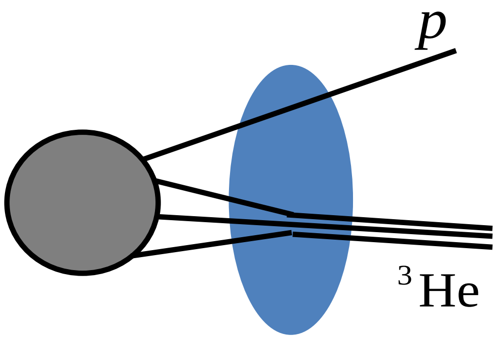
$$D_r(\mathbf{r}) = \left(\frac{1}{4\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R^2}\right)$$

$$\sqrt{\frac{3}{2}} \approx 1.22$$

$$D_{4r}(\mathbf{r}) = \left(\frac{3}{8\pi R^2}\right)^{3/2} \exp\left(-\frac{3\mathbf{r}^2}{8R^2}\right)$$

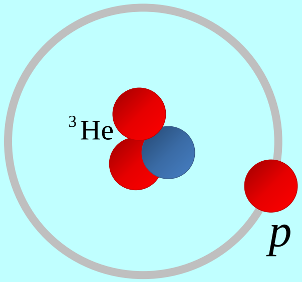
Final state interaction
& factorization

$$R(\mathbf{q}) = \int d^3r\, D_{4r}(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2$$



S. Bazak & St. Mrówczyński, Eur. Phys. J. A **56**, 193 (2020)

Resonance ^{4}Li



^{4}Li

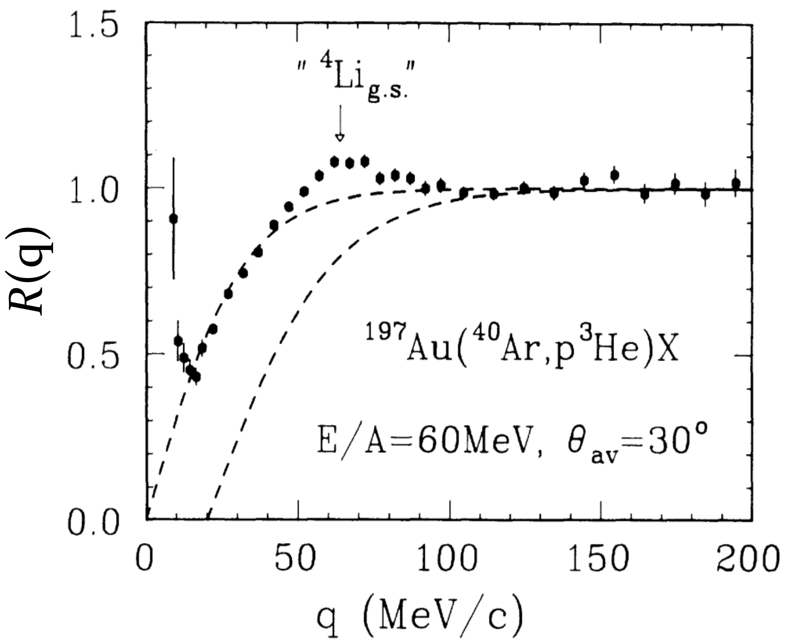
$^4\mathrm{Li} \to {}^3\mathrm{He} + p$

$\Gamma = 6\ \mathrm{MeV}$

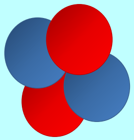
$m = m_{^3\mathrm{He}} + m_p + 4.1\ \mathrm{MeV}$

