Jet quenching in glasma

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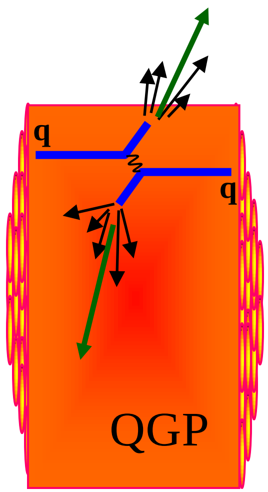
In collaboration with **Margaret Carrington, Alina Czajka & ….**

Happy Birthday!

Jet quenching

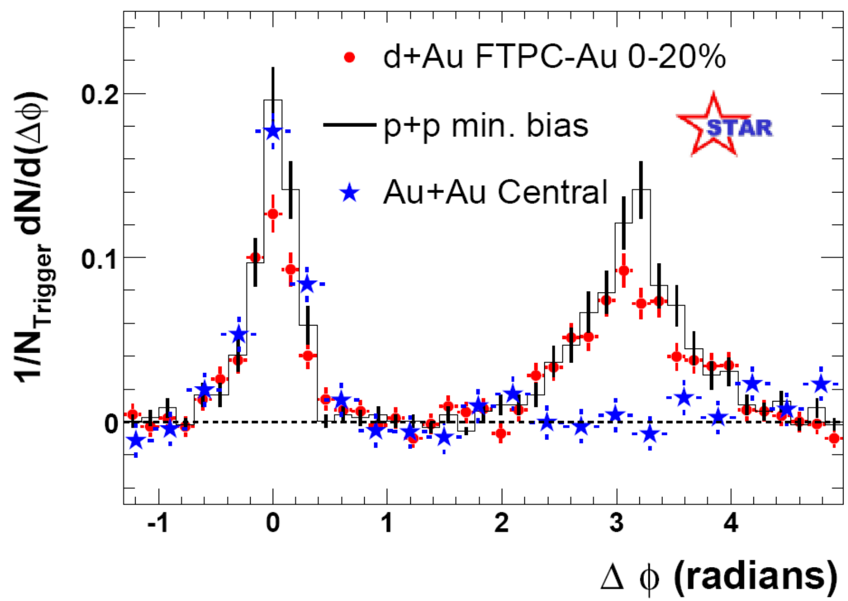
trigger

near-side



away-side

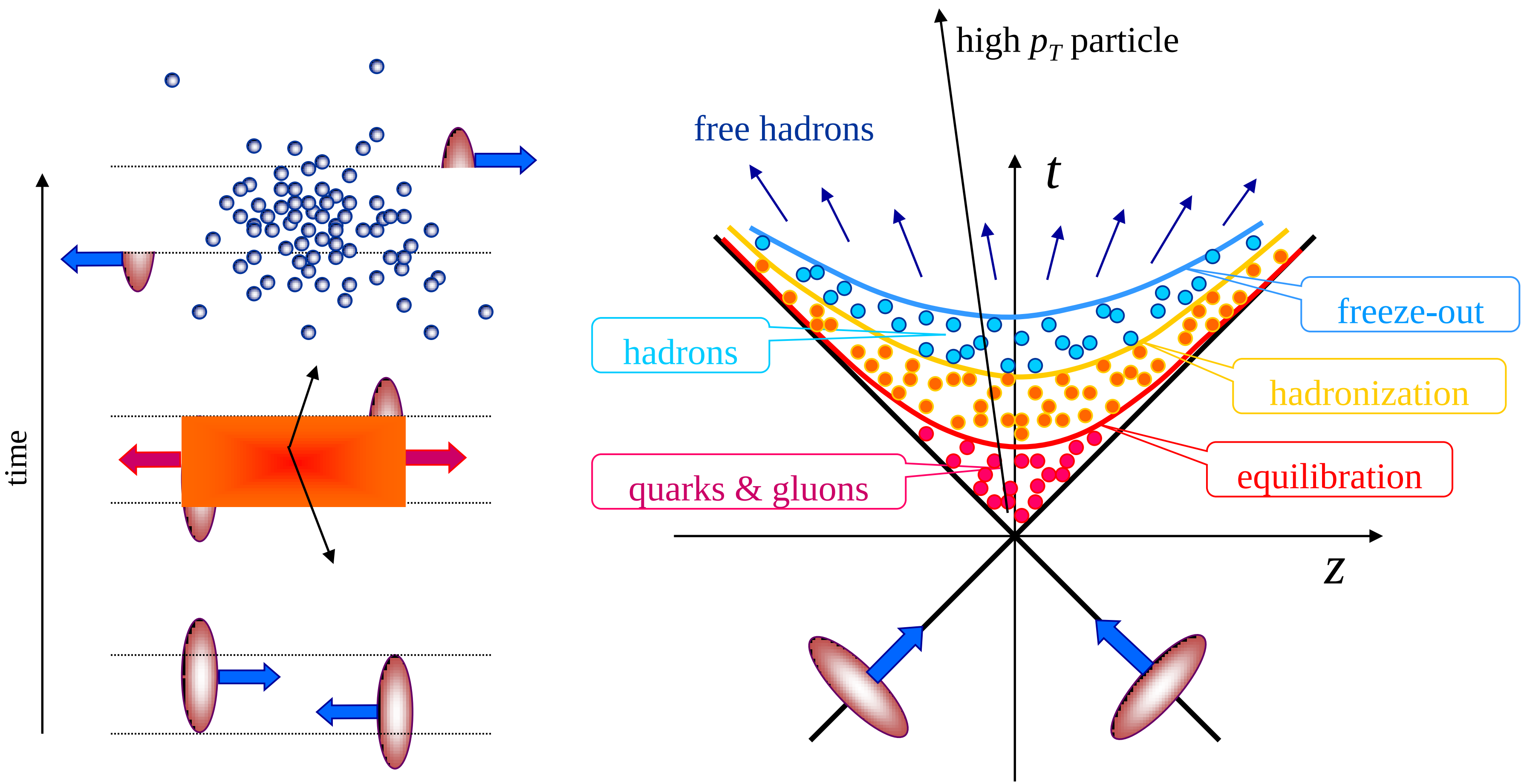
near-side away-side



Away-side jet is suppressed
in central collisions!

