Event-by-event fluctuations – the future of ion physics

X Polish Workshop on Relativistic Heavy-Ion Collisions
“Unreasonable effectiveness of statistical approaches to high-energy collisions”
December 13-15, 2013, Kielce, Poland
Fluctuations and correlations:

• May serve as a signature of the onset of deconfinement

Close to the phase transition Equation of State changes rapidly which can impact energy dependence of fluctuations

• Can help to locate the critical point of strongly interacting matter

Analogy to critical opalescence – enlarged fluctuations close to the critical point. For strongly interacting matter maximum of CP signal expected when freeze-out happens near CP

http://www.msm.cam.ac.uk/doitpoms/tlplib/solid-solutions/videos/laser1.mov
NA61/SHINE heavy ion program (continuation of NA49 efforts)

The most interesting region of the phase diagram is accessible at the SPS

- Onset of deconfinement at \( \cong 30 \text{A GeV} \)  
  PRC 77, 024903 (2008)
- Critical point? Example: \((T^{\text{CP}}, \mu_B^{\text{CP}}) = (162\pm2, 360\pm40) \text{ MeV}\)  
  JHEP 0404, 050 (2004)

Comprehensive scan in the whole SPS energy range (13A-158A GeV) with light and intermediate mass nuclei

- Search for the critical point
  Search for a maximum of CP signatures: fluctuations of N, average \( p_T \), etc., intermittency, when system freezes out close to CP

- Study of the properties of the onset of deconfinement
  Search for the onset of the horn/kink/step in collisions of light nuclei; additional analysis of fluctuations and correlations (azimuthal, particle ratios, etc.)

Estimated (NA49) and expected (NA61) chemical freeze-out points according to  

PRC 73, 044905 (2006)
History

How we were measuring chemical, $p_T$, and multiplicity fluctuations (in NA49)

Fig. from http://en.wikipedia.org/wiki/Big_History
Chemical (particle type) fluctuations

\( \sigma_{\text{dyn}} \) measure of particle ratio fluctuations \((K/\pi, p/\pi, K/p)\)

\[ \sigma^2_{\text{dyn}} \sim \frac{1}{N_w} \] (PRC 81, 034910 (2010), PRC 84, 014904 (2011))

\( \Rightarrow \sigma_{\text{dyn}} (K/\pi) \) (increase at lower SPS energies) and \( \sigma_{\text{dyn}} (p/\pi) \) fully reproduced in multiplicity scaling model (PRC 81, 034910 (2010); J. Phys. G38, 124096 (2011))

\( \sigma_{\text{dyn}} (K/p) \) – not understood as due to multip. scaling (change of sign close to the onset of deconf. energy); see \( \Phi_{p,K} \) later

\( \sigma_{\text{dyn}} \) easy for interpretation

Older NA49 results NOT corrected for the effect of misidentification

Multiplicity fluctuations

scaled variance of multiplicity distribution \( \omega \) (intensive – not dependent on \( N_w \))

Proper normalization (\( \omega=1 \) for Poisson)

NA49 results NOT corrected for detector inefficiencies and trigger bias

Transverse momentum fluctuations

\( \Phi_{pT} \) measure (strongly intensive – not dependent on \( N_w \) and its fluctuations)

Lack of proper normalization

NA49 results corrected for detector inefficiencies but NOT corrected for trigger bias
Modern times
How we are measuring chemical, $p_t$, and multiplicity fluctuations

Fig. from http://letsbuildateamfast.blogspot.com/2012/10/importance-of-using-online-render-farm.html
Multiplicity and chemical fluctuations of identified particles

- Instead of $\sigma_{\text{dyn}}$ new strongly intensive measure $\Phi$

$$\Phi_{ij} = \frac{\sqrt{\langle N_i \rangle \langle N_j \rangle}}{\langle N_i \rangle + \langle N_j \rangle} \cdot \left[ \sqrt{\Sigma[N_i, N_j]} - 1 \right]$$

$$\Sigma[N_i, N_j] = C_\Sigma^{-1} \left[ \langle N_i \rangle \omega[N_j] + \langle N_j \rangle \omega[N_i] - 2 \left( \langle N_i N_j \rangle - \langle N_i \rangle \langle N_j \rangle \right) \right]$$

$$C_\Sigma = \langle N_i \rangle + \langle N_j \rangle$$

For Poisson multip. distrib. $\omega = 1$

**Intensive measure**: in WNM

$\omega$ independent of $N_W$ but dependent on fluctuations of $N_W$

- New “identity method”

In experiment chemical fluctuations of identified particles multiplicities may be distorted by incomplete particle identification

Results on chemical fluctuations in NA49 and NA61 presented below are **corrected for misidentification** using the unfolding procedure of the identity method:


- For independent particle emission $\Phi = 0$

**Strongly intensive measure**: in WNM

$\Phi$ independent of $N_W$ and fluctuations of $N_W$

Results on chemical fluctuations in NA49 and NA61 presented below are **corrected for misidentification** using the unfolding procedure of the identity method:


Fluctuations cannot be corrected for the limited acceptance → results are presented in NA61 acceptance (https://edms.cern.ch/document/1237791/1)
• $\omega_{p+p}$ and $\omega_p < 1$ probably due to baryon number conservation. $\omega_p$ and $\omega_{p+p}$ similar (small fraction of antiprotons)

• $\omega_K > 1$ probably due to strangeness conservation. $\omega_{K^+}$ close to 1 and $< \omega_K$, which suggests that strangeness conservation contributes to $\omega_K$

• Increase of $\omega_\pi$ with energy reflecting increase of $\omega_{Nch}$ measured in full phase-space (see PR 351, 161 (2001)). $\omega_{\pi^+} < \omega_\pi$ possibly due to charge conservation

• $\omega_\pi$ and $\omega_{Nch}$ similar at higher energies (at lowest energies the fraction of protons is significant)

• HSD, EPOS, UrQMD predictions are similar to experimental results
** φ measure of chemical fluctuations: comparison of p+p with central Pb+Pb (NA49) collisions **

- $\Phi_{\pi(p+p)}$ and $\Phi_{\pi^0}$ < 0 most probably due to charge conservation and resonance decays (PR C70, 064903 (2004)). Similar tendency for NA61 p+p and NA49 Pb+Pb.