$m = 3749.7\ \mathrm{MeV}$

$s = 2$



J. Pochodzala et al. Phys. Rev. C **35**, 1695 (1987)



^{4}He

$r_{\mathrm{RMS}} = 1.68\ \mathrm{fm}$

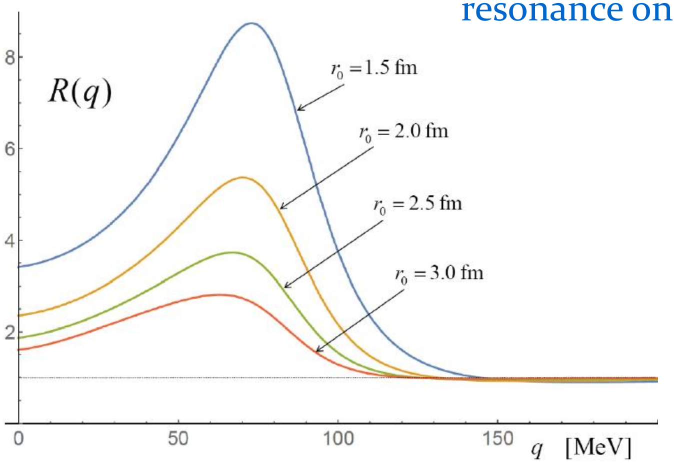
$\varepsilon_B = 28.3\ \mathrm{MeV}$

$m = 3727.4\ \mathrm{MeV}$

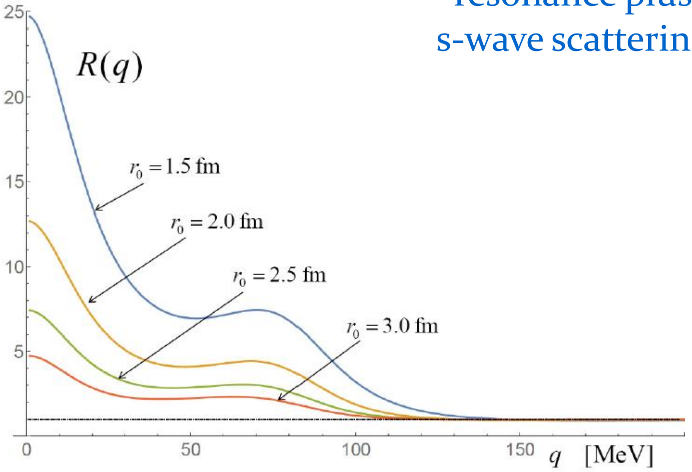
$s = 0$

M. Stefaniak for HADES Collaboration, arXiv:2402.09280

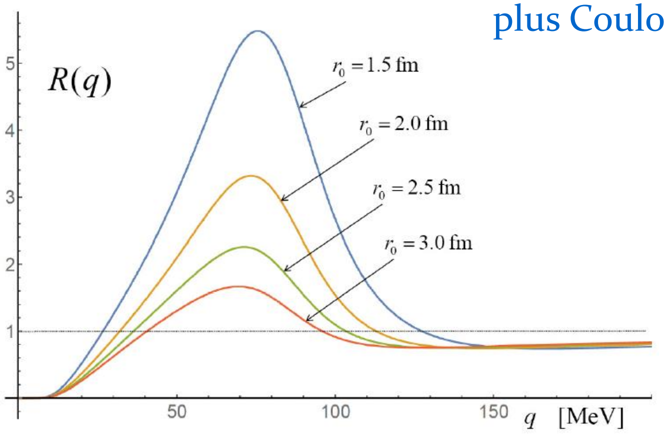
Correlation function *p*-^{3}He



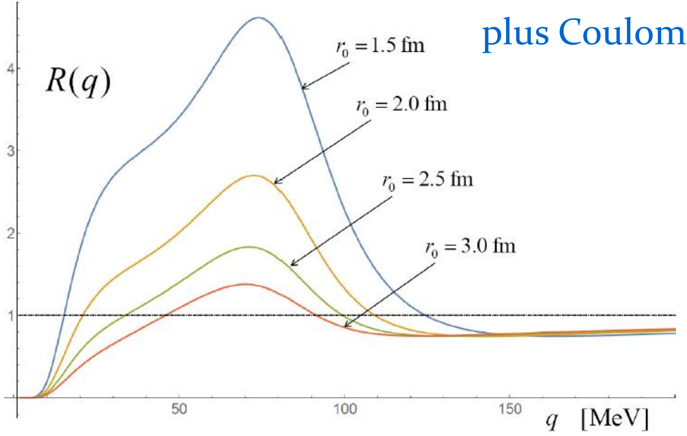
resonance only



resonance plus s-wave scattering



resonance plus Coulomb



resonance plus s-wave scattering plus Coulomb

S. Bazak & St. Mrówczyński, Eur. Phys. J. A **56**, 193 (2020)

How to measure yield of ^{4}Li

$$\boxed{\frac{dN_{\mathrm{Li}}}{d\mathbf{p}} = S_R \frac{dN_p}{d\mathbf{p}} \frac{dN_{^3\mathrm{He}}}{d\mathbf{p}}}$$

p - momentum per nucleon

$$S_R \equiv \int d^3q \, R_R(\mathbf{q})$$

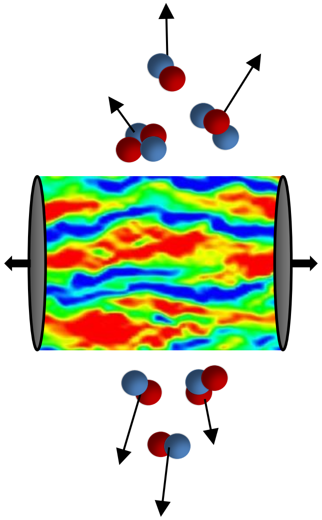
correlation function where only the ^{4}Li resonance contributes

Conclusions

- The correlation function of light nuclei depends on their production mechanism.
- Light nuclei should not be treated as point-like objects.
- The correlation functions of light nuclei can reveal an existence of various nuclear resonances.