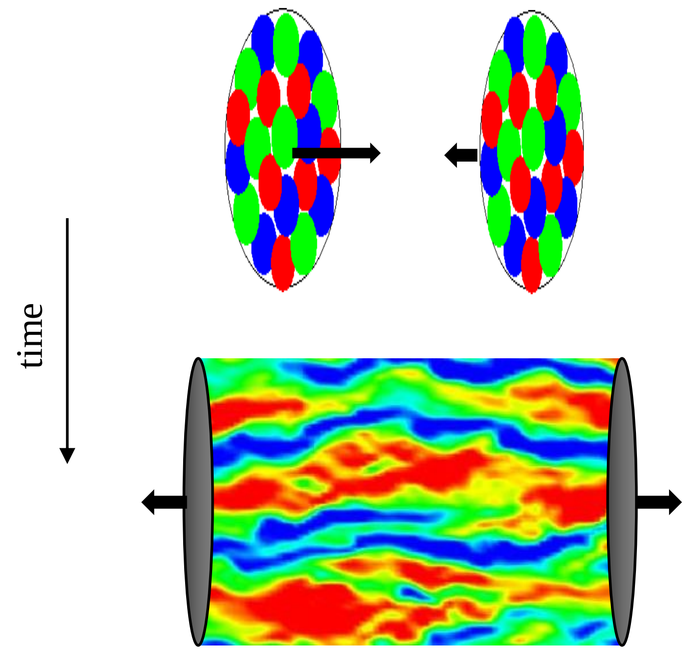
J. Adams et al. [STAR Collaboration], Nucl. Phys. **A757**, 102 (2005)

Relativistic heavy-ion collisions



Glasma – the earliest phase of matter in relativistic heavy-ion collisions

Within the framework of CGC (Color Glass Condensate), color charges confined in the colliding nuclei generate **glasma** – the system of strong mostly classical chromodynamic fields which evolve towards equilibrium.

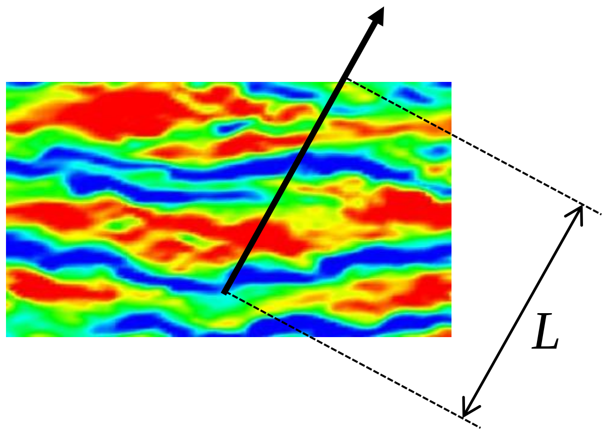


Glasma

lifetime ~ 0.1 fm/*c*
energy density ~ 1 TeV/fm^3

Jet quenching in glasma

How hard probes propagate through the glasma?



$$\frac{dE}{dx},\ \hat{q}\ ?$$

$\frac{dE}{dx}$ - collisional energy loss

$\hat{q}$ - transverse momentum broadening

$\frac{dE^{\mathrm{rad}}}{dx} = -\frac{1}{8}\alpha_s N_c \hat{q} L$ - radiative energy loss

Fokker-Planck equation

- Transport of hard probes can be described using the Fokker-Planck equation.

$$\boxed{\underbrace{\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)}_{\text{drift}} n(t,\mathbf{r},\mathbf{p}) = \underbrace{\left(\nabla_p^i X^{ij}(\mathbf{v})\nabla_p^j + \nabla_p^i Y^i(\mathbf{v})\right)}_{\text{collisions}} n(t,\mathbf{r},\mathbf{p})}$$

$n(t,\mathbf{r},\mathbf{p})$ - distribution function of hard probes $\qquad \mathbf{v}\equiv\frac{\mathbf{p}}{E_{\mathbf{p}}},\quad \nabla_p^i\equiv\frac{\partial}{\partial p_i}$

$$X^{ij}(\mathbf{v}),\ Y^i(\mathbf{v}) \quad\Rightarrow\quad \begin{cases} \dfrac{dE}{dx} = -\dfrac{\mathrm{v}^i}{\mathrm{v}} Y^i(\mathbf{v}) & \text{collisional energy loss} \\[2ex] \hat{q} = \dfrac{2}{\mathrm{v}}\left(\delta^{ij} - \dfrac{\mathrm{v}^i \mathrm{v}^j}{\mathrm{v}^2}\right) X^{ji}(\mathbf{v}) & \text{momentum broadening} \end{cases}$$

$$n(t,\mathbf{r},\mathbf{p}) = n_{\text{eq}}(\mathbf{p}) \sim e^{-\frac{E_{\mathbf{p}}}{T}} \qquad \Leftrightarrow \qquad \boxed{Y^j(\mathbf{v}) = \frac{\mathrm{v}^i}{T} X^{ij}(\mathbf{v})}$$

solves FK equation

Fokker-Planck equation of hard probes in glasma

$$\boxed{\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)n(t,\mathbf{r},\mathbf{p})=\left(\nabla_p^i X^{ij}(\mathbf{v})\nabla_p^j+\nabla_p^i Y^i(\mathbf{v})\right)n(t,\mathbf{r},\mathbf{p})}$$

$$\begin{cases} X^{ij}(\mathbf{v})=\dfrac{g}{N_c}\displaystyle\int_0^t dt'\Big\{\big\langle E^i(t,\mathbf{r})E^j(t',\mathbf{r}')\big\rangle+\varepsilon^{jkl}\mathrm{v}^k\big\langle E^i(t,\mathbf{r})B^l(t',\mathbf{r}')\big\rangle \\ \qquad +\varepsilon^{ikl}\mathrm{v}^k\big\langle B^l(t,\mathbf{r})E^j(t',\mathbf{r}')\big\rangle+\varepsilon^{ikl}\varepsilon^{jmn}\mathrm{v}^k\mathrm{v}^m\big\langle B^l(t,\mathbf{r})B^n(t',\mathbf{r}')\big\rangle\Big\} \\ Y^j(\mathbf{v})=\dfrac{\mathrm{v}^i}{T}X^{ij}(\mathbf{v}) \end{cases}$$

- The collision term is given by field correlators $\langle E^iE^j\rangle,\langle B^iE^j\rangle,\langle B^iB^j\rangle$

St. Mrówczyński, European Physical Journal A **54**, 43 (2018)

Equilibrium QGP – fields as a noise

space-time translational invariance & isotropy

$$\boxed{\left\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int\frac{d^3k}{(2\pi)^3}\, e^{-i\left(\omega(t-t')-\mathbf{k}(\mathbf{r}-\mathbf{r}')\right)}\underbrace{\left\langle E_a^i E_b^j\right\rangle_{\omega,\mathbf{k}}}_{\text{flucuation spectrum}}}$$

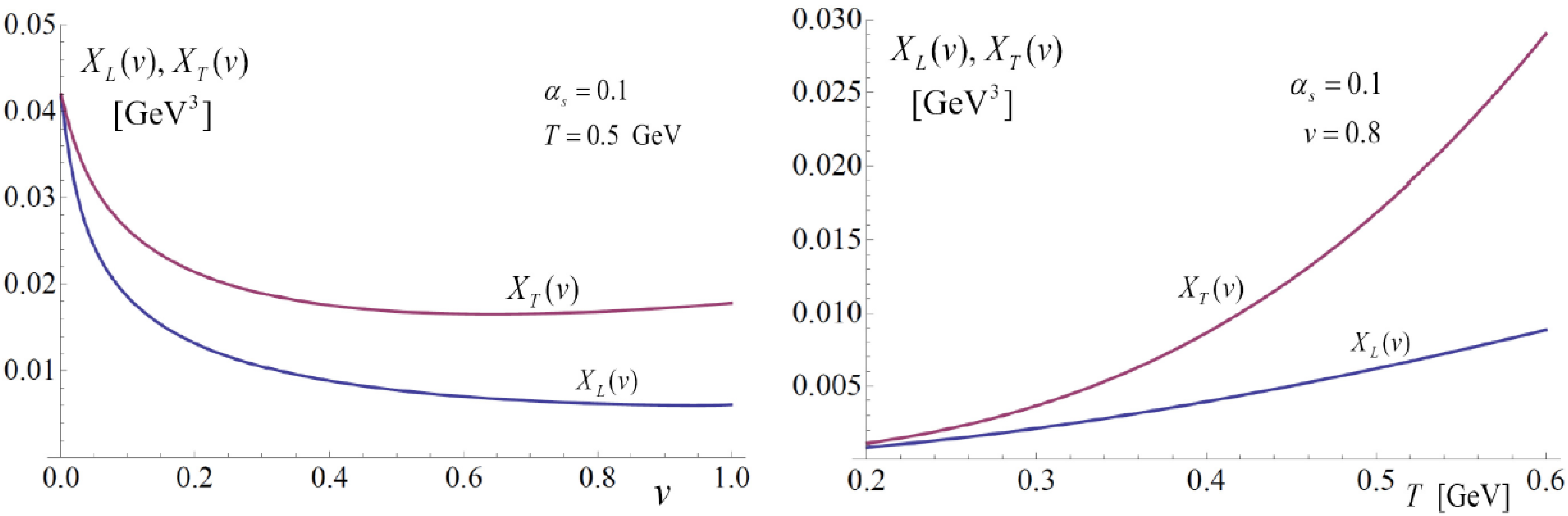
▶ $$\left\langle E_a^i E_b^j\right\rangle_{\omega,\mathbf{k}} = 2\delta^{ab}\frac{\omega^4}{e^{\beta|\omega|}-1}\left[\frac{k^i k^j}{\mathbf{k}^2}\frac{\mathrm{Im}\,\varepsilon_L(\omega,\mathbf{k})}{|\omega^2\varepsilon_L(\omega,\mathbf{k})|^2} + \left(\delta^{ij}-\frac{k^i k^j}{\mathbf{k}^2}\right)\frac{\mathrm{Im}\,\varepsilon_T(\omega,\mathbf{k})}{|\omega^2\varepsilon_T(\omega,\mathbf{k})-\mathbf{k}^2|^2}\right]$$

$\varepsilon_{L,T}(\omega,\mathbf{k})$ - chromodielectric functions

▶ $$X^{ij}(\mathbf{v}) \equiv X_L(\mathrm{v})\frac{\mathrm{v}^i\mathrm{v}^j}{\mathbf{v}^2} + X_T(\mathrm{v})\left(\delta^{ij}-\frac{\mathrm{v}^i\mathrm{v}^j}{\mathbf{v}^2}\right), \qquad Y^j(\mathbf{v}) = \frac{\mathrm{v}^i}{T}X^{ij}(\mathbf{v}) = \frac{\mathrm{v}^i}{T}X_L(v)$$

St. Mrówczyński, Physical Review D **77**, 105022 (2008)

Fokker-Planck equation of equilibrium QGP



Quantitative agreement with $X_L(\mathrm{v})$ & $X_T(\mathrm{v})$ obtained from the Boltzmann collision term by means of the diffusive approximation.

The standard FP equation is reproduced!

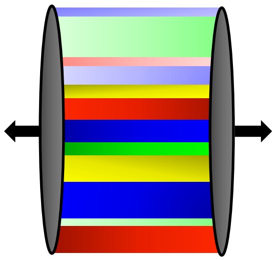
B. Svetitsky, Physical Review D **37**, 2484 (1988)

Rough estimate

Density of energy accumulated in the fields

$$\varepsilon_{\text{field}} = \frac{1}{2}\Big(\big\langle E_a^i E_a^i \big\rangle + \big\langle B_a^i B_a^i \big\rangle\Big)$$

E & *B* fields along the axis *z*



Density of energy released in a central collision

$$\varepsilon_{\text{coll}} = \frac{c_{\text{inel}} A \sqrt{s}}{\pi R_A^2 l}$$

$$\varepsilon_{\text{coll}} = \varepsilon_{\text{field}} \quad \Longrightarrow \quad \begin{cases} -\dfrac{dE}{dx} \sim (0 \div 10) \quad \left[\dfrac{\text{GeV}}{\text{fm}}\right] \\ \hat{q} \sim 10 \quad \left[\dfrac{\text{GeV}^2}{\text{fm}}\right] \end{cases}$$

$$c_{\text{inel}} = 0.5, \quad A = 200, \quad \sqrt{s} = 5\,\text{TeV}, \quad l = 1\,\text{fm}$$

St. Mrówczyński, European Physical Journal A **54**, 43 (2018)

Realistic calculations in proper time expansion

R. J. Fries, J. I. Kapusta, and Y. Li, arXiv:nucl-th/0604054
G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Proper time τ is treated as a small parameter $\tau << Q_s^{-1}$

Fully analytic approach

M. Carrington, A. Czajka & St. Mrówczyński, Nuclear Physics A **1001**, 121914 (2020)
M. Carrington, A. Czajka & St. Mrówczyński, European Physical Journal A **58**, 5 (2022)
M. Carrington, A. Czajka & St. Mrówczyński, Physical Review C **106**, 034904 (2022)
M. Carrington, A. Czajka & St. Mrówczyński, Physics Letters B **834**, 137464 (2022)
M. Carrington, A. Czajka & St. Mrówczyński, Physical Review C **106**, 034904 (2022)
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)
M. Carrington & St. Mrówczyński, Acta Physica Polonica B **55**, 4-A3 (2024)

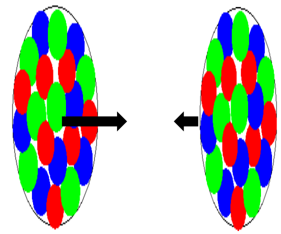
Color Glass Condensate

Classical Yang-Mills equation

$$\boxed{D_\mu F^{\mu\nu}(x) = j^\nu(x)}$$

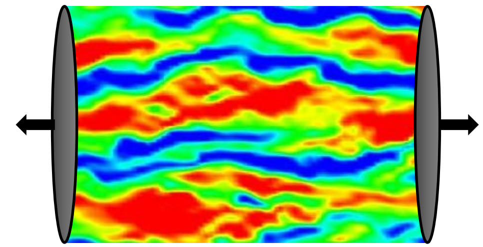
$$j^\mu(x) = j_1^\mu(x) + j_2^\mu(x)$$

$$j_{1,2}^\mu(x) = \pm\delta^{\mu\mp}\delta(x^\pm)\rho_{1,2}(\mathbf{x}_\perp)$$



β_1^i β_2^i

pre-collision potentials

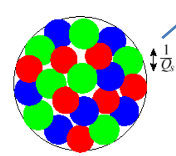


$\alpha, \alpha_\perp^i$

post-collision potentials

Ansatz of gauge potentials

$$\begin{cases} A^+(x) = \Theta(x^+)\Theta(x^-)x^+\alpha(\tau,\mathbf{x}_\perp) \\ A^-(x) = -\Theta(x^+)\Theta(x^-)x^-\alpha(\tau,\mathbf{x}_\perp) \\ A^i(x) = \Theta(x^+)\Theta(x^-)\alpha_\perp^i(\tau,\mathbf{x}_\perp) \\ \qquad +\Theta(-x^+)\Theta(x^-)\beta_1^i(\mathbf{x}_\perp) + \Theta(x^+)\Theta(-x^-)\beta_2^i(\mathbf{x}_\perp) \end{cases}$$



Boundary condition

$$\begin{cases} \alpha(0,\mathbf{x}_\perp) = \beta_1^i(\mathbf{x}_\perp) + \beta_2^i(\mathbf{x}_\perp) \\ \alpha_\perp^i(0,\mathbf{x}_\perp) = -\dfrac{ig}{2}[\beta_1^i(\mathbf{x}_\perp), \beta_2^i(\mathbf{x}_\perp)] \end{cases}$$

Gauge condition

$$x^+A^- + x^-A^+ = 0$$

E. Iancu, R. Venugopalan, in *Quark-Gluon Plasma 3*,
ed. by R.C. Hwa, X.-N. Wang (World Scientific, Singapore, 2004

Proper time expansion

$$\alpha(\tau,\mathbf{x}_\perp)=\sum_{n=0}^{\infty}\tau^n\alpha_{(n)}(\mathbf{x}_\perp),\qquad \alpha_\perp^i(\tau,\mathbf{x}_\perp)=\sum_{n=0}^{\infty}\tau^n\alpha_{\perp(n)}^i(\mathbf{x}_\perp)$$

Proper time τ is treated as a small parameter $\tau << Q_s^{-1}$

Yang-Mills equations for the expanded potentials are solved recursively

$$\alpha_{(n)}=\alpha_{\perp(n)}^i=0 \quad \text{for} \quad n=1,3,5,\ldots$$

0th order - oboundary conditions

$$\begin{cases}\alpha_{(0)}=-\dfrac{ig}{2}[\beta_1^i,\beta_2^i]\\ \alpha_{\perp(0)}^i=\beta_1^i+\beta_2^i\end{cases}$$

Post-collision potentials are expressed through pre-collision potentials

2nd order

$$\begin{cases}\alpha_{(2)}=-\dfrac{ig}{16}[D^j,[D^j,[\beta_1^i,\beta_2^i]]]\\ \alpha_{\perp(2)}^i=\dfrac{ig}{4}\varepsilon^{zij}\varepsilon^{zkl}[D^j,[\beta_1^k,\beta_2^l]]\end{cases}$$

$$D^i\equiv\partial^i-ig(\beta_1^i+\beta_2^i)$$

Fully analytic approach!

R. J. Fries, J. I. Kapusta, and Y. Li, arXiv:nucl-th/0604054
G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Proper time expansion cont.

Chromoelectric and chromomagnetic fields

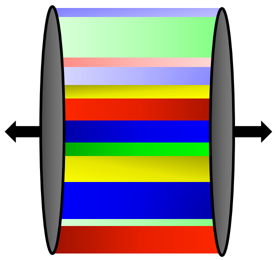
$$E^i = F^{i0}, \quad B^i = \frac{1}{2}\varepsilon^{ijk}F^{kj}$$

Zeroth order

$$\mathbf{E}_{(0)} = (0,0,E), \qquad \mathbf{B}_{(0)} = (0,0,B)$$

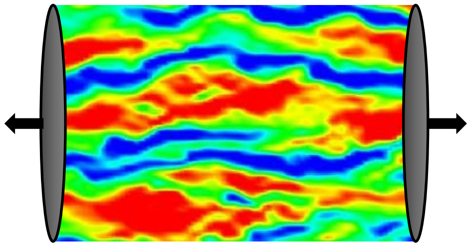
$$E^z_{(0)}(\mathbf{x}_\perp) = -ig[\beta_1^i(\mathbf{x}_\perp), \beta_2^i(\mathbf{x}_\perp)]$$

$$B^z_{(0)}(\mathbf{x}_\perp) = -ig\varepsilon^{zij}[\beta_1^i(\mathbf{x}_\perp), \beta_2^j(\mathbf{x}_\perp)]$$



E & *B* fields along the axis *z*

At higher orders transverse fields show up



R. J. Fries, J. I. Kapusta, and Y. Li, arXiv:nucl-th/0604054

G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Field correlators

The correlators

$$\left\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle, \quad \left\langle E_a^i(t,\mathbf{r})B_b^j(t',\mathbf{r}')\right\rangle, \quad \left\langle B_a^i(t,\mathbf{r})B_b^j(t',\mathbf{r}')\right\rangle$$

are expressed through

$$\sum \partial^i \partial^j \left\langle \beta_a^k(\mathbf{x}_\perp)\beta_b^l(\mathbf{y}_\perp)\ldots\beta_c^m(\mathbf{z}_\perp)\right\rangle$$

IR regulator $m = \Lambda_{\mathrm{QCD}}$

In covariant gauge $\partial_\mu \beta^\mu = 0$

$$-\nabla^2\beta^+(\mathbf{x}_\perp) = \rho(\mathbf{x}_\perp) \quad \Rightarrow \quad \beta^+(\mathbf{x}_\perp) = \frac{1}{2\pi}\int d^2x'_\perp K_0(m\,|\,\mathbf{x}_\perp - \mathbf{x}'_\perp\,|)\rho(\mathbf{x}'_\perp)$$

The potentials are transformed from the covariant to light-cone gauge

Wick theorem

$$\left\langle \rho_a^k(\mathbf{x}_\perp)\rho_b^l(\mathbf{y}_\perp)\ldots\rho_c^m(\mathbf{z}_\perp)\right\rangle = \sum\prod\left\langle \rho_a^i(\mathbf{x}_\perp)\rho_b^j(\mathbf{y}_\perp)\right\rangle$$

Glasma graph approximation

$$\left\langle \beta_a^k(\mathbf{x}_\perp)\beta_b^l(\mathbf{y}_\perp)\ldots\beta_c^m(\mathbf{z}_\perp)\right\rangle = \sum\prod\left\langle \beta_a^i(\mathbf{x}_\perp)\beta_b^j(\mathbf{y}_\perp)\right\rangle = \sum\prod B_{ab}^{ij}(\mathbf{x}_\perp - \mathbf{y}_\perp)$$

Basic correlator

$$B^{ij}_{ab}(\mathbf{x}_\perp,\mathbf{y}_\perp) \equiv \left\langle \beta^i_a(\mathbf{x}_\perp)\beta^j_b(\mathbf{y}_\perp)\right\rangle = \int d^2x'_\perp\, d^2y'_\perp \cdots\cdots \left\langle \rho^i_a(\mathbf{x}'_\perp)\rho^j_b(\mathbf{y}'_\perp)\right\rangle$$

$$\left\langle \rho^i_a(\mathbf{x}_\perp)\rho^j_b(\mathbf{y}_\perp)\right\rangle = g^2\mu\,\delta^{ab}\delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp)$$

color charge surface density

$$\mu = g^{-4}Q_s^2$$

$$B^{ij}_{ab}(\mathbf{x}_\perp,\mathbf{y}_\perp) \equiv \delta^{ab}\left(\delta^{ij}C_1(r) - \hat{r}^i\hat{r}^jC_2(r)\right)$$

$$\mathbf{r} \equiv \mathbf{x}_\perp - \mathbf{y}_\perp, \quad r \equiv |\mathbf{r}|, \quad \hat{r}^i \equiv \frac{r^i}{r}$$

$$\begin{cases} C_1(r) \equiv \dfrac{m^2K_0(mr)}{g^2N_c\left(mrK_1(mr)-1\right)}\left\{\exp\left[\dfrac{g^4N_c\mu\left(mrK_1(mr)-1\right)}{4\pi m^2}\right]-1\right\} \\[2ex] C_2(r) \equiv \dfrac{m^3rK_1(mr)}{g^2N_c\left(mrK_1(mr)-1\right)}\left\{\exp\left[\dfrac{g^4N_c\mu\left(mrK_1(mr)-1\right)}{4\pi m^2}\right]-1\right\} \end{cases}$$

$$\underset{r \ll m^{-1}}{\approx} \quad \#\log(mr)$$

UV regularization required

$$r > Q_s^{-1}$$

J. Jalilian-Marian, A. Kovner, L. D. McLerran and H. Weigert, Physical Review D **55**, 5414 (1997)

Fokker-Planck Equation

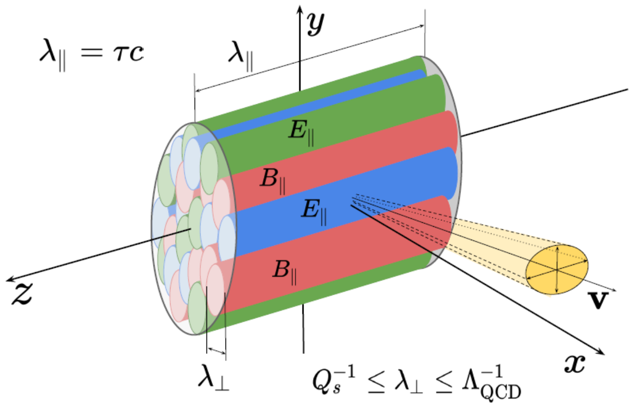
$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)n(t,\mathbf{r},\mathbf{p})=\left(\nabla_p^i X^{ij}(\mathbf{v})\nabla_p^j+\nabla_p^i Y^i(\mathbf{v})\right)n(t,\mathbf{r},\mathbf{p})$$

$$\begin{cases} X^{ij}(\mathbf{v})=\dfrac{g}{N_c}\displaystyle\int_0^t dt'\Big\{\big\langle E^i(t,\mathbf{r})E^j(t',\mathbf{r}')\big)\big\rangle+\varepsilon^{jkl}\mathrm{v}^k\big\langle E^i(t,\mathbf{r})B^l(t',\mathbf{r}')\big)\big\rangle \\ \qquad +\varepsilon^{ikl}\mathrm{v}^k\big\langle B^l(t,\mathbf{r})E^j(t',\mathbf{r}')\big)\big\rangle+\varepsilon^{ikl}\varepsilon^{jmn}\mathrm{v}^k\mathrm{v}^m\big\langle B^l(t,\mathbf{r})B^n(t',\mathbf{r}')\big)\big\rangle\Big\} \\ Y^j(\mathbf{v})=\dfrac{\mathrm{v}^i}{T}X^{ij}(\mathbf{v}) \end{cases}$$

The correlators are computed order by order.

$$\begin{cases} \hat{q}=\dfrac{2}{\mathrm{v}}\left(\delta^{ij}-\dfrac{\mathrm{v}^i\mathrm{v}^j}{\mathrm{v}^2}\right)X^{ji}(\mathbf{v}) \\ \dfrac{dE}{dx}=-\dfrac{\mathrm{v}^i}{\mathrm{v}}Y^i(\mathbf{v}) \end{cases}$$

Hard probes in glasma - $\hat{q}$



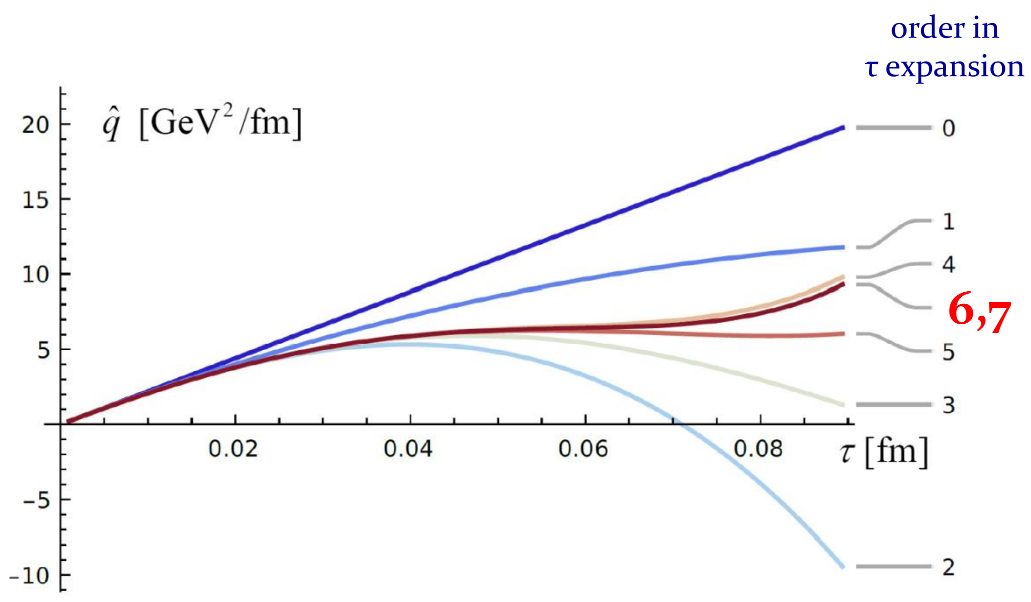
$$\hat{q} = \frac{2}{\mathrm{v}}\left(\delta^{ij} - \frac{\mathrm{v}^i \mathrm{v}^j}{\mathrm{v}^2}\right) X^{ji}(\mathbf{v})$$

$N_c = 3, \quad g = 1$

$Q_s = 2$ GeV

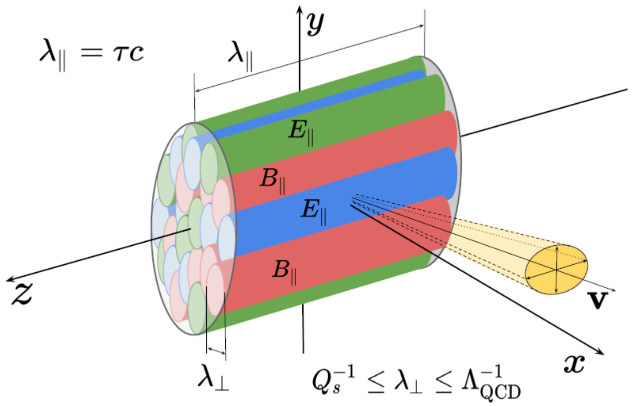
$m = 0.2$ GeV

$\mathrm{v} = \mathrm{v}_\perp = 1$

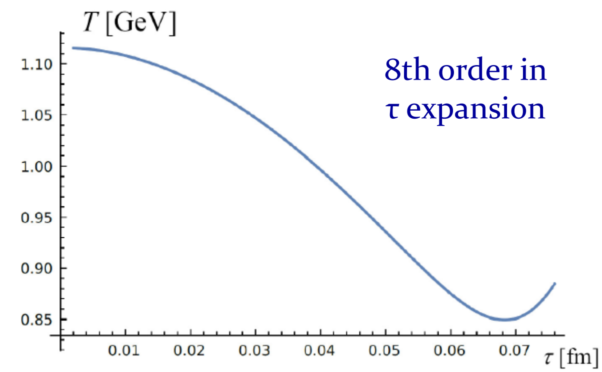


M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Hard probes in glasma - $\frac{dE}{dx}$



$$\frac{dE}{dx} = -\frac{1}{T}\frac{v^i v^i}{v} X^{ij}(\mathbf{v})$$

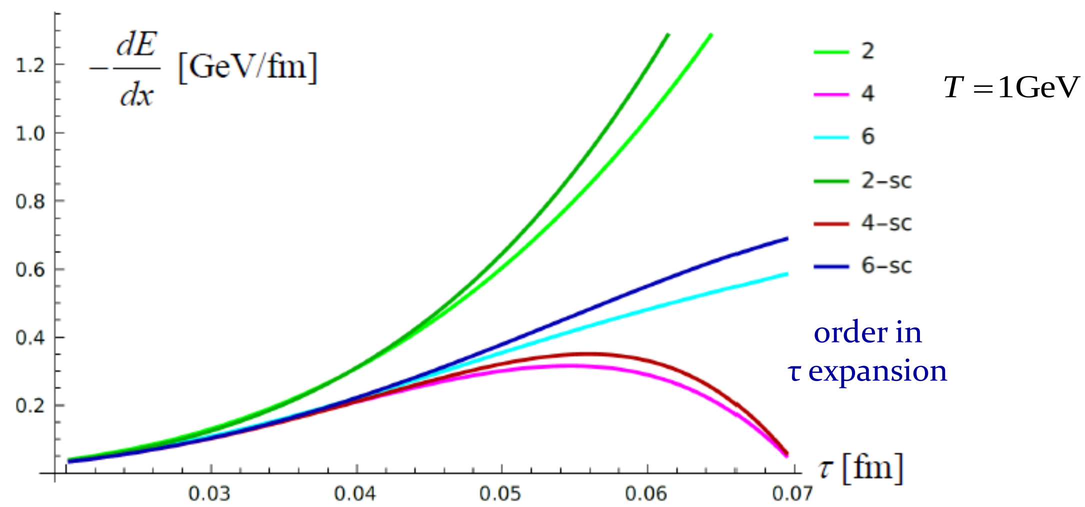


$N_c = 3, \quad g = 1$

$Q_s = 2$ GeV

$m = 0.2$ GeV

$v = v_\perp = 1$



M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Glasma impact on jet quenching

Glasma

$\hat{q}_{\max} = 6 \ \mathrm{GeV}^2/\mathrm{fm}$

$t_{\max} = 0.06 \ \mathrm{fm}$

Equilibrium QGP

$\hat{q} = 3T^3$

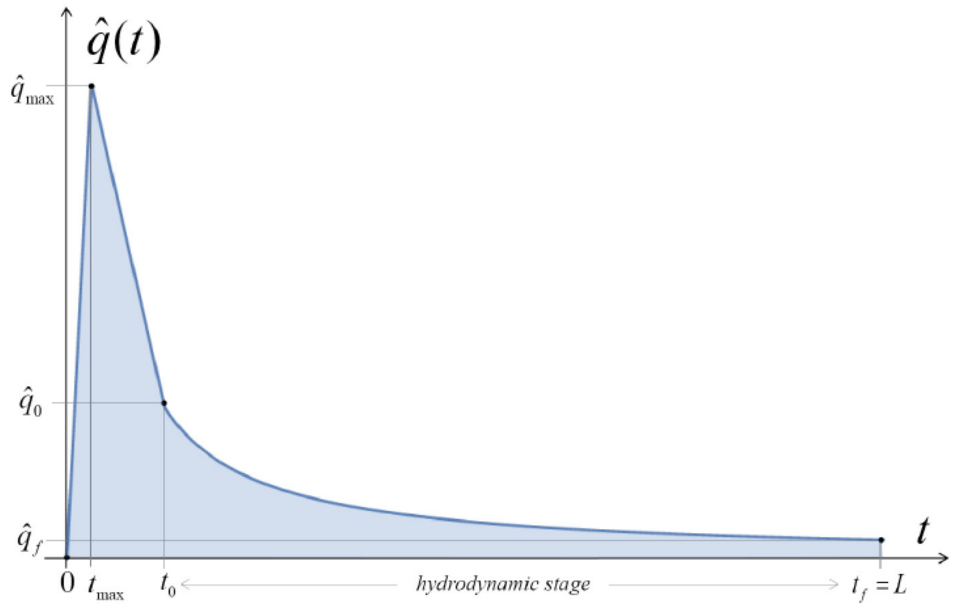
$t_0 = 0.6 \ \mathrm{fm}$

$T_0 = 450 \ \mathrm{MeV}$

$\hat{q}_0 = 1.4 \ \mathrm{GeV}^2/\mathrm{fm}$

$T = T_0 \left(\frac{t_0}{t} \right)^{1/3}$

$L = 10 \ \mathrm{fm}$



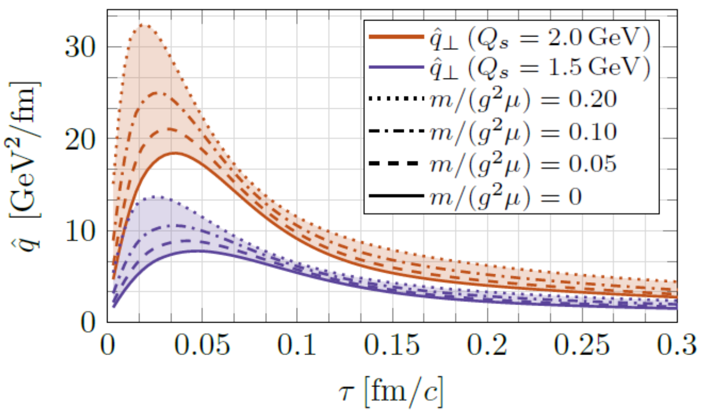
$$\Delta p_T^2 \Big|^{\text{non-eq}} = \int_0^{t_0} dt\, \hat{q}(t)$$

$$\Delta p_T^2 \Big|^{\text{eq}} = \int_{t_0}^{L} dt\, \hat{q}(t)$$

$$\frac{\Delta p_T^2 \Big|^{\text{non-eq}}}{\Delta p_T^2 \Big|^{\text{eq}}} = 0.93$$

S. Cao et al. [JETSCAPE], Physical Review C **104**, 024905 (2021),
C. Shen, U. Heinz, P. Huovinen and H. Song, Physical Review C **84**, 044903 (2011).

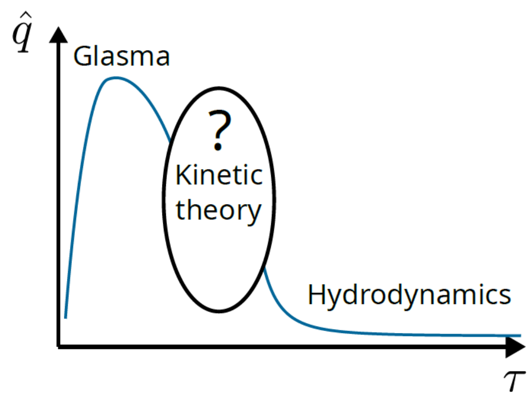
Glasma impact on jet quenching cont.



Full simulations of glasma

A. Ipp, D.I. Műller and D. Schuh, Phys. Lett. B **810**, 135810 (2020)

D. Avramescu, V. Băran, V. Greco, A. Ipp, D.I. Műller & M.Ruggieri, Phys. Rev. D **107**, 114021 (2023)



Kinetic theory interpolates between glasma and equilibrium QGP

K. Boguslavski, A. Kurkela, T.Lappi, F. Lindenbauer & J.Peuron, Phys. Lett. B **850**, 138525 (2024)

Summary & Conclusions

- The Fokker-Planck equation of hard probes in glasma is derived.
- The known case of equilibrium plasma is reproduced.
- The correlators of glasma field are computed up to τ^7.
- The momentum broadening and energy loss in the glasma are significantly bigger than in equilibrated QGP.
- In spite of its short lifetime, the glasma significantly contributes to the jet quenching.