- In p+p $\Phi_{\pi K}$ > 0 probably due to strangeness conservation ($\Phi_{\pi K}$ close to 0 supports this interpretation). For p+p $\Phi_{\pi K}$ slightly increases with energy; such effect not visible for NA49 Pb+Pb.

- Very weak increase of $\Phi_{(p+p)K}$ with energy in p+p data, whereas for Pb+Pb $\Phi_{(p+p)K}$ decreases with energy (high momentum part removed from NA49 Pb+Pb data). For both systems $\Phi_{(p+p)K}$ crosses zero at middle SPS energies. No energy dependence of $\Phi_{pK^+}$.

- EPOS and UrQMD model predictions are similar to measurements in p+p.
**\(p_T\) and multiplicity fluctuations of non-identified particles**

- New strongly intensive measures \(\Delta\) and \(\Sigma\) (here applied to \(p_T\) fluct.) \(\rightarrow\) PRC 88, 024907 (2013)
- Novel method of correcting (NA61) results (\(\omega, \Delta^{XN}, \Sigma^{XN}, \Phi_{p_T}\)) for non-target interactions, detector inefficiencies, and trigger bias (see later)

\[
\Phi_{p_T} \equiv \sqrt{\frac{\langle X^2 \rangle}{\langle N \rangle} - \frac{2\langle X \rangle \langle NX \rangle}{\langle N \rangle^2} + \frac{\langle X \rangle^2 \langle N^2 \rangle}{\langle N \rangle^3}} - \sqrt{\frac{\langle X_2 \rangle}{\langle N \rangle} - \frac{\langle X \rangle^2}{\langle N \rangle^2}}
\]

\[
X = \sum_{i=1}^{N} p_{T_i} \quad X_2 = \sum_{i=1}^{N} (p_{T_i}^2)
\]

important relation:
\[
\Phi_{p_T} = \sqrt{\frac{p_T}{\omega(p_T)} \left( \sqrt{\Sigma^{XN}} - 1 \right)}
\]

\(\Delta^{XN}\) uses only first two moments: \(\langle N \rangle, \langle X \rangle, \langle X^2 \rangle, \langle N^2 \rangle\)

\(\Sigma^{XN}\) uses also correlation term:
\(\langle XN \rangle - \langle X \rangle \langle N \rangle\)

thus \(\Delta\) and \(\Sigma\) can be sensitive to several physics effects in different ways
## Transverse momentum fluctuations

<table>
<thead>
<tr>
<th>unit</th>
<th>No fluctuations; N = const.</th>
<th>Independent Particle Model (IPM)</th>
<th>Model of Independent Sources (MIS); for example WNM (N_S ≡ N_W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi_{p_T} )</td>
<td>MeV/c</td>
<td>( \Phi_{p_T} = -\sqrt{p_T} \omega(p_T) )</td>
<td>( \Phi_{p_T} = 0 )</td>
</tr>
<tr>
<td>( \Delta^{X_N} )</td>
<td>dimensionless</td>
<td>( \Delta^{X_N} = 0 )</td>
<td>( \Delta^{X_N} = 1 )</td>
</tr>
<tr>
<td>( \Sigma^{X_N} )</td>
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</tr>
</tbody>
</table>

\( \Delta \) and \( \Sigma \) are dimensionless and have scale which allows for a quantitative comparison of fluctuations of different, in general dimensional, extensive quantities.

## Multiplicity fluctuations

<table>
<thead>
<tr>
<th>unit</th>
<th>No fluct.; N = const.</th>
<th>Poisson N distribution</th>
<th>Model of Independent Sources (MIS); for example WNM (N_S ≡ N_W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega )</td>
<td>dimensionless</td>
<td>( \omega = 0 )</td>
<td>( \omega = 1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( &lt;n&gt; ) - mean multiplicity from a single source ( \omega_{N_S} ) - fluctuations in N_S</td>
</tr>
</tbody>
</table>
“Know your reference”

- What does the elliptic flow coefficient $v_2 = 0.1$ mean?  
- It means that 50% more particles are emitted “in plane” than “out of plane”. Huge effect!

- What does the $\Phi_{pT} = 10 \text{ MeV}/c$ mean?  
- Nothing! We do not know whether it is a large or a small effect. Especially when the magnitudes of $\Phi_{pT}$ from several “trivial” effects (BE statistics, resonance decays, etc.) are not estimated.

- What does the $\Sigma^{xN} = 1.1$ mean?  
- It means that (for this specific combination of moments $\to \Sigma$ quantity) we measure 10% deviation from IPM (fluctuations are 10% larger than in IPM)

Similar advantage for $\omega \to$ here Poisson N distribution (instead of IPM) used as the reference: $\omega_N = 0$ for $N = \text{const.}$ and $\omega_N = 1$ for Poisson N distribution. Thus for any $P(N)$ distribution:  
$\omega_N > 1$ (or $\omega_N \gg 1$) corresponds to “large” (or “very large”) fluctuations of N,  
$\omega_N < 1$ (or $\omega_N \ll 1$) corresponds to “small” (or “very small”) fluctuations of N

$\Delta$ and $\Sigma$ measures – keep the advantages of both $\Phi$ (they are strongly intensive) and $\omega$ (they are properly normalized)
Several effects were studied for new $\Delta^{XN}$ and $\Sigma^{XN}$ measures:

1. IPM, MIS, source-by-source $T$ fluctuations (example of MIS), event-by-event (global) $T$ fluctuations, $M(p_T)$ vs $N$ correlation → arXiv:1309.7878

S-by-s $T$ fluct. (MIS) $\Delta^{XN} = \Sigma^{XN} > 1$ (≈1.2 for Boltzmann $p_T$ distrib. with $<T>$=150 MeV/c)

E-by-e $T$ fluct.: for fixed $\sigma_T$ $\Delta^{XN} \uparrow \Sigma^{XN} \uparrow$ when $<N_s> \uparrow$

   for fixed $<N_s>$ $\Delta^{XN} \uparrow \Sigma^{XN} \uparrow$ when $\sigma_T \uparrow$

2. Quantum effects → arXiv:1308.0752; and 3)

Ideal Bose and Fermi gases within GCE:

$\Delta^{XN}$, Bose $< \Delta^{XN}$, Boltz $= 1 < \Delta^{XN}$, Fermi

$\Sigma^{XN}$, Fermi $< \Sigma^{XN}$, Boltz $= 1 < \Sigma^{XN}$, Bose

Similar analysis done for $\Phi_{p_T}$ (belongs to $\Sigma$-"family"): $\Phi_{p_T}^{Boltz} = 0$, $\Phi_{p_T}^{Bose} > 0$, $\Phi_{p_T}^{Fermi} < 0$

→ PLB 439, 6 (1998); PLB 465, 8 (1999)

4. system size and energy dependence using UrQMD

→ PRC 88, 024907 (2013)

One of conclusions (supported by UrQMD tests):
$\Delta$ and $\Sigma$ measure deviations from MIS in different ways ⇒ in the analysis of experimental data a simultaneous measurement of both quantities would be highly desirable
Methods of analyzing $p_T$ and $N$ fluctuations in NA61 $p+p$

- **Acceptance** should be **defined** and described → note, it is different (wider!) than that one for chemical fluctuations (where acceptance had to be limited to regions where inclusive dE/dx fit was possible)

- **Correction for contamination of non-target interactions** → based on special events with removed target

- **Correction for detector inefficiencies and losses of inelastic events** (trigger bias) → performed by use of processed through Geant (+fully reconstructed) samples of EPOS events

- correction for non-target interactions for $\Phi_{p_T}$, $\Delta X_N$, $\Sigma X_N$ (not shown here), $N$ and $\omega$ is negligible

- correction for detector inefficiencies and trigger bias changes results significantly
Transverse momentum fluctuations in inelastic p+p collisions (NA61)

Results within the full NA61 acceptance

$\Phi_{pT}$ and $\Sigma^{XN}$ - the same “family” of strongly intensive measures (the same moments used)

$\Sigma^{XN}$ shows fluctuations above IPM predictions and $\Delta^{XN}$ below IPM

Possible explanations of $\Sigma^{XN}>1$, $\Delta^{XN}<1$ and $\Phi_{pT}>0$

1. BE statistics
   $\rightarrow$ arXiv:1308.0752; PRC 88, 024907 (2013); PLB 439, 6 (1998); PLB 465, 8 (1999)

2. Average $p_T$ per event
   $M(p_T)$ versus N correlation
   $\rightarrow$ arXiv:1309.7878
Comparison of $p_T$ fluctuations for NA49 A+A and NA61 p+p collisions in the same (NA49) acceptance

- **Forward-rapidity**
  \[ 1.1 < y_\pi < 2.6; \]
  \[ y_p < y_{\text{beam}} - 0.5 \]

- **Common** (for all energies)
  limited azimuthal angle

- Similar behaviour for Pb+Pb and p+p; difference only for negatively charged particles

Due to smaller acceptance magnitudes of p+p points are smaller then those on prev. page

For NA61 only stat. errors shown
● **Forward-rapidity**

$1.1 < y_\pi < 2.6$

● **Wide azimuthal angle** – nearly as available at 158 GeV/c

Increase of $\Phi_{pT}$ ~ two times larger for all charged than for negatively charged particles (as predicted for CP)

Predictions for $\Sigma^{XN}$ and $\Delta^{XN}$ at CP not available yet

Details of CP predictions (curves) for $\Phi_{pT}$

→ NP A830, 547C-550C (2009)

For NA61 only stat. errors shown
Multiplicity fluctuations of non-identified particles in inelastic p+p collisions (NA61)

- Increase of $\omega$ with energy reflecting increase of $\omega_{\text{Nch}}$ measured in full phase-space (see PR 351, 161 (2001))
- $\omega_{+/−} < \omega_{\text{Nch}}$ possibly due to charge conservation

... and comparison with NA49 Pb+Pb within the same (NA49) acceptance

- Forward-rapidity $1.0 < y_{π} < y_{\text{beam}}$
  (but similar tendency for $\omega$ observed also in wider $y$: $0.0 < y_{π} < y_{\text{beam}}$)
- Energy dependent azimuthal angle acceptance → as available in NA49 detector

Difference between Pb+Pb and p+p → violation of the Wounded Nucleon Model
Multiplicity fluctuations in NA49 were measured also in a wider rapidity range: $0 < y < y_{\text{beam}}$ (energy dependent azimuthal angle acceptance) but the tendency is similar to that one shown at $1 < y < y_{\text{beam}}$ (previous page).

The difference between NA49 p+p and NA61 p+p is due to the absence of trigger bias correction in NA49.
Comparison with models, it is what about “Unreasonable effectiveness (or not) of statistical approaches to high-energy collisions”

Example for the most intuitive variable - $\omega$

Comparison of NA61 p+p with NA49 1% most central Pb+Pb at the top SPS energy

Negatively charged particles are almost not influenced by resonance decays

Negatively charged

$0 < y < y_{\text{beam}}$

1% PbPb (NA49)

158A GeV

$<N_W>$
Predictions of WNM (Wounded Nucleon Model)

\[ \omega(A+A) = \omega^{N_w=\text{const}}(A+A) + \frac{\langle N \rangle_{A+A}}{\langle N_w \rangle} \cdot \omega_w \]

For \( N_w = \text{const.} \)
\[ \omega_w = 0 \]
\[ \omega(A+A) = \omega(N+N) \]

For fluctuating \( N_w \)
\[ \omega_w > 0 \]
\[ \omega(A+A) > \omega(N+N) \]

\[ \omega(A+A) < \omega(N+N) \]

Falsification of Wounded Nucleon Model via results on fluctuations

WNM already falsified by spectra and yields, but here:

Why predictions of WNM are so important? String models are essentially based on WNM

WNM + isospin effect → under investigations; \( V(N^-) \) and \( \langle N^- \rangle \) results from p+p and n+p will be used to predict \( \omega \) for Pb+Pb (limited NA49 acceptance should be also taken into account)
Predictions of IB-GCE (Grand Canonical Ensemble, Ideal Boltzmann)

\[ \omega(A+A) = \omega_{V=\text{const}}(A+A) + \frac{\langle N \rangle_{A+A}}{V} \cdot \omega_v \]

\[ \omega_v \] - volume fluctuations

For neg. charged \( \omega_{V=\text{const}}(A+A) \approx 1 \)

For \( V = \text{const.} \)

\[ \omega_v = 0 \]

\[ \omega(A+A) \approx 1 \]

For fluctuating \( V \)

\[ \omega_v > 0 \]

\[ \omega(A+A) > 1 \]

\[ \omega(A+A) < 1 \] forbidden in IB-GCE!

IB-GCE is falsified by Pb+Pb point; see also NA49 older results (low → top SPS energies) compared to models – Fig. 4 in PRC 76, 024902 (2007)

p+p result alone can be interpreted as an evidence of volume fluctuations in p+p!
GCE and CE (and MCE) are close to each other in the limit of large volumes ...
(called: thermodynamical equivalence of all statistical ensembles)

\[ z \sim \text{single particle partition function (} \sim V) \]

For large systems (\( z \gg 1 \))
\[ <N_{+/-}>_{\text{IB-CE}} \approx <N_{+/-}>_{\text{IB-GCE}} = z \]

For small systems (\( z \ll 1 \))
\[ <N_{+/-}>_{\text{IB-CE}} \approx z^2 \ll <N_{+/-}>_{\text{IB-GCE}} = z \]

... but this is true for average multiplicities, **not** for fluctuations !!

**Average multiplicities** – difference between IB-GCE and IB-CE only for small systems

**Scaled variance of multiplicity distribution** – difference between IB-GCE and IB-CE remains even for large systems

Example for IB-CE with charge conservation (Q=0); results for single charge only

FIG. 1. The ratio of \( \langle N_{\pm}\rangle_{\text{c.e.}} \) to \( \langle N_{\pm}\rangle_{\text{g.c.e.}} \) (5)

FIG. 2. The scaled variances of \( N_{\pm} \) calculated within the g.c.e., \( \omega^\pm_{\text{g.c.e.}} = 1 \) (14), and c.e., \( \omega^\pm_{\text{c.e.}} \) (15).
Predictions of IB-CE (Canonical Ensemble, Ideal Boltzmann)

For $V = \text{const.}$

$$\omega(A+A) \leq 1 \quad \text{(see also Fig. below)}$$

For $V \to 0$

$$\omega(A+A) \uparrow 1 \quad \text{(single charge)}$$

For more detailed calculations within GCE, CE and MCE, including quantum effects (FD, BE), resonance decays and the influence of limited acceptance → see PRC 76, 024902 (2007)
Summary

Why “event-by-event analysis of fluctuations is the future of ion physics”

New tools and methods:

- **Identity method** to correct chemical fluctuations and multiplicity fluctuations of identified particles on misidentification effect; can be applied to many different fluctuation measures
- Method of **correcting fluctuation measures** (now applied to $p_T$ and N fluctuations of non-identified particles) on non-target interactions (important for p+p), detector inefficiencies, and trigger bias

New measures:

- **Strongly intensive** measure $\Phi$, instead of old $\sigma_{\text{dyn}}$, used to measure chemical fluctuations in NA61 p+p and NA49 Pb+Pb collisions. In future the analysis of “chemical” $\Delta$ and $\Sigma$ planned
- Strongly intensive and **properly normalized new measures $\Delta$ and $\Sigma$** used in NA49 and NA61 to calculate $p_T$ fluctuations
Observations and interpretation of new results:

- **Conservation laws** play important role in chemical and multiplicity fluctuations in p+p. Fluctuations of bar(p)+p with respect to K show different behavior in p+p (NA61) and Pb+Pb (NA49) but in both systems they change sign at middle SPS energies. Models (EPOS, HSD, UrQMD) nicely reproduce tendency of NA61 p+p data.

- **ω value for Pb+Pb** falsify IB-GCE model (known for years), but new (corrected) NA61 results on p+p, compared with NA49 Pb+Pb, falsify also WNM model.

Reminder of theoretical findings:
GCE formulation of Stat. Model → problems for small systems (multiplicities)
GCE formulation of Stat. Model → problems even for large systems (fluctuations)

- The values of $\Sigma^{XN}$ (the same “family” as $\Phi_{pT}$) are higher than 1 (1 = Independent Particle Model), and values of $\Delta^{XN}$ are smaller than 1; similar behaviour for NA61 p+p and NA49 Pb+Pb (in the same NA49 acceptance)

- Magnitudes of $p_T$ fluctuations in p+p at 20-158 GeV/c are significant in the acceptance of NA61 and much smaller when additional cuts, as used in the energy scan of Pb+Pb in NA49 (forward-rapidity), are applied. But **NA61 acceptance for fluctuation analysis can be enlarged towards mid-rapidity** due to installation of He beam pipes (they reduce the number of $\delta$-electrons in VTPCs). Moreover, in NA61 Pb+Pb collisions are also planned!
Comparison of $\Phi_{p_T}$ between p+p and Pb+Pb collisions (NA49: PR C79, 044904 (2009)) in the NA49 phase-space cuts. No significant difference is observed.

NA61: p+p in NA49 acceptance
NA49: 7.2% Pb+Pb

No indications of CP in NA49 Pb+Pb and in NA61 p+p

We are waiting for the results from Be+Be, Ar+Ca, and Xe+La.
Backup
Fixed target exp. in the north area of the CERN SPS
Based on the upgraded NA49 detector; started in 2007
NA61 ion program – continuation of NA49 (search for CP, study of the properties of the onset of deconfinement, study high $p_T$ particles)

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Four large volume **Time Projection Chambers (TPCs)**: VTPC-1, VTPC-2 (inside superconducting magnets), MTPC-L, MTPC-R; measurement of $dE/dx$ and $p$. **Time of Flight (ToF)** detector walls.

- **Projectile Spectator Detector (PSD)** for centrality measurement (energy of projectile spectators) and determination of reaction plane; *resolution of 1 nucleon (!) in the studied energy range* (important for fluctuation analysis).

- **Helium beam pipes** inside VTPC-1 and VTPC-2 (to reduce $\delta$-electrons).

- **Z-detector** (measures ion charge for on-line selection of secondary ions, **A-detector** (measures mass composition of secondary ion beam).

- Low Momentum Particle Detector (**LMPD**) for centrality determination in $p+A$; measures target nucleus spectators.

Large acceptance: $\approx 50\%$
High momentum resolution: 
$\sigma(p)/p^2 \approx 10^{-4} (\text{GeV/c})^{-1}$ (at full $B=9 \text{T} \cdot \text{m}$)
ToF walls resolution: 
ToF-L/R: $\sigma(t) \approx 60 \text{ ps}$; ToF-F: $\sigma(t) \approx 120 \text{ ps}$
Good particle identification: 
$\sigma(dE/dx)/<dE/dx> \approx 0.04$; $\sigma(m_{\text{inv}}) \approx 5 \text{ MeV}$
High detector efficiency: $> 95\%$
Event rate: 70 events/sec
Chemical (particle type) fluctuations

- $\sigma_{\text{dyn}}$ measure of dynamical particle ratio fluctuations ($K/\pi$, $p/\pi$, $K/p$)

- E-by-e fit of particle multiplicities required in NA49
- Mixed events used as reference
- $\sigma^2_{\text{dyn}} \sim 1/N_w$ (PRC 81, 034910 (2010), PRC 84, 014904 (2011))

Relative width (of $K/\pi$, $p/\pi$, $K/p$)

$$\sigma = \frac{\text{RMS}}{\text{Mean}} \cdot 100\%$$

$$\sigma_{\text{dyn}} = \text{sign} \left( \sigma^2_{\text{data}} - \sigma^2_{\text{mixed}} \right) \sqrt{\left| \sigma^2_{\text{data}} - \sigma^2_{\text{mixed}} \right|} \quad \sigma_{\text{dyn}}^2 \approx |\nu_{\text{dyn}}|$$

- $K/p$: dynamical fluctuations change sign close to the onset of deconfinement energy
- $K^+/p$ – contrib. from res. production suppressed
Scaling of particle ratio fluctuations

$\sigma_{\text{dyn}}$ can be separated [PRC 81, 034910 (2010)] into
- correlation strength term
- term purely dependent on multiplicities

In case of unchanged correlations (invariant correlation strength) the general expectation is:

$$\sigma_{\text{dyn}} \propto \sqrt{\frac{1}{\langle A \rangle} + \frac{1}{\langle B \rangle}}$$

$A, B = N_K, N_\pi, N_p, ...$

- Scaling works very well for $K/\pi$ and $p/\pi$ fluctuations
- The change of sign in $K/p$ fluctuations excludes any simple scaling based on average multiplicities. The above scaling assumed invariant correlation strength => underlying correlation between kaons and protons is changing with energy


Please note: the difference between STAR and NA49 for $K/\pi$ and $K/p$ (not shown here) already understood as due to acceptance → NA49, arXiv:1310.3428
Centrality dependence of event-by-event particle ratio fluctuations

$\sqrt{s_{_{NN}}} = 17.3$ GeV

Fixed physics (energy), varying volume (system size)
Absolute values rise towards peripheral collisions as in STAR (shown for K/π fluctuations at $\sqrt{s_{_{NN}}} = 62$ and 200 GeV, PRL 103, 092301 (2009)) and UrQMD

The same multiplicity scaling seems to hold: (compatible with hypothesis that at constant energy underlying correlations are not significantly changed by variation of the system size)

$\sigma_{_{dyn}} \propto \sqrt{\frac{1}{\langle A \rangle} + \frac{1}{\langle B \rangle}}$

NA49: to be published; Kresan, QM2011 poster
Energy and centrality dependence of particle ratio fluctuations on one scale

The same dependence on multiplicities is observed for $K/\pi$ and $p/\pi$ fluctuations

$$\sigma_{\text{dyn}} \propto \sqrt{\frac{1}{\langle A \rangle} + \frac{1}{\langle B \rangle}}$$

No common scaling of energy and centrality dependence for $K/p$ fluctuations
Problems with $\sigma_{\text{dyn}}$? Let's test both $\sigma_{\text{dyn}}$ (or $\nu_{\text{dyn}}$) and $\Phi/\psi$ on fast generators

$$
\nu_{\text{dyn}}(\text{Particle}_1, \text{Particle}_2) = \frac{\langle N_1(N_1-1) \rangle}{\langle N_1 \rangle^2} + \frac{\langle N_2(N_2-1) \rangle}{\langle N_2 \rangle^2} - 2 \frac{\langle N_1 N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle}
$$

$$
\nu_{\text{dyn}} \approx \text{sgn}(\sigma_{\text{dyn}}) \sigma^2_{\text{dyn}}
$$

$z = x - \bar{x}$; $\bar{x}$ - inclusive average
event variable $Z_x = \sum_{i=1}^N (x_i - \bar{x})$

$\Phi_x = \sqrt{\frac{\langle Z_x^2 \rangle}{\langle N \rangle}} - \sqrt{Z_x^2}$

$\Psi_x = \frac{\langle Z_x^2 \rangle}{\langle N \rangle} - Z_x^2$

Old known quantities now used for chemical fluctuations:
$\Phi$ and $\psi$ - strongly intensive measures of fluctuations (do not depend on volume and volume fluctuations)

$\Phi_{\text{chemical}} (p_T, \phi \rightarrow x)$

Here for system composed by kaons and pions we use

$x = 1$ for kaons

$x = 0$ for pions

Simulation of independent particle production

Grebieszkow, unpublished

![Graphs showing distributions of $\Phi/\Psi$ and $\nu_{\text{dyn}}$](image.png)
\(v_{\text{dyn}}\) and thus \(\sigma_{\text{dyn}}\) are not intensive measures.

Ratio fluctuations scale roughly as the inverse of the accepted multiplicity \(\sigma_{\text{dyn}}^2 \sim 1/\langle N_{\text{accepted}} \rangle\).

\(\Rightarrow\) Rise toward low \(\sqrt{s}\) in \(K/\pi\) fluct. due to low multiplicity rather than due to deconfinement (as originally believed).

Moreover: all existing chemical fluctuation measures are sensitive to non-perfect particle identification :(

Solution: identity method (→ see Gaźdicki, Grebieszkow, Maćkowiak, Mrówczyński, PR C83, 054907 (2011)). Advantages: e-by-e fits of particle ratios not required (only global \(dE/dx\) fits), mixed events as reference not required, effect of limited \(dE/dx\) resolution can be corrected in a model independent way.

\(x_i\) (assumed ID) replaced by identity \(w_i(dE/dx) = \rho_i(dE/dx) / \rho(dE/dx)\) measuring the probability that the particle is pion or kaon or proton or electron, etc.

Original idea developed and improved in: PR C84, 024902 (2011), PR C86, 044906 (2012) and currently applied to NA49 and NA61 data (M. Maćkowiak-Pawłowska, A. Rustamov).
Identity method

In experiment chemical fluctuation of identified particles multiplicities may be distorted by the incomplete particle identification.

The identity method allows to obtain second and third moments (pure and mixed) of identified particle multiplicity distribution corrected for misidentification effect.

Using \( dE/dx \) fit a particle identity is calculated as:

\[
W_i = \frac{\rho_i(dE/dx)}{\rho(dE/dx)} ,
\]

where \( \rho_i \) - function fitted to \( i^{th} \) particle type and \( \rho \) - function fitted to total \( dE/dx \) distribution in a given phase-space bin (i: \( \pi, p, K \))
Event quantity $W_i$ defined as:

$$W_i = \sum w_i,$$

where summation runs over all particles in an event.

Once, detector response ($\rho_i$) and $W$ distributions are known the identity method is used to obtain moments of identified particle multiplicity distributions.

$$\rho_i, W_i, \ldots \longrightarrow <N_i^2>, <N_{i,j}>$$

For perfect particle identification $W_i$ distribution equals the multiplicity distribution.

For particles with larger PID contamination (like K) $W_i$ distribution gets smoother.

Example for $p+p$ at $\sqrt{s_{NN}} = 17.3$ GeV

See PR C84, 024902 (2011), PR C86, 044906 (2012) for the details of the matrix used in calculations.
Multiplicity fluctuations in p+p increase linearly with $\langle N_{ch} \rangle$ in full phase-space (reflection of KNO scaling) - Phys. Rept. 351, 161 (2001)

$$\omega^{acc} = \left( \omega^4 - 1 \right) p + 1$$
$$p = \frac{\langle N^{acc} \rangle}{\langle N \rangle}$$

← valid if no correlations in momentum space
Common NA61/SHINE and NA49 acceptance for chemical fluctuations

In order to compare p+p (NA61) and Pb+Pb (NA49) results the common acceptance for the chemical fluctuation analysis was defined. Low particle multiplicity in p+p interactions limits the acceptance to the region in which track statistics is sufficient for the dE/dx fits.

\[ \sqrt{s_{NN}} = 7.6 \text{ GeV} \]

\[ \sqrt{s_{NN}} = 17.3 \text{ GeV} \]

Colored region marks common acceptance used for comparison of p+p and Pb+Pb results (scattered points indicate acceptance used for Pb+Pb analysis only).

For details see https://edms.cern.ch/document/1237791/1.
Scaled variance $\omega$ of multiplicity distribution

- Intensive measure
- For Poisson multiplicity distribution $\omega = 1$
- In Wounded Nucleon Model (superposition):
  \[ \omega(A+A) = \omega(N+N) + \langle n \rangle \omega_W \]
  \( \langle n \rangle \) - mean multiplicity of hadrons from a single source
  \( = \langle N \rangle_{A+A} / \langle N_w \rangle \)
  $\omega_W$ - fluctuations in $N_w$
  $\omega$ is strongly dependent on $N_w$ fluctuations

\[ \omega = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle} \]

Therefore for A+A we show only 1% most central data

$\Phi_x$ measure (ZPC 54, 127 (1992)) of fluctuations ($x=p_T, \phi, Q$)

- In superposition models $\Phi_x(A+A) = \Phi_x(N+N)$
- For independent particle emission $\Phi_x = 0$
- In superposition models $\Phi_x$ is independent of $N_w$ and $N_w$ fluctuations (strongly intensive)
System size dependence  
(p+p, C+C, Si+Si, and Pb+Pb)  
of average $p_T$ and multiplicity 
fluctuations at 158A GeV

Energy dependence of average 
$p_T$ and multiplicity fluctuations 
for central Pb+Pb

For energy dependence of $\Phi_{p_T}$ important cut on $y^*$ to get rid of artificial effect of event-by-event centrality fluctuations while studying only forward-rapidity $\rightarrow$ for details see separate paper KG, PRC 76, 064908 (2007)
Average $p_T$ and $N$ fluctuations: dependence on phase diagram coordinates

Maximum of $\Phi_{p_T}$ and $\omega$ observed for C+C and Si+Si

Data are consistent with the CP$_2$ predictions


Up to now strategy in fluctuation analysis → acceptance described, but results NOT corrected for detector effects (two-track resolution) and trigger bias (only $\Phi_{p_T}$ was corrected for TTR)
Notation: $\Delta/\Sigma[P_{T_i}, N] \equiv \Delta/\Sigma^{XN}$

Model of Independent Sources (MIS) reduced to Independent Particle Model (IPM)
Each event composed by a given number of identical single sources.
For each source the number of particles generated from the Poisson distribution with a mean value of 5.
Particle $p_T$ generated from exp. $m_T$ spectrum with inverse slope $T=150$ MeV.
Number of sources composing an event was either constant (circles) or selected from Poisson (triangles) or from Negative Binomial distribution (squares). For Negative Binomial distribution its dispersion $\sqrt{\text{Var}(N_S)}$ was large and taken to be equal $<N_S>/2$.

Confirmation that these measures are intensive (circles) and strongly intensive (triangles, squares). For these simulations $\Phi_{pT} = 0.$
For each source the number of particles from Poisson with a mean value of 5.

Particle $p_T$ generated from exp. $m_T$ spectrum with average inv. slope $<T>=150$ MeV. $T$ generated separately for each single source (source-by-source $T$ fluctuations $\rightarrow$ MIS) from Gaussian shape with dispersion $\sigma_T=25$ MeV. Number of sources composing an event generated from the Poisson distribution.

Lines $\rightarrow$ analytical calculations for $m_T$ exponential shape (see the paper);
solid line for pion mass and dashed line for massless particles

Positive signal $\Phi_{p_T} > 0$ ($\approx 24$ MeV/c, not shown), $\Delta^{XN}$ and $\Sigma^{XN} > 1$; the measures are strongly intensive.
source-by-source $T$ fluctuations replaced by event-by-event (global) $T$ fluctuations. For each event $T$ generated from Gaussian shape with dispersion $\sigma_T = 25$ MeV.

Lines → analytical calculations for $m_T$ exponential shape (see the paper);
solid line for pion mass and dashed line for massless particles

Strong dependence of $\Delta^{XN}$ and $\Sigma^{XN}$ on the number of sources for event-by-event $T$ fluctuations (the same observation for $\Phi_{pT}$ – not shown)
The same as previous page.

**Event-by-event T fluctuations.**

T varied from event to event following Gaussian distribution with dispersion $\sigma_T$. In order to avoid negative T values only events within $T=150 \pm 3\sigma_T$ MeV were accepted.

The number of sources composing an event was generated from the Poisson distribution with a mean value of 100.

Lines → analytical calculations for $m_T$ exponential shape (see the paper); solid line for pion mass and dashed line for massless particles

The values of all fluctuation measures (also for $\Phi_{p_T}$ which is not shown) increase when event-by-event "temperature" fluctuations are stronger (higher $\sigma_T$)
Previous slides → the same behaviour and magnitudes of $\Delta^{XN}$ and $\Sigma^{XN}$
The example that those two measures can be different → see calculations within UrQMD 3.3 model

\[ M(p_T) \] – average transverse momentum per event

Known from years **correlation between** \( M(p_T) \) **and** \( N \) **in elementary interactions.**

Here such a correlation taken from \( p+p \) at 158 GeV/c (forward-rapidity): NA49, PRC 70, 034902 (2004). **\(<M(p_T)>, \text{versus } N \text{ values}\)** from NA49 (red triangles in right panel) used as 2T values in fast generator where \( \frac{dN}{dm_T} = C \cdot m_T \exp\left(-\frac{m_T}{T}\right) \)

\[
\Delta^{XN} = 0.8158 \pm 0.0051 \\
\Sigma^{XN} = 1.0075 \pm 0.0018 \\
\Phi_{p_T} = 0.82 \pm 0.19 \text{ MeV/c}
\]
Fig. 3: The (a) $\Delta[P_T,N]$ and (b) $\Sigma[P_T,N]$ for the pion gas as the functions of $T$. The solid lines correspond to $\mu_\pi = 0$ and dotted lines to $\mu_\pi = 100$ MeV. The horizontal dashed lines show the Boltzmann approximation (12) equal to 1.

Fig. 1. $\Phi_2$ measure of $p_{\perp}$-fluctuations in the hadron gas as a function of temperature for four values of the chemical potential. The resonances are either neglected (dashed lines) or taken into account (solid lines). The most upper dashed and solid lines correspond to $\mu = 70$ MeV, the lower ones to $\mu = 0$, etc.
NA49 azimuthal acceptance is limited. Detector is left-right symmetric. Acceptance for positive and negative particles is the same, provided the azimuthal angle for one charge is reflected.

To allow quantitative comparison of $\Phi_\phi$ (azimuthal fluct.) for pos. and neg. charged particles we rotated particles of one charge.

FIG. 2: NA49 ($\phi, p_T$) acceptance of positively and negatively charged particles for $2.0 < y^*_\pi < 2.2$ at 80A GeV central $Pb + Pb$ collisions.

System size dependence
$p+p$, $C+C(15.3\%)$, $Si+Si(12.2\%)$, $Pb+Pb(5\%$ or MB) at 158A GeV

FIG. 3: Examples of NA49 ($\phi, p_T$) acceptance of charged particles with the azimuthal angle of negatively charged particles reflected (see the text for details). The solid lines represent the analytical parametrization of acceptance used for further analysis.

Energy scan
7.2% $Pb+Pb$

2.0 < $y_\pi$ < 2.2

See PRC 70, 034902 (2004) and PRC 79, 044904 (2009) for detailed parametrization of acceptance regions (particles inside black lines) in each rapidity bin.
Note: for $\Phi_{pT}$ analysis (and now also for $\Delta^{XN}$ and $\Sigma^{XN}$) we use:

Forward-rapidity only $(1.1 < y_\pi < 2.6)$, $0.005 < p_T < 1.5$ GeV/c
Limited azimuthal acceptance (much more limited in case of energy scan)
Energy dependence of $\Delta^{XN}$ and $\Sigma^{XN}$ measures of $p_T$ fluctuations

Lines – UrQMD 1.3
- NA49 shows tendency similar to UrQMD predictions
NA49 published
($\Phi_{p_T}$)

FIG. 12. (Color online) $\Phi_{p_T}$ as a function of energy for the 7.2% most central Pb + Pb interactions. Data points are corrected for limited two-track resolution. Errors are statistical only. Systematic errors are given in Table IV.

FIG. 16. (Color online) Comparison of $\Phi_{p_T}$ as a function of energy from data (data points, corrected for limited two-track resolution) with UrQMD model calculations (black lines) with acceptance restrictions as for the data. The panels represent results for all charged (left), negatively charged (center), and positively charged particles (right).
System size dependence of $\Delta^{XN}$ and $\Sigma^{XN}$ measures of $p_T$ fluctuations

- UrQMD 3.3
- NA49 data: maximum for peripheral Pb+Pb
- UrQMD (6 cent. of Pb+Pb): no significant system size dependence, only a small maximum for $\Delta^{XN}$ in Pb+Pb(6)
FIG. 8. $\Phi_{p_T}$ versus mean number of wounded nucleons $\langle N_w \rangle$. Data points were corrected for limited two-track resolution. Errors are statistical only. Systematic error is smaller than 1.6 MeV/c.

FIG. 10. $\Phi_{p_T}$ versus mean number of wounded nucleons calculated using the HIJING model with geometrical acceptance cuts included (black lines) and without geometrical acceptance restrictions (gray lines). Results are compared to data (points) corrected for limited two-track resolution (the markers are the same as in Fig. 8). The panels represent: all charged, negatively charged, and positively charged particles. Data points contain both short and long range correlations. The effects of short range correlations are not incorporated in the HIJING model.
Influence of corrections on NA61 p+p results

- correction for non-target interactions is negligible
- correction for detector inefficiencies and trigger bias changes results significantly
Comparison of NA61 p+p with NA49 A+A

In NA49:

- **$p_T$ fluctuations** (energy dependence for 7.2% central Pb+Pb, and system size dependence for p+p, C+C, Si+Si, and Pb+Pb at $\sqrt{s_{NN}} = 17.3$ GeV) **were measured in forward-rapidity only** $1.1 < y_\pi < 2.6$ (azimuthal angle was “narrow”- common for all energies or “wide” - for system size dependence at $\sqrt{s_{NN}} = 17.3$ GeV)

- Complete **system size dependence of multiplicity fluctuations** (p+p, C+C, Si+Si, and Pb+Pb at $\sqrt{s_{NN}} = 17.3$ GeV) **was shown for forward-rapidity only** $1.1 (1.0) < y_\pi < 2.6$ ($y_{beam}$) (“wide” azimuthal angle; almost complete at low $p_T$)

- **Energy dependence of multiplicity fluctuations** (7.2% central Pb+Pb) **was measured for** $0 < y_\pi < 1, \ 1 < y_\pi < y_{beam}$, and $0 < y_\pi < y_{beam}$ (azimuthal angle was strongly dependent on energy: “narrow” for low SPS energies, “wide” for top SPS)
Energy dependence of $p_T$ fluctuations:
NA61 p+p within NA49 Pb+Pb selection cuts

- In NA49 because of high density of tracks, analysis of $p_T$ fluctuations was limited to forward-rapidity region ($1.1 < y_\pi < 2.6$)
- common azimuthal acceptance for all energies

By applying NA49 cuts $\Phi_{p_T}$ in p+p decreases (mainly because of narrower rapidity range). NA61 plans to extend the physics program to repeat and complement NA49 Pb+Pb measurements. **The new He beam pipes reduce the number of $\delta$-electrons in VTPCs by a factor of 10 and allow to extend the acceptance towards mid-rapidity.**
Forward-rapidity
$1.1 < y_\pi < 2.6$

Wide azimuthal angle – nearly as available at 158 GeV/c

Predictions within **CMC model** [NP A693, 799 (2001); NP A761, 149 (2005)] (mixture of their sigmas and indep. produced pions) for Si+Si at 158A GeV in NA49 acc: $\Delta^{XN} \sim 0.76$ and $\Sigma^{XN} \sim 1.35$ → but should not be directly comp. with Stephanov *et al.* (different correlation length).
Forward-rapidity
$1.1 < y < 2.6$

Wide azimuthal angle – *nearly* as available at 158 GeV/c

Forward-rapidity
$1.0 < y < y_{\text{beam}}$

Wide az. angle – as available at 158 GeV/c

For p+p points → acceptance as for $p_T$ fluct. analysis (above)
Wide az. angle – as available at 158 GeV/c

C+C, Si+Si, Pb+Pb
- Forward-rapidity
  \(1.0 < y_\pi < y_{\text{beam}}\)

p+p
- Forward-rapidity
  \(1.1 < y_\pi < 2.6\)

Wide az. angle – nearly as available at 158 GeV/c
FIG. 1. (Color online) The scaled variances for negatively charged particles, $\omega^-$, both primordial and final, along the chemical freeze-out line for central Pb + Pb (Au + Au) collisions. Different lines present the GCE, CE, and MCE results. Symbols at the lines for final particles correspond to the specific collision energies pointed out in Table I. The arrows show the effect of resonance decays.

FIG. 4. (Color online) The scaled variances for negative (top) and positive (bottom) hadrons along the chemical freeze-out line for central Pb + Pb collisions at the SPS energies. The points show the preliminary data of NA49 [14]. Total (statistical + systematic) errors are indicated. The statistical model parameters $T$, $\mu_B$, and $\gamma_S$ at different SPS collision energies are presented in Table I. Lines show the GCE, CE, and MCE results calculated with the NA49 experimental acceptance according to Eq. (22).

older NA49 data;
$1 < y_\pi < y_{beam}$ + az. angle restrictions