Two very different cases of producing light nuclei

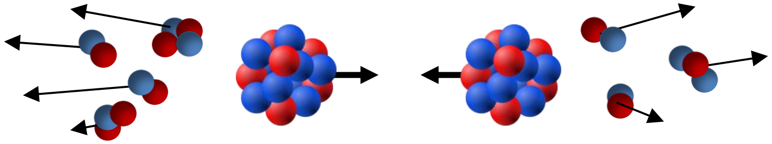
Genuine production



$y \approx 0$

hard process

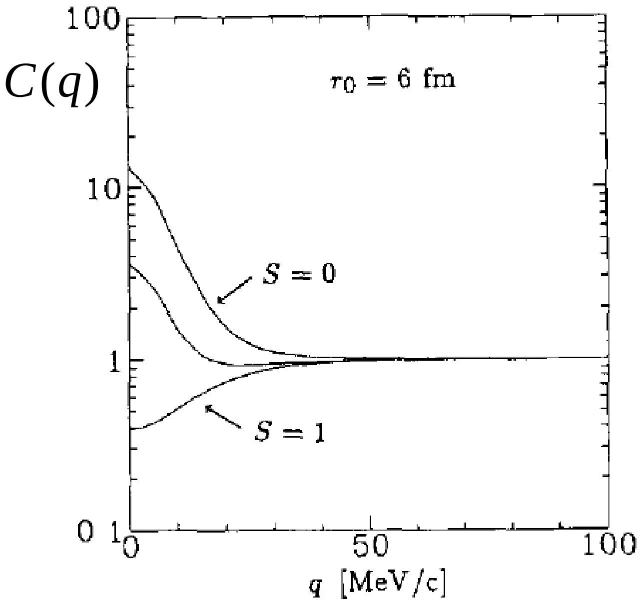
Shattering of incoming nuclei



$y \approx y_t$ $y \approx y_p$

soft process

n-p correlation function



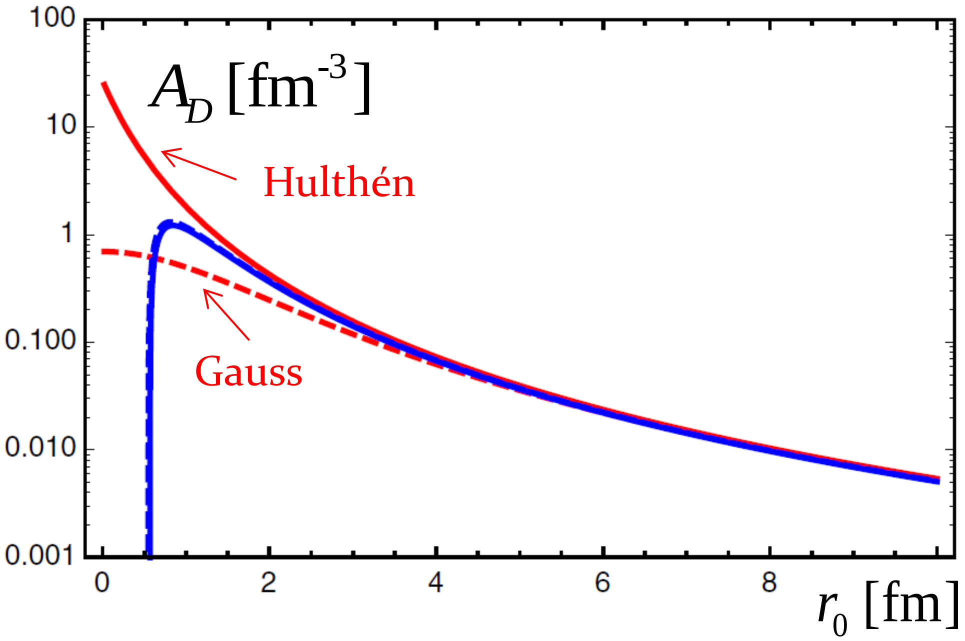
$$S(\mathbf{r}) = \left(\frac{1}{2\pi r_0^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2r_0^2}\right)$$

Lednicky-Lyuboshitz formula

St. Mrówczyński, Phys. Lett. B **277**, 43 (1992)

Sum rule due to completeness of quantum states

$$\int \underline{d^3\mathbf{q}\big(C_1(\mathbf{q}) - C_0(\mathbf{q})\big)} = -\underline{A_D}$$



R. Maj & St. Mrówczyński, Phys. Rev. C **101**, 014901 (2020)
R. Maj & St. Mrówczyński, Phys. Rev. C **71**, 044905 (2005)
St. Mrówczyński, Phys. Lett. B **345**, 393 (1